

Coordinated Expansion Planning for Multi-infeed LCC-HVDC Systems Considering Generalized Short-circuit Ratio Constraints

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Abstract—Line-commutated converter based high-voltage direct current (LCC-HVDC) systems may diminish the power grid strength, potentially precipitating static voltage stability issues. To maintain the required level of power grid strength while minimizing the total system cost, this paper proposes a coordinated expansion planning framework for multi-infeed LCC-HVDC (MI-HVDC) systems that explicitly incorporates generalized short-circuit ratio (gSCR) constraints. Specifically, the gSCR requirement is innovatively formulated as a semidefinite constraint, and the proposed framework is reformulated as a mixed-integer semidefinite programming (MISDP) model to guarantee the global optimality. Then, to address the computational intractability of the MISDP model, a generalized Benders decomposition (GBD)-based approach is employed for efficient solution. Time-domain simulations conducted in PSCAD/EMTDC demonstrate the effectiveness of the proposed framework in enhancing the voltage stability of MI-HVDC systems.

Index Terms—Line-commutated converter (LCC), high-voltage direct current (HVDC), multi-infeed LCC-HVDC (MI-HVDC), generalized short-circuit ratio (gSCR), mixed-integer semidefinite programming (MISDP), voltage stability, expansion planning.

I. INTRODUCTION

OVER the past several decades, the high-voltage direct current (HVDC) transmission technology has been increasingly deployed, leveraging its superior capabilities in long-distance, large-capacity, and cross-regional power delivery [1]. In particular, the line-commutated converter based high-voltage direct current (LCC-HVDC) gains prominence

due to its technical maturity and cost-effectiveness. However, the integration of multiple LCC-HVDC links into electrically proximate AC buses gives rise to multi-infeed LCC-HVDC (MI-HVDC) systems [2]. Such configurations often precipitate the static voltage stability issues at both the sending- and receiving-end grids [3]–[6]. Consequently, developing a reliable yet cost-effective expansion planning framework for MI-HVDC systems has become a critical research frontier [7].

The conventional expansion planning methods for transmission system primarily focus on optimizing the economic efficiency while mitigating multiple uncertainties. For instance, the $N-1$ reliability criteria is embedded into the expansion planning objectives for HVDC systems to achieve a balance between the economic costs and operational reliability in [8]. Considering the long- and short-term uncertainties and credible contingencies, a multi-stage robust transmission expansion planning framework for hybrid AC/DC systems is presented in [9]. Considering the generation variability of intermittent renewable energy sources, an adaptive robust method is proposed in [10] to derive the optimal transmission expansion planning scheme with robustness against long- and short-term uncertainties. Integrating the fluctuating and uncertain characteristics of renewable energy, a coordinated planning method between power to hydrogen supply chains and HVDC links is studied in [11].

However, while the aforementioned studies have made significant contributions to HVDC systems, a critical aspect remains insufficiently addressed: the inherent voltage stability challenges introduced by multiple LCC-HVDC injections. Unlike high-voltage AC (HVAC) and voltage source converter based high-voltage direct current (VSC-HVDC) technologies, LCC-HVDC converters depend on the AC voltage for commutation, which renders converters highly vulnerable to commutation failures and necessitates substantial reactive power support [5]. These operational characteristics intrinsically compromise the static voltage stability of both sending- and receiving-end AC systems. Consequently, it is imperative to incorporate stability constraints into the expansion planning framework for MI-HVDC system to ensure the reliable operation [7].

The short-circuit ratio (SCR) and its derivatives are wide-

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ly recognized as effective metrics for quantifying the grid strength of LCC-HVDC systems. The existing literature has extensively explored to enhance the SCR of LCC-HVDC systems through the device allocation and system reinforcement. For instance, an SCR-constrained coordinated expansion planning model for wind-solar-thermal-storage is established in [12], utilizing a dual-layer optimization algorithm to bolster the voltage support capability. Simulation results demonstrate its efficacy in strengthening the voltage support strength of LCC-HVDC systems. In [13], by embedding the multi-infeed SCR (MI-SCR) constraints [14], a cooperative-game-based wind generation capacity planning method considering LCC-HVDC injections is proposed to balance the economic benefits and the operational security of wind farms. In [15], an allocation method for synchronous condensers considering SCR requirements is proposed to enhance the grid strength of weak power systems with high penetration of intermittent renewable energy sources. The time-domain simulation results demonstrate the stronger voltage stability of the optimized system. In [16], a capacity planning method of grid-following converters is proposed to enhance the generalized SCR (gSCR) [17] of multi-converter systems.

Despite these advancements, several critical challenges remain systematically unaddressed.

1) Power system dispatching strategies, particularly unit commitment (UC), substantially influence the SCR, as the dynamic impedance of synchronous generators is a decisive factor in determining the SCR [18].

2) SCR exhibits significant sensitivity to power system topology, which undergoes dynamic changes during ongoing system expansion planning [19]. Neglecting these interdependencies may result in insufficient grid strength reinforcement and diminished cost-effectiveness in the resulting expansion schemes.

In fact, there are two major obstacles to addressing the aforementioned two challenges: the inherent nonconvexity of the SCR constraints (including but not limited to SCR, MI-SCR, and gSCR) and the nonlinear correlation between the SCR metrics and UC control variables [20]. These intrinsic

characteristics obstruct the explicit mathematical formulation of SCR expressions in making decisions [21]. To tackle this problem, several meta-heuristic algorithms [22], [23] and data-driven methods [20], [21] have been introduced. For instance, the genetic algorithm (GA) is applied to optimize the allocation of synchronous condensers under SCR constraints in [22]. In [23], solutions generated by three different meta-heuristic algorithms are compared, and the one yielding the highest SCR value is selected for the expansion planning scheme of weak grids. In [20], a set of linear functions is trained to approximate the SCR constraints in the linear form, which are subsequently incorporated into a mixed-integer linear programming (MILP) based UC model. In [21], an artificial neural network is trained to formulate the nonlinear correlation between the gSCR value and UC control variables as a set of mixed-integer linear expressions, and a generalized spectral decomposition-based algorithm is then proposed to transform the gSCR constraint to a set of mixed-integer linear constraints. Despite the valuable work done by the above research, there are still limitations.

1) The meta-heuristic algorithms normally do not have unique solutions and are sensitive to preset parameters [23].

2) The data-driven methods are likely to generate physically infeasible results and cannot guarantee out-of-sample performance [24].

Given the aforementioned background, a coordinated expansion planning framework for MI-HVDC systems considering gSCR constraints is proposed in this paper, which has a rigorous global optimality. First, the conventional gSCR constraints based on eigenvalue inequality are reformulated into a semidefinite form to mitigate their inherent nonconvexity. Second, a mixed-integer semidefinite programming (MISDP) model is developed for the proposed framework to integrate these constraints, thereby ensuring the global optimality. Then, owing to the computational complexity and intractability of the MISDP model, the generalized Benders decomposition (GBD)-based approach [25] is employed for higher efficiency. The motivations and the solutions of this paper can be summarized in Table I. The primary contributions of this paper are as follows.

TABLE I
MOTIVATIONS AND SOLUTIONS OF THIS PAPER

Motivation		Solution
Practical context	Deficiency of existing research	
Static voltage stability challenges introduced by LCC-HVDC injections	1) Conventional expansion planning methods overlook the grid strength of MI-HVDC systems [9]-[12] 2) Existing related research does not consider the impact of UC results and system topology on SCR [13]-[17]	A coordinated expansion planning framework for MI-HVDC systems considering gSCR constraints
Nonlinearity of gSCR constraints and difficulty in solving MISDP model	1) Meta-heuristic algorithms cannot ensure an optimal solution and the obtained results can be unstable [22], [23] 2) Data-driven methods may generate physically infeasible results and have poor generalization [20], [21]	1) A semidefinite form of gSCR constraints and MISDP model 2) A GBD-based approach for MISDP model

1) A semidefinite form of gSCR constraints is formulated to replace the expression of conventional gSCR constraints. Distinct from existing approximation methods, this reformulation avoids lossy relaxations, thereby ensuring significantly

improved solution accuracy.

2) An MISDP model is built for the proposed framework, thereby guaranteeing the attainment of global optimality.

3) A GBD-based approach is introduced to decompose

and solve the MISDP model using the industry-standard commercial solvers. In contrast to meta-heuristic algorithms, the GBD-based approach ensures the solution optimality and uniqueness.

The rest of this paper is organized as follows. Section II introduces the semidefinite form of gSCR constraints and MISDP model for the proposed framework. Section III introduces the GBD-based approach. Section IV conducts numerical case studies. Section V concludes this paper.

II. SEMIDEFINITE FORM OF GSCR CONSTRAINTS AND MISDP MODEL FOR PROPOSED FRAMEWORK

A. Semidefinite Form of gSCR Constraints

In this paper, the gSCR is used as a metric of strength assessment for MI-HVDC systems. According to the definitions and experimental results in [26], a system is considered weak when the gSCR is below 3, and classified as very weak when the gSCR is less than 2, with its boundary value errors not exceeding 5%. Consequently, this paper adopts the boundary value of gSCR as 3.

1) Expression of Conventional gSCR Constraints

The conventional gSCR constraints of MI-HVDC systems can be formulated as:

$$gSCR = \min EIG(\text{diag}^{-1}(\mathbf{P}_{\text{HVDC}})\mathbf{B}_{\text{Kr}}) \quad (1)$$

$$gSCR \geq \alpha \quad (2)$$

where $gSCR$ is the gSCR value of the system; α is the boundary value of gSCR; $EIG(\cdot)$ is the eigenvalue extraction function; $\mathbf{P}_{\text{HVDC}} = [P_{\text{HVDC}}^1, P_{\text{HVDC}}^2, \dots, P_{\text{HVDC}}^i, \dots, P_{\text{HVDC}}^D]$, P_{HVDC}^i is the rate power capacity of LCC-HVDC at bus i , and D is the number of LCC-HVDCs; and $\mathbf{B}_{\text{Kr}}$ is the Kron-reduced admittance matrix, which represents the electrical coupling strength between multiple LCC-HVDCs and can be defined by (3).

$$\mathbf{B}_{\text{Kr}} = \mathbf{B}_{\text{HH}} - \mathbf{B}_{\text{HL}}\mathbf{B}_{\text{LL}}^{-1}\mathbf{B}_{\text{LH}} \quad (3)$$

where $\mathbf{B}_{\text{HH}}$, $\mathbf{B}_{\text{HL}}$, $\mathbf{B}_{\text{LL}}$, and $\mathbf{B}_{\text{LH}}$ are the submatrices of the admittance matrix $\mathbf{B}'$, and the subscripts H and L represent the buses with LCC-HVDC terminals and the remaining buses, respectively.

The admittance matrix $\mathbf{B}'$ is an $N \times N$ symmetric positive definite matrix (N is the number of buses), whose element can be formulated as:

$$B'_{ij} = \begin{cases} -u_{ij}^{\text{Brh}} b_{ij} & i \neq j \\ \sum_{g \in G_i} u_{i,t,g}^{\text{SG}} b_{i,g} + \sum_{k \in V_i} u_{i,k}^{\text{Brh}} b_{i,k} & i = j \end{cases} \quad (4)$$

where G_i is the set of synchronous generators at bus i ; V_i is the set of lines connected to bus i ; $u_{i,t,g}^{\text{SG}}$ is a Boolean variable that represents the on/off status of synchronous generator g at bus i at time t ; u_{ij}^{Brh} is the construction decision variable of transmission line (i,j) ; b_{ij} and b_{ik} are the admittances of transmission lines (i,j) and (i,k) , respectively; and $b_{i,g}$ is the admittance of synchronous generator g at bus i .

Note that the matrix $\mathbf{B}_{\text{LL}}$ in (3) is invertible, and the proof is as follows.

$\mathbf{B}_{\text{LL}}$ can be reformulated as the sum of a shunt admittance matrix $\mathbf{D}'$ with all $D'_{ii} \geq 0$ and a branch admittance matrix $\mathbf{L}'$:

$$\mathbf{B}_{\text{LL}} = \mathbf{L}' + \mathbf{D}' \quad (5)$$

Note that $\mathbf{L}'$ is a generalized Laplacian matrix since $L'_{ii} = -\sum_{i \neq j} L'_{ij}$ and $L'_{ii} \geq 0$, and we have:

$$\mathbf{L}' \geq \mathbf{0} \quad (6)$$

where $\geq$ is the symbol for positive semidefinite matrix.

Besides, for a transmission system without isolated buses, there exists:

$$L'_{ii} + D'_{ii} > 0 \quad (7)$$

Combining (5)-(7), we have:

$$\mathbf{B}_{\text{LL}} = \mathbf{L}' + \mathbf{D}' \succ \mathbf{0} \quad (8)$$

where $\succ$ is the symbol for positive definite matrix.

Therefore, $\mathbf{B}_{\text{LL}}$ is an invertible matrix.

Besides, to enhance the clarity of both the proof procedure and the physical interpretations of $\mathbf{B}_{\text{HH}}$, $\mathbf{B}_{\text{HL}}$, $\mathbf{B}_{\text{LL}}$, $\mathbf{B}_{\text{LH}}$, $\mathbf{L}'$, and $\mathbf{D}'$, a simplified example based on the Garver 6-bus system is provided in Supplementary Material A.

However, the conventional gSCR constraints are not only nonlinear but also nonconvex, which is not friendly to traditional commercial solvers. This is mainly due to: ① the eigenvalue extraction function $EIG(\cdot)$, inverse operation $\text{diag}^{-1}(\mathbf{P}_{\text{HVDC}})$, and the product operation $\text{diag}^{-1}(\mathbf{P}_{\text{HVDC}})\mathbf{B}_{\text{Kr}}$ in (1); and ② the inverse operation $\mathbf{B}_{\text{LL}}^{-1}$ in (3).

2) Semidefinite Form

To tackle the problem mentioned above, a semidefinite form of gSCR constraints is proposed in this part.

Combining (1) and (2), the gSCR constraints can be formulated as:

$$\text{diag}^{-1}(\mathbf{P}_{\text{HVDC}})\mathbf{B}_{\text{Kr}} - \alpha \mathbf{I} \geq \mathbf{0} \quad (9)$$

where $\mathbf{I}$ is the identity matrix. Left-multiplying (9) by $\text{diag}(\mathbf{P}_{\text{HVDC}})$ yields:

$$\mathbf{B}_{\text{Kr}} - \alpha \cdot \text{diag}(\mathbf{P}_{\text{HVDC}}) \geq \mathbf{0} \quad (10)$$

Then, introducing (3) into (10) yields:

$$(\mathbf{B}_{\text{HH}} - \alpha \cdot \text{diag}(\mathbf{P}_{\text{HVDC}})) - \mathbf{B}_{\text{HL}}\mathbf{B}_{\text{LL}}^{-1}\mathbf{B}_{\text{LH}} \geq \mathbf{0} \quad (11)$$

According to (8), $\mathbf{B}_{\text{LL}}$ is a strictly positive definite matrix. Then, we can apply the Schur complement lemma [27], namely, for any symmetric matrix $\begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix}$, if $\mathbf{Z} \geq \mathbf{0}$, we have:

$$\mathbf{X} - \mathbf{Y}\mathbf{Z}^{-1}\mathbf{Y}^T \geq \mathbf{0} \Leftrightarrow \begin{bmatrix} \mathbf{X} & \mathbf{Y} \\ \mathbf{Y}^T & \mathbf{Z} \end{bmatrix} \geq \mathbf{0} \quad (12)$$

Combining the above statement, for the gSCR constraints in (11) and the condition (12), we can reformulate the gSCR constraints as:

$$\begin{bmatrix} \mathbf{B}_{\text{HH}} - \alpha \cdot \text{diag}(\mathbf{P}_{\text{HVDC}}) & \mathbf{B}_{\text{HL}} \\ \mathbf{B}_{\text{LH}} & \mathbf{B}_{\text{LL}} \end{bmatrix} \geq \mathbf{0} \quad (13)$$

Thus, we obtain the semidefinite form of gSCR constraints (13) and eliminate the nonconvexity and nonlinearity in the conventional gSCR constraints.

B. MISDP Model for Proposed Framework

1) Objective Function

The objective is to minimize the total cost c incurred during the planning period and can be formulated as:

$$\min c = c^{\text{Ope}} + c^{\text{Inv}} \quad (14)$$

where c^{Ope} is the annualized operation cost; and c^{Inv} is the annualized investment cost.

$$c^{\text{Ope}} = 365 \sum_{o \in O} \pi_o \cdot \left\{ \sum_{t \in T} \Delta t \left[\sum_{g \in G^{\text{Ope}}} \left[c_g^{\text{M}} u_{o,t,g}^{\text{SG}} + c_g^{\text{P}} p_{o,t,g}^{\text{SG}} + c_g^{\text{down}} (1 - u_{o,t,g}^{\text{up}}) + c_g^{\text{up}} u_{o,t,g}^{\text{up}} \right] + \sum_{h \in H^{\text{Ope}}} c_h^{\text{HVDC}} p_{o,t,h}^{\text{HVDC}} + \sum_{i=1}^N c_i^{\text{LS}} p_{o,t,i}^{\text{LS}} \right] \right\} \quad (15)$$

$$c^{\text{Inv}} = \sum_{l \in L^{\text{Inv}}} c_l u_l^{\text{Brh}} + \sum_{g \in G^{\text{Inv}}} c_g u_g^{\text{SG,P}} \quad (16)$$

where O is the scenario set indexed by o ; π_o is the probability of scenario o ; Δt is the dispatching period; T is the set of periods; G^{Ope} and H^{Ope} are the operational sets of synchronous generators and LCC-HVDC injection buses, respectively; c_g^{M} , c_g^{P} , c_g^{up} , and c_g^{down} are the fixed, variable, start-up, and shut-down costs of synchronous generator g , respectively; $p_{o,t,g}^{\text{SG}}$ is the active power output of synchronous generator g in scenario o at time t ; $u_{o,t,g}^{\text{SG}}$ and $u_{o,t,g}^{\text{up}}$ are the binary variables indicating whether the synchronous generator g in scenario o at time t is on and start-up, respectively; c_h^{HVDC} and $p_{o,t,h}^{\text{HVDC}}$ are the cost of LCC-HVDC injection bus h and the active power of LCC-HVDC injection bus h in scenario o at time t , respectively; c_i^{LS} and $p_{o,t,i}^{\text{LS}}$ are the penalty term of bus i and the amount of load shedding of bus i in scenario o at time t , respectively; L^{Inv} and G^{Inv} are the candidate planning sets of transmission lines and synchronous generators, respectively; c_l and c_g are the investment costs of transmission line l and synchronous generator g , respectively; and $u_g^{\text{SG,P}}$ is the planning decision variable of synchronous generator g .

It is worth mentioning that, since the locations of LCC-HVDC injection buses are typically fixed during earlier and higher-level planning stages and the proposed framework focuses on the internal planning of the receiving-end transmission systems, these injection buses are treated as predetermined inputs in the expansion planning model.

2) Constraints

The power balance constraint of bus i in scenario o at time t can be formulated as:

$$\sum_{l \in L_i} p_{o,t,l}^{\text{Line}} = p_{o,t,i}^{\text{LS}} - p_{o,t,i}^{\text{Load}} + \sum_{g \in G_i} p_{o,t,g}^{\text{SG}} + \sum_{h \in H_i} p_{o,t,h}^{\text{HVDC}} \quad (17)$$

where $p_{o,t,l}^{\text{Line}}$ is the power flow of line l in scenario o at time t ; $p_{o,t,i}^{\text{Load}}$ is the load demand of bus i in scenario o at time t ; and L_i and H_i are the sets of transmission lines and LCC-HVDC injection buses connected to bus i , respectively.

Given the high X/R ratio in transmission systems, the linear power flow constraints are adopted here and can be formulated as:

$$p_{o,t,l}^{\text{Line}} + (\theta_{o,t,l,1} - \theta_{o,t,l,2}) b_l = 0 \quad \forall l \in L \quad (18)$$

$$-M(1 - u_l^{\text{Brh}}) \leq p_{o,t,l}^{\text{Line}} + (\theta_{o,t,l,1} - \theta_{o,t,l,2}) b_l \quad \forall l \in L^{\text{Inv}} \quad (19)$$

$$p_{o,t,l}^{\text{Line}} - (\theta_{o,t,l,1} - \theta_{o,t,l,2}) b_l \leq M(1 - u_l^{\text{Brh}}) \quad \forall l \in L^{\text{Inv}} \quad (20)$$

$$-\hat{p}_l^{\text{Line}} \leq p_{o,t,l}^{\text{Line}} \leq \hat{p}_l^{\text{Line}} \quad \forall l \in L \quad (21)$$

$$-u_l^{\text{Brh}} \hat{p}_l^{\text{Line}} \leq p_{o,t,l}^{\text{Line}} \leq u_l^{\text{Brh}} \hat{p}_l^{\text{Line}} \quad \forall l \in L^{\text{Inv}} \quad (22)$$

$$\theta_{o,t,\text{slack}} = 0 \quad (23)$$

where L is the set of existing transmission lines; M is a large constant; $\theta_{o,t,l,1}$ and $\theta_{o,t,l,2}$ are the voltage angles of two buses at the ends of line l in scenario o at time t ; $\theta_{o,t,\text{slack}}$ is the voltage angle of the slack node in scenario o at time t ; u_l^{Brh} and b_l are the construction decision variable and admittance of transmission line l , respectively; and $\hat{p}_l^{\text{Line}}$ is the capacity of transmission line l .

The UC constraints of synchronous generators involve the bounded output constraint ((24)), the unit placement constraint ((25)), the maximum and minimum start-up and shut-down time constraints ((26) and (27)), and the limited ramp-down and ramp-up characteristic constraints ((28) and (29)).

$$u_{o,t,g}^{\text{SG}} \check{p}_g^{\text{SG}} \leq p_{o,t,g}^{\text{SG}} \leq u_{o,t,g}^{\text{SG}} \hat{p}_g^{\text{SG}} \quad \forall g \quad (24)$$

$$u_{o,t,g}^{\text{SG}} \leq u_g^{\text{SG,P}} \quad \forall g \in G^{\text{Inv}} \quad (25)$$

$$\sum_{\tau=t-T_g^{\text{up}}}^{t-1} u_{o,\tau,g}^{\text{SG}} \geq T_g^{\text{up}} (u_{o,t,g}^{\text{up}} - u_{o,t-1,g}^{\text{up}}) \quad \forall g, \forall t \in T \quad (26)$$

$$\sum_{\tau=t-T_g^{\text{down}}}^{t-1} u_{o,\tau,g}^{\text{SG}} \geq T_g^{\text{down}} (u_{o,t,g}^{\text{up}} - u_{o,t-1,g}^{\text{up}}) \quad \forall g, \forall t \in T \quad (27)$$

$$p_{o,t,g}^{\text{SG}} - p_{o,t-1,g}^{\text{SG}} \leq \Delta t \left[u_{o,t,g}^{\text{SG}} R_g^{\text{up}} + (1 - u_{o,t-1,g}^{\text{SG}}) R_g^{\text{start}} \right] \quad \forall g \quad (28)$$

$$p_{o,t,g}^{\text{SG}} - p_{o,t-1,g}^{\text{SG}} \geq \Delta t \left[u_{o,t,g}^{\text{SG}} R_g^{\text{down}} + (1 - u_{o,t-1,g}^{\text{SG}}) R_g^{\text{shut}} \right] \quad \forall g \quad (29)$$

where $\check{p}_g^{\text{SG}}$ and $\hat{p}_g^{\text{SG}}$ are the minimum and maximum active power outputs of synchronous generator g , respectively; T_g^{up} and T_g^{down} are the minimum start-up and shut-down time of synchronous generator g , respectively; R_g^{up} and R_g^{down} are the maximum ramp-up and ramp-down speeds of synchronous generator g , respectively; and R_g^{start} and R_g^{shut} are the start-up and shut-down capacities of synchronous generator g , respectively.

The bound constraint of LCC-HVDC injection bus h is:

$$0 \leq p_{o,t,h}^{\text{HVDC}} \leq P_{\text{HVDC}}^h \quad (30)$$

The load shedding constraint of bus i is:

$$0 \leq p_{o,t,i}^{\text{LS}} \leq p_{o,t,i}^{\text{Load}} \quad (31)$$

The semidefinite form of gSCR constraints is expressed as (13).

The equipment investment constraints involve the financial constraints and the limited construction number constraints:

$$c^{\text{Inv}} \leq K_{\text{max}}^{\text{Inv}} \quad (32)$$

$$\sum_{l \in L^{\text{Inv}}} u_l^{\text{Brh}} \leq K_{\text{max}}^{\text{Brh}} \quad (33)$$

$$\sum_{g \in G^{\text{Inv}}} u_g^{\text{SG,P}} \leq K_{\text{max}}^{\text{SG}} \quad (34)$$

where $K_{\max}^{\text{Inv}}$ is the maximum investment cost; and $K_{\max}^{\text{Brh}}$ and $K_{\max}^{\text{SG}}$ are the maximum construction numbers of transmission lines and synchronous generators, respectively.

III. GBD-BASED APPROACH

Although the MISDP model for the proposed framework has global optimality, it is not friendly to most commercial solvers, which will greatly compromise its practicality and portability. To tackle this problem, a GBD-based approach is proposed to solve the MISDP model.

For readability, the MISDP model is simplified in the compact form as:

$$\begin{cases} \min \{ \text{Tr}(\mathbf{M}^T \mathbf{u}) + \text{Tr}(\mathbf{N}^T \mathbf{v}) \} \\ \text{s.t. } \text{Tr}(\mathbf{A}_i^T \mathbf{u}) + \text{Tr}(\mathbf{D}_i^T \mathbf{v}) \leq C_i \quad i \in [1, m_1] \\ \text{Tr}(\mathbf{E}_i^T \mathbf{u}) + \text{Tr}(\mathbf{F}_i^T \mathbf{v}) = J_i \quad i \in [1, m_2] \\ \mathbf{O}\mathbf{u} + \mathbf{Q}\mathbf{v} \geq \mathbf{0} \\ \mathbf{R}\mathbf{v} \leq \mathbf{S} \\ \mathbf{u} \geq \mathbf{0} \\ \mathbf{v} \geq \mathbf{0} \\ \mathbf{v} \leq \mathbf{1} \end{cases} \quad (35)$$

where $\mathbf{u}$ and $\mathbf{v}$ are the continuous and discrete variable vectors, respectively; $\mathbf{A}_i$, $\mathbf{D}_i$, $\mathbf{E}_i$, $\mathbf{F}_i$, $\mathbf{O}$, $\mathbf{Q}$, $\mathbf{M}$, $\mathbf{N}$, $\mathbf{R}$, and $\mathbf{S}$ are the coefficient matrices; C_i and J_i are the constants; m_1 is the number of inequality constraints; m_2 is the number of equality constraints; and $\text{Tr}(\cdot)$ is the trace operation.

However, it is worth mentioning that the heavy computational burden of solving the MISDP model in (35) necessitates a computationally friendly optimization method.

In this section, the GBD-based approach decomposes the primal MISDP problem into a semidefinite programming (SDP) sub-problem that contains only continuous variables and an MILP master problem that includes all discrete variables. In this way, the primal MISDP problem can be solved iteratively [27], offering higher computational efficiency than directly solving the original problem.

The whole process of the GBD-based approach implements the following four steps.

1) Step 1: Set Initial Values of Discrete Variables

For higher solution efficiency, the initial values of discrete variables are set to approximate the optimal values. Relax the primal MISDP problem to the SDP problem as:

$$\begin{cases} \min \{ \text{Tr}(\mathbf{M}^T \mathbf{u}) + \text{Tr}(\mathbf{N}^T \mathbf{v}') \} \\ \text{s.t. } \text{Tr}(\mathbf{A}_i^T \mathbf{u}) + \text{Tr}(\mathbf{D}_i^T \mathbf{v}') \leq C_i \quad i \in [1, m_1] \\ \text{Tr}(\mathbf{E}_i^T \mathbf{u}) + \text{Tr}(\mathbf{F}_i^T \mathbf{v}') = J_i \quad i \in [1, m_2] \\ \mathbf{O}\mathbf{u} + \mathbf{Q}\mathbf{v}' \geq \mathbf{0} \\ \mathbf{R}\mathbf{v}' \leq \mathbf{S} \\ \mathbf{u} \geq \mathbf{0} \\ \mathbf{v}' \geq \mathbf{0} \\ \mathbf{v}' \leq \mathbf{1} \end{cases} \quad (36)$$

where $\mathbf{v}'$ is the relaxed form of $\mathbf{v}$. Then, the initial values of the optimal discrete variable vector, denoted as $\bar{\mathbf{v}}_0$, can be obtained as:

$$\bar{\mathbf{v}}_0 = \lfloor \bar{\mathbf{v}}' + 0.5 \rfloor \quad (37)$$

where $\lfloor \cdot \rfloor$ is the floor function; and $\bar{(\cdot)}$ is the symbol for the optimal value.

2) Step 2: Solve SDP Sub-problem

Taking the k^{th} iteration as an example, by fixing $\mathbf{v}$, the primal MISDP problem can be converted into an SDP sub-problem (38) and can be solved with well-established SDP techniques [15].

$$\begin{cases} \min \text{Tr}(\mathbf{M}^T \mathbf{u}_k) \\ \text{s.t. } \text{Tr}(\mathbf{A}_i^T \mathbf{u}_k) \leq C_i - \text{Tr}(\mathbf{D}_i^T \bar{\mathbf{v}}_k) \quad i \in [1, m_1] \\ \text{Tr}(\mathbf{E}_i^T \mathbf{u}_k) = J_i - \text{Tr}(\mathbf{F}_i^T \bar{\mathbf{v}}_k) \quad i \in [1, m_2] \\ \mathbf{O}\mathbf{u}_k \geq -\mathbf{Q}\bar{\mathbf{v}} \\ \mathbf{u}_k \geq \mathbf{0} \end{cases} \quad (38)$$

where the subscript k is the iteration index.

The SDP sub-problem is typically reformulated as its dual problem for guaranteeing its feasibility [28]:

$$\begin{cases} \max \left\{ -\sum_{i=1}^{m_1} \lambda_{k,i}^1 (C_i - \text{Tr}(\mathbf{D}_i^T \bar{\mathbf{v}}_k)) - \text{Tr}(\mathbf{Z}_k^T \mathbf{Q}\bar{\mathbf{v}}_k) - \sum_{i=1}^{m_2} \lambda_{k,i}^2 (J_i - \text{Tr}(\mathbf{F}_i^T \bar{\mathbf{v}}_k)) \right\} \\ \text{s.t. } \lambda_{k,i}^1 \geq 0 \quad i \in [1, m_1] \\ \mathbf{Z}_k \geq \mathbf{0} \\ \mathbf{M}^T + \sum_{i=1}^{m_1} \lambda_{k,i}^1 \mathbf{A}_i^T + \sum_{i=1}^{m_2} \lambda_{k,i}^2 \mathbf{E}_i^T - \mathbf{Z}_k^T \mathbf{O} \geq \mathbf{0} \end{cases} \quad (39)$$

where $\lambda_{k,i}^1$, $\lambda_{k,i}^2$, and $\mathbf{Z}_k$ are the related dual variables for (38).

The feasible region of the dual sub-problem (39) is invariant to $\bar{\mathbf{v}}_k$, and the optimal solution of (39) is either bounded or unbounded.

3) Step 3: Generate Sets of Benders Cuts and Update Upper Bound

The Benders cuts and the upper bound are generated conditionally based on the properties of the optimal solution of (39).

1) If the optimal solution of (39) is bounded, the Benders optimality cuts and the upper bound are obtained as (40) and (41), respectively.

$$\begin{aligned} SP \geq & -\sum_{i=1}^{m_1} \bar{\lambda}_{k,i}^1 (C_i - \text{Tr}(\mathbf{D}_i^T \mathbf{v}_{k+1})) - \text{Tr}(\bar{\mathbf{Z}}_k^T \mathbf{Q}\mathbf{v}_{k+1}) - \\ & \sum_{i=1}^{m_2} \bar{\lambda}_{k,i}^2 (J_i - \text{Tr}(\mathbf{F}_i^T \mathbf{v}_{k+1})) \end{aligned} \quad (40)$$

$$\begin{aligned} UB_k = & -\sum_{i=1}^{m_1} \bar{\lambda}_{k,i}^1 (C_i - \text{Tr}(\mathbf{D}_i^T \bar{\mathbf{v}}_k)) - \text{Tr}(\bar{\mathbf{Z}}_k^T \mathbf{Q}\bar{\mathbf{v}}_k) - \\ & \sum_{i=1}^{m_2} \bar{\lambda}_{k,i}^2 (J_i - \text{Tr}(\mathbf{F}_i^T \bar{\mathbf{v}}_k)) \end{aligned} \quad (41)$$

where SP is the auxiliary variable of MILP master problem; and UB_k is the upper bound in the k^{th} iteration.

2) If the optimal solution of (39) is unbounded, the upper bound is obtained as (42), and by solving the slack problem of (38), the Benders feasibility cut is generated as (43).

$$UB_k = UB_{k-1} \quad (42)$$

$$\begin{cases} \min \left\{ \sum_{i=1}^{m_1} s_{1,i} + \mathbf{1}^T \mathbf{s}_2 \right\} \\ \text{s.t. } \text{Tr}(\mathbf{A}_i^T \mathbf{u}_k) - s_{1,i} \leq C_i - \text{Tr}(\mathbf{D}_i^T \bar{\mathbf{v}}_k) \quad i \in [1, m_1] \\ \text{Tr}(\mathbf{E}_i^T \mathbf{u}_k) = J_i - \text{Tr}(\mathbf{F}_i^T \bar{\mathbf{v}}_k) \quad i \in [1, m_2] \\ \mathbf{O} \mathbf{u}_k + \text{diag}(\mathbf{s}_2) \geq -\mathbf{Q} \bar{\mathbf{v}}_k \\ \mathbf{u}_k \geq \mathbf{0} \\ \mathbf{s}_2 \geq \mathbf{0} \\ s_{1,i} \geq 0 \quad i \in [1, m_1] \end{cases} \quad (43)$$

where $\mathbf{1}$ is an all-ones vector; and $s_{1,i}$ and $\mathbf{s}_2$ are the slack variable and vector, respectively. For the feasibility and clarity of the SDP sub-problem, (43) is typically addressed by solving its dual problem (39), which will not be discussed further in this context.

By solving (43), the Benders feasibility cut in the k^{th} iteration can be formulated as:

$$0 \geq - \sum_{i=1}^{m_1} \tilde{\lambda}_{k,i}^1 (C_i - \text{Tr}(\mathbf{D}_i^T \mathbf{v}_{k+1})) - \text{Tr}(\tilde{\mathbf{Z}}_k^T \mathbf{Q} \mathbf{v}_{k+1}) - \sum_{i=1}^{m_2} \tilde{\lambda}_{k,i}^2 (J_i - \text{Tr}(\mathbf{F}_i^T \mathbf{v}_{k+1})) \quad (44)$$

where $\tilde{\lambda}_{k,i}^1$, $\tilde{\lambda}_{k,i}^2$, and $\tilde{\mathbf{Z}}_k$ are the dual optimal solutions of (43).

Thereafter, the sets of Benders cuts can be updated as:

$$\begin{cases} BO \leftarrow BO \cup \{(40)\} & (39) \text{ is bounded} \\ BF \leftarrow BF \cup \{(44)\} & (39) \text{ is unbounded} \end{cases} \quad (45)$$

where BO and BF are the sets of Benders optimality and feasibility cuts, respectively. Besides, for better numerical stability, depending on the specific problem, no-good cuts can be added to improve the numerical stability.

4) Step 4: Solve MILP Master Problem and Update Lower Bound

Combining the results mentioned above, an MILP master problem can be formulated as:

$$\begin{cases} \min \{ \mathbf{N} \mathbf{v}_{k+1} + SP \} \\ \text{s.t. } \{ SP, \mathbf{v}_{k+1} \} \in BO \\ \mathbf{v}_{k+1} \in BF \\ \mathbf{R} \mathbf{v}_{k+1} \leq \mathbf{S} \\ \mathbf{v}_{k+1} \in \{ \mathbf{0}, \mathbf{1} \} \end{cases} \quad (46)$$

By solving (46), the optimal values of the discrete variable vector in the $(k+1)^{\text{th}}$ iteration $\bar{\mathbf{v}}_{k+1}$ can be obtained, and the lower bound in the k^{th} iteration LB_k can be expressed as:

$$LB_k = \overline{SP} \quad (47)$$

where $\overline{SP}$ is the optimal solution of (46).

$\mathbf{u}$ and $\mathbf{v}$ will be iteratively updated via Steps 2-4, until the presetting termination criterion δ is reached:

$$\frac{UB_k - LB_k}{UB_k} \leq \delta \quad (48)$$

Given the steps mentioned above, the whole process of the GBD-based approach can be summarized in Supplementary Material B Fig. B1.

IV. CASE STUDIES

A. Settings

1) Test system: as shown in Fig. 1, a modified IEEE 39-bus system is served for demonstrating the proposed framework, with 39 buses, 10 synchronous generators, 37 transmission lines, 9 corridors without transmission lines, and 3 LCC-HVDC terminals included. The three LCC-HVDC terminals with 1100 MW capacities are connected to Buses 6, 18, and 29, respectively. Besides, three types of synchronous generators (types 1-3) are considered [21], with their technical parameters shown in Table II. According to [17], an empirical value of 3 is recommended for the boundary values of gSCR in MI-HVDC systems, which is also adopted in this work. The annualized investment cost rate and the investment cost of transmission lines are borrowed from [29].

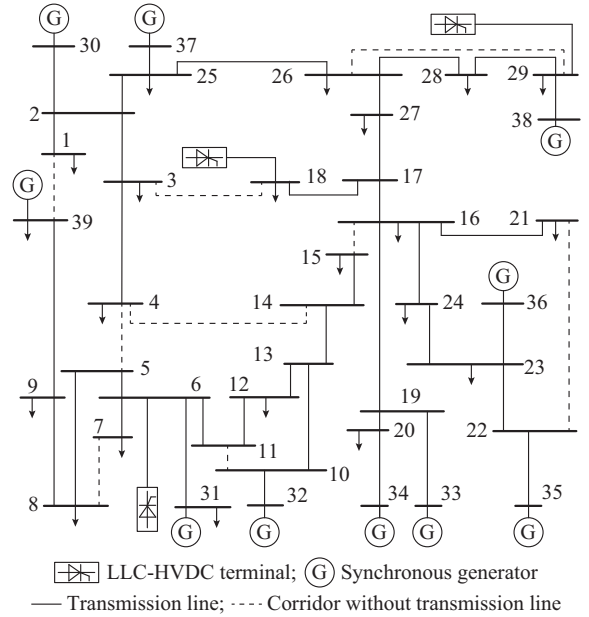


Fig. 1. Modified IEEE 39-bus system.

TABLE II
TECHNICAL PARAMETERS OF THREE TYPES OF SYNCHRONOUS GENERATORS

Type	$\hat{p}_g^{\text{SG}}$ (MW)	$\check{p}_g^{\text{SG}}$ (MW)	$b_{t,g}$ (p.u.)	T_g^{up} (hour)	T_g^{down} (hour)	c_g (CNY)	c_g^{M} (CNY/h)	c_g^{P} (CNY/MWh)	c_g^{up} (CNY)	c_g^{down} (CNY)
Type 1	600	240	25.00	10	18	2.22×10^9	1700	300	2.5×10^5	5×10^4
Type 2	500	175	22.22	10	16	1.85×10^9	1500	350	2.0×10^5	3×10^4
Type 3	350	105	11.24	4	6	1.30×10^9	1000	400	1.0×10^5	2×10^4

2) Environment: all the programs in this paper are implemented by Python 3.8 and the CVXPY 1.5.2 package and run on a desktop computer equipped with 2 Intel Xeon Platinum 8581C CPUs and 256 GB RAM. And the commercial solvers used in this paper are Gurobi 12.0.3 (for the MILP master problem) and Mosek 11.0 (for the SDP sub-problem). The relevant code and simulation data are available in [30].

B. Expansion Planning Results

To verify the effectiveness of the proposed framework, four groups are demonstrated and compared in this subsection.

1) Group 1: coordinated expansion planning considering gSCR constraints and UC results by solving the MISDP model through the GBD-based approach.

2) Group 2: coordinated expansion planning considering UC results without gSCR constraints [9]–[14], [31], [32].

3) Group 3: coordinated expansion planning considering gSCR constraints and UC results by GA with a hybrid selection strategy (roulette wheel and elitism selection). The mutation type used here is random resetting; the crossover type is uniform crossover; and the key hyperparameters of GA are shown in Table III.

TABLE III
HYPERPARAMETERS OF GA

Hyperparameter	Value	Rationale
Population size	50	Number of candidate solutions
Crossover probability	0.6	Probability of combining parental traits to produce offspring
Mutation probability	0.01	Probability of a genetic change per generation
Elitism ratio	0.2	Proportion of top individuals preserved between generations
The maximum iteration without improvement	20	Stopping criterion after no better solutions found consecutively

4) Group 4: coordinated expansion planning considering gSCR constraints by solving the MISDP model through the GBD-based approach.

The expansion planning results are shown in Table IV. The relevant statistical results of expansion planning are presented in Table V. As illustrated in Table IV and Table V, the significant differences emerge among the four groups in terms of the system configuration, cost, and grid strength.

In order to meet the gSCR requirement, Group 1 deploys a more robust infrastructure than Group 2, including 3 additional transmission lines and 3 extra synchronous generators. Besides, Group 1 considers all corridors without transmission lines, whereas Group 2 omits several, including corridors 3-18, 7-8, and 26-29. This phenomenon directly verifies the significant effect of system topology on gSCR level.

Furthermore, although the total cost of Group 1 is slightly higher than that of Group 2, Group 1 achieves the crucial balance of cost-effectiveness and grid strength. In contrast, Group 3 results in the highest total cost while remaining compliant with the gSCR requirement ($\text{min gSCR} = 3.01$), which can be attributed to the suboptimal convergence of GA, leading to a significant overshoot in operational expenses.

TABLE IV
EXPANSION PLANNING RESULTS

Equipment	Result			
	Group 1	Group 2	Group 3	Group 4
Transmission line	1-39(1), 2-3(2), 2-30(1), 3-4(1), 3-18(2), 4-5(1), 6-7(1), 7-8(1), 16-19(2), 19-33(2), 20-34(1), 21-22(1), 23-24(1), 23-36(1), 25-37(1), 26-29(1)	1-2(1), 1-39(1), 2-3(2), 4-5(2), 5-6(1), 5-8(1), 20-34(1), 21-22(2), 22-35(1), 23-24(1), 23-36(1), 25-37(1), 28-29(1)	1-39(2), 2-3(1), 3-4(1), 3-18(1), 4-5(1), 6-7(2), 7-8(1), 12-13(1), 14-15(1), 15-16(1), 16-17(1), 19-33(1), 20-34(1), 21-22(1), 23-24(1), 23-24(1), 26-29(1)	1-39(1), 2-3(1), 2-30(1), 3-18(1), 4-5(1), 6-7(1), 7-8(1), 13-14(1), 16-19(2), 19-33(1), 20-34(1), 21-22(1), 23-24(1), 23-36(1), 26-29(1)
	Bus 30(2), Bus 31(2), Bus 33(2), Bus 34(1), Bus 35(1), Bus 36(1), Bus 37(2), Bus 39(3)	Bus 30(2), Bus 31(3), Bus 32(1), Bus 33(1), Bus 34(3), Bus 35(3), Bus 36(2), Bus 37(3), Bus 38(1), Bus 39(3)	Bus 30(2), Bus 31(3), Bus 33(2), Bus 34(2), Bus 35(2), Bus 36(2), Bus 37(2), Bus 38(1)	Bus 30(2), Bus 31(3), Bus 32(2), Bus 33(1), Bus 34(2), Bus 35(1), Bus 36(1), Bus 37(2), Bus 39(2)
	Bus 31(1), Bus 33(1), Bus 34(1), Bus 38(2), Bus 39(3)	Bus 39(3)	Bus 30(1), Bus 32(1), Bus 33(1), Bus 34(1), Bus 39(1)	Bus 38(1), Bus 39(3)
	Bus 31(1), Bus 32(1), Bus 34(1), Bus 35(1), Bus 36(2), Bus 38(1), Bus 39(2)	Bus 31(1), Bus 39(2)	Bus 30(1), Bus 31(1), Bus 32(1), Bus 33(2), Bus 34(2), Bus 35(1), Bus 39(1)	Bus 32(1), Bus 38(1)

Note: numbers in brackets indicate the planned number of the corresponding installations.

TABLE V
RELEVANT STATISTICAL RESULTS OF EXPANSION PLANNING

Group	Annualized investment cost (CNY)	Annualized operation cost (CNY)	Total annualized cost (CNY)	Total cost (CNY)	min gSCR
Group 1	1.32×10^{10}	4.69×10^{10}	6.01×10^{10}	1.20×10^{12}	3.27
Group 2	1.25×10^{10}	4.50×10^{10}	5.75×10^{10}	1.15×10^{12}	1.40
Group 3	1.27×10^{10}	9.17×10^{10}	1.04×10^{11}	2.08×10^{12}	3.01
Group 4	1.13×10^{10}	4.97×10^{10}	6.10×10^{10}	1.22×10^{12}	2.76

Group 4, which neglects the start-stop cycles of synchronous generators, not only exhibits an inadequate gSCR ($\text{min gSCR} = 2.76$) in practice but also incurs a higher total cost compared with Group 1, highlighting the economic and technical risks of oversimplified operational constraints.

In summary, Group 1 demonstrates a better configuration by achieving the relatively low total cost, while securing the highest gSCR level. These results collectively validate the efficacy of the proposed framework in conducting the cost-ef-

fective expansion planning for MI-HVDC systems, successfully reconciling economic objectives with grid strength requirements.

C. Time-domain Simulation Verification

To verify the efficacy of the proposed framework in enhancing the voltage stability, the time-domain simulations are performed for both Group 1 and Group 2. The obtained MI-HVDC systems are implemented in PSCAD/EMTDC, where the LCC-HVDC converters are modeled based on the CIGRE HVDC benchmark (the key parameters are shown in Supplementary Material C Table CI) [33]. For a comprehensive assessment of the voltage stability under the planned configurations of Group 1 and Group 2, the voltage dips with a duration of 0.05 s are applied at the receiving-end buses (Buses 6, 18, and 29) of LCC-HVDC links in the MI-HVDC system. The corresponding voltage, reactive power, and extinction angle responses are then analyzed to evaluate the system voltage stability. According to [17], a stable system should exhibit a smooth and rapid recovery of voltage following the disturbance. The time-domain simulation results under voltage dips at Buses 6, 18, and 29 are presented in Figs. 2-4, respectively.

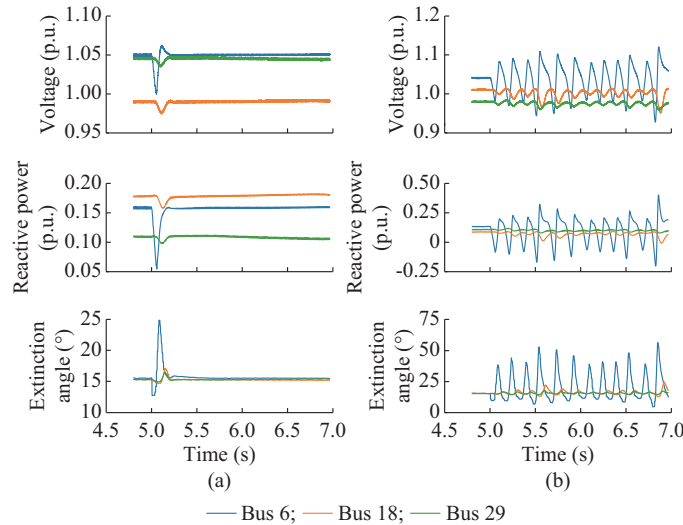


Fig. 2. Time-domain simulation results under voltage dip at Bus 6. (a) Group 1. (b) Group 2.

As shown in Fig. 2, a voltage dip at Bus 6 induces voltage responses at all other receiving-end buses. In Group 1, both voltage and reactive power recover within approximately 0.3 s, accompanied only by a minor transient overvoltage (about 0.02 p.u.) at Bus 6 and no sustained oscillations. Besides, the extinction angle remains above 0 throughout the simulation, indicating no commutation failure. In contrast, although Group 2 also avoids the commutation failure, its voltage and reactive power exhibit sustained oscillations without signs of attenuation after the voltage dip is cleared. These observations demonstrate the higher voltage stability of Group 1 compared with that of Group 2.

Figure 3 shows that both Group 1 and Group 2 recover rapidly after the voltage dip at Bus 18 is cleared, demonstrating a comparable robustness to this disturbance.

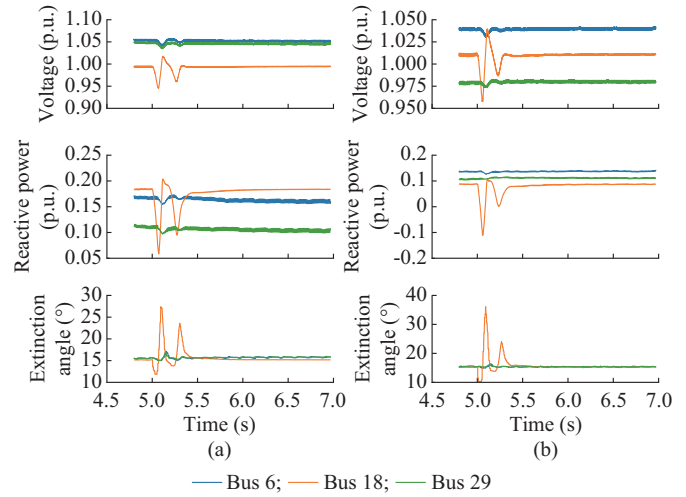


Fig. 3. Time-domain simulation results under voltage dip at Bus 18. (a) Group 1. (b) Group 2.

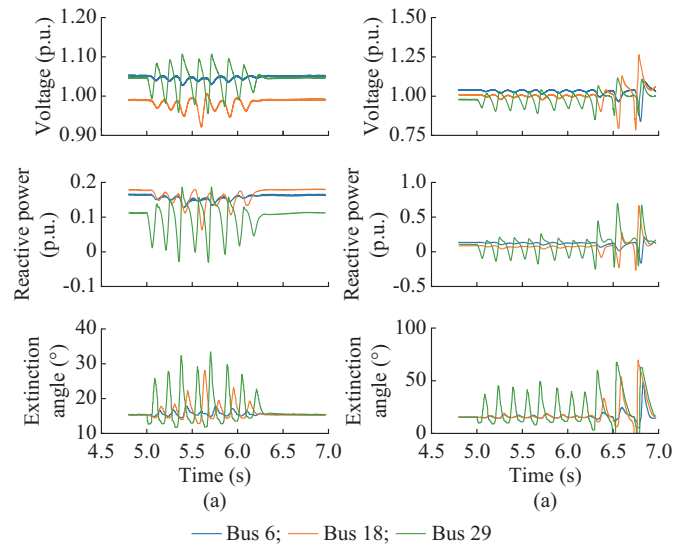


Fig. 4. Time-domain simulation results under voltage dip at Bus 29. (a) Group 1. (b) Group 2.

Figure 4 shows that after the voltage dip at Bus 29 is cleared, the voltage and reactive power in Group 1 stabilize within approximately 1.3 s after a short oscillatory transient. Group 2, however, displays continuous oscillations with increasing amplitude. The instability ultimately induces a commutation failure at the LCC-HVDC station of Bus 29 at 6.5 s, leading to a further escalation of system instability.

Based on the above time-domain simulation results, the configuration of Group 1 exhibits overall enhanced voltage stability and greater grid strength than that of Group 2. These outcomes validate the effectiveness of the proposed framework, which incorporates gSCR constraints and UC results into the expansion planning for MI-HVDC system.

Furthermore, to investigate the impact of modular multi-level converter based high-voltage direct current (MMC-HVDC), the LCC-HVDC links in Fig. 1 are replaced with MMC-HVDC links, and the corresponding time-domain simulation results are presented in Supplementary Material D [34]-[36].

D. Real System Analysis

To further validate the effectiveness and scalability of the proposed framework, a practical 56-bus transmission system within the Eastern China Power Grid, incorporating two LCC-HVDC links, is modeled in Fig. 5. A comparative analysis is then conducted by optimizing the system separately using Group 1 and Group 2. The key parameters for the LCC-HVDC links are set according to the specifications provided in Supplementary Material C Table CI. The relevant statistical results of expansion planning for the practical 56-bus transmission system are summarized in Table VI.

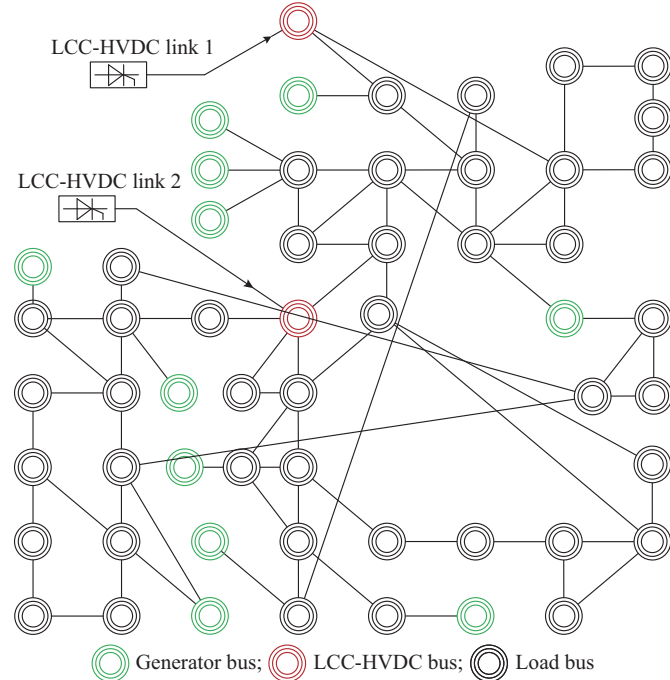


Fig. 5. Topology of practical 56-bus transmission system.

TABLE VI
RELEVANT STATISTICAL RESULTS OF EXPANSION PLANNING FOR
PRACTICAL 56-BUS TRANSMISSION SYSTEM

Statistical result	Group 1	Group 2
Number of planned transmission lines	18	15
Number of planned synchronous generators	26	21
Cost of adding transmission lines (CNY)	1.83×10^{11}	1.24×10^{11}
Cost of adding synchronous generators (CNY)	8.7×10^{10}	6.68×10^{10}
Cost of LCC-HVDC link (CNY)	1.1×10^9	1.1×10^9
Operation cost (CNY)	2.66×10^{12}	2.57×10^{12}
Total cost (CNY)	2.93×10^{12}	2.76×10^{12}
min gSCR	3.05	1.88

As summarized in Table VI, Group 1 incorporates more transmission lines and synchronous generators compared with Group 2. Despite a 5.8% higher total cost, Group 1 achieves a higher improvement at the gSCR level than Group 2, corresponding to a significant enhancement in grid

strength.

The time-domain simulations under voltage dips at receiving-end buses of LCC-HVDC links 1 and 2 are presented in Figs. 6 and 7, respectively.

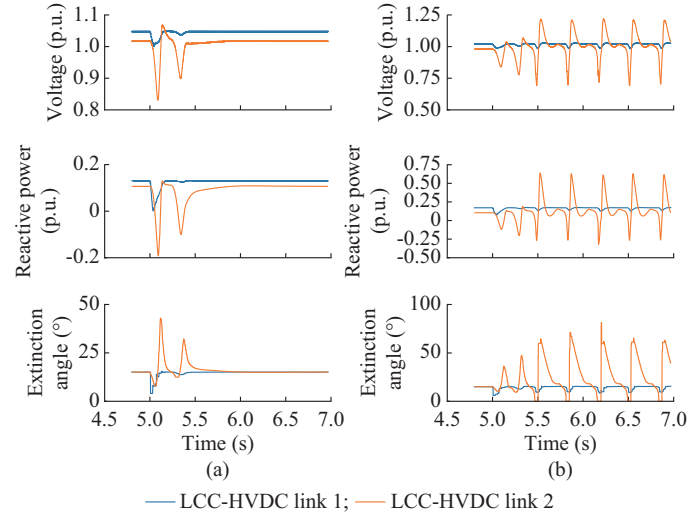


Fig. 6. Time-domain simulations under voltage dip at receiving-end buses of LCC-HVDC link 1. (a) Group 1. (b) Group 2.

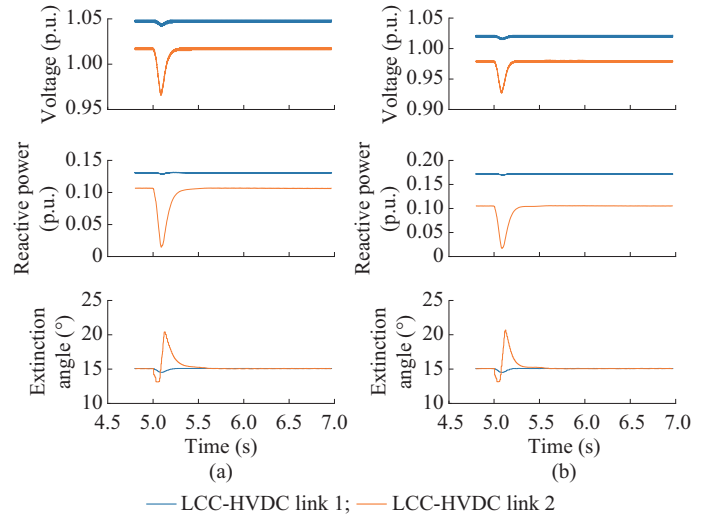


Fig. 7. Time-domain simulations under voltage dip at receiving-end buses of LCC-HVDC link 2. (a) Group 1. (b) Group 2.

As shown in Fig. 6, a voltage dip at receiving-end buses of LCC-HVDC link 1 leads to corresponding voltage responses at those of LCC-HVDC link 2. In Group 1, both voltage and reactive power recover within approximately 0.4 s, accompanied only by a minor transient overvoltage (about 0.02 p.u.) at LCC-HVDC link 2 and no sustained oscillations. Besides, the extinction angle remains above 0 throughout the simulation, indicating no commutation failure. In contrast, Group 2 displays continuous oscillations with growing amplitudes. The instability ultimately induces a commutation failure at the LCC-HVDC station of LCC-HVDC link 2 at 5.5 s, further aggravating system instability.

Figure 7 reveals that both Group 1 and Group 2 recover rapidly after the voltage dip at receiving-end buses of LCC-HVDC link 2 is cleared, demonstrating a comparable robust-

ness to this disturbance.

Besides, it is noteworthy that, regardless of which receiving-end bus the voltage dip occurs at, the voltage fluctuation at LCC-HVDC link 2 is always more severe than that at LCC-HVDC link 1. This phenomenon can be attributed to the fact that the receiving-end bus of LCC-HVDC link 2 is electrically farther from the synchronous generators than that of LCC-HVDC link 1, and the local AC system provides weaker support. Consequently, the voltage at the receiving-end bus of LCC-HVDC link 2 is more susceptible to disturbances. Furthermore, the electrical coupling between the two buses allows disturbances originating at the receiving-end bus of LCC-HVDC link 1 to propagate readily to the more vulnerable LCC-HVDC link 2, exerting a substantial impact.

In summary, Group 1 demonstrates significantly enhanced voltage stability at a moderate additional cost, whereas Group 2 exhibits inconsistent recovery and weaker resilience. These results confirm the efficacy of the proposed framework in achieving a well-balanced expansion planning for MI-HVDC systems that successfully reconciles the economic and stability objectives.

E. Further Analysis

1) Global Optimality Analysis of MISDP Model

The global optimality of the MISDP model can be validated by establishing the exact equivalence between the semidefinite form of gSCR constraints and the conventional gSCR constraints [37]. Accordingly, the following three gSCR constraints are compared in this subsection.

- 1) gSCR1: the conventional gSCR constraints formulated as (1) and (2).
- 2) gSCR2: the semidefinite form of gSCR constraints formulated as (13).
- 3) gSCR3: training an artificial neural network (ANN) $f(\cdot)$ to capture the nonlinear relationship between the gSCR value and the relevant decision variables, and then modeling the gSCR constraint as a set of mixed-integer linear constraints.

$$gSCR3 = \min f(\mathbf{P}_{HVDC}, u_l^{Brh}, u_g^{SG}) \quad (49)$$

Specifically, the synchronous generator and transmission lines are fixed at their initial state. P_{HVDC}^i increases from 2 with a step size of 0.02 for a total of 400 variation steps, and the values of the three gSCR constraints are recorded, as shown in Fig. 8. Besides, it is worth noting that the boundary values for gSCR1 and gSCR3 are 3, while the boundary value for gSCR2 is 0.

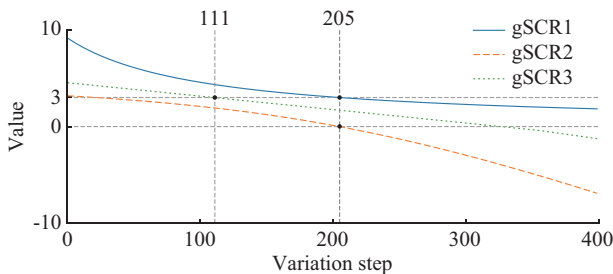


Fig. 8. Values of three gSCR constraints with variation steps of P_{HVDC}^i .

As illustrated in Fig. 8, the moment gSCR2 reaches its boundary value coincides exactly with that of gSCR1 reaching its own, which validates the accuracy of the semidefinite form of gSCR constraints and the global optimality of the MISDP model. In contrast, for gSCR3, the ANN-based method fails to adequately capture the gSCR requirement through its linear constraints due to the limited fitting accuracy. As a result, when gSCR3 reaches its boundary value, gSCR1 has not yet reached its own.

Given the analysis mentioned above, the semidefinite form of gSCR constraints is entirely equivalent to the conventional gSCR constraints, thereby ensuring the global optimality of the MISDP model.

2) Computational Performance Analysis of GBD-based Approach

To validate the effectiveness and scalability of the GBD-based approach for solving the MISDP model, a comparative analysis is performed against the GA. Besides, the outer approximation based branch-and-bound (OAB&B) approach, a well-established one for solving the MISDP model with theoretical global optimality [38], is employed as a benchmark to evaluate the accuracy of both GBD-based approach and GA. The evaluation comprehensively considers multiple performance metrics, including optimal value, gap to OAB&B optimum, total number of iterations, and solution time. The comparison is performed on four widely-used benchmark power systems: the Garver 6-bus, IEEE RTS 24-bus, IEEE 39-bus, and CSEE 88-bus systems. The OAB&B approach is implemented using Julia 1.11.3 and the JuMP 1.24.0 package. The resulting optimization problems are solved, with Gurobi 12.0.3 for the MILP master problem and MOSEK 11.0 for the SDP sub-problem. A relative gap tolerance of 2.5% is set. All experiments are conducted on a desktop computer with the same specifications as outlined in Section IV-A-2).

TABLE VII
RELEVANT STATISTICAL RESULTS OF PERFORMANCE ANALYSIS

System	Approach	Optimal value	Gap (%)	Total number of iterations	Solution time (s)
Garver 6-bus	OAB&B	7.64×10^6			45.77
	GA	1.04×10^7	35.86	33	1547.16
	GBD-based	7.64×10^6	0	49	244.07
IEEE RTS 24-bus	OAB&B	1.25×10^7			1145.01
	GA	3.26×10^7	160.80	58	17702.32
	GBD-based	1.25×10^7	0	110	3568.13
IEEE 39-bus	OAB&B	6.01×10^7			5286.44
	GA	1.04×10^8	73.04	94	38060.83
	GBD-based	6.01×10^7	0	346	13586.73
CSEE 88-bus	OAB&B	Memory overflow			
	GA	Infinity		20	17555.96
	GBD-based	5.14×10^7		1463	45690.49

As shown in Table VII, although the GBD-based approach incurs longer solution time compared with the OAB&B approach when applied to the Garver 6-bus, IEEE RTS 24-bus, and IEEE 39-bus systems, it attains the same optimal solu-

tion and demonstrates precision far exceeding that of the GA. However, in the CSEE 88-bus system, the OAB&B approach fails due to memory overflow, as it must retain all integer and continuous variables in each solving iteration. In contrast, the proposed framework succeeds by decomposing the problem and addressing these variable sets separately, despite an extended runtime. Besides, due to the vast solution space, the GA also fails to find a feasible solution for the CSEE 88-bus system and is prematurely halted after 20 iterations, having reached the “the maximum iteration without improvement” limit in Table III. These observations validate the global optimality of GBD-based approach with high computational efficiency when solving the MISDP model for MI-HVDC system.

V. CONCLUSION

This paper presents a coordinated expansion planning framework for MI-HVDC systems considering the gSCR constraints, aiming to minimize the total system cost while adhering to prescribed gSCR thresholds. To ensure the computational tractability and guarantee global optimality, the gSCR constraints are reformulated in a semidefinite form, resulting in an MISDP model for the expansion planning problem. Subsequently, a GBD-based approach is proposed to solve the MISDP model efficiently. Case studies demonstrate that the expansion planning results derived from the proposed framework exhibit superior voltage stability.

Notwithstanding its contributions, this study has two primary limitations: it relies on a set of deterministic typical scenarios to approximate renewable generation variability, and its scope is currently restricted to MI-HVDC systems. Consequently, the future work will generalize the proposed framework by incorporating stochastic modeling of renewable energy and extending its applicability to multi-infeed MMC-HVDC and hybrid HVDC configurations.

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