

Convex Hull Method for Real-time Pricing in Smart Grid with Multiple Generating Units via Smoothing Technique

Zeyi Zhu, Yan Gao, Xiaodong Ding, and Youmeng He

Abstract—The increasing electricity demand imposes higher requirements on the power system to generate electricity and balance the supply and demand. The unit commitment problem is critical in power systems for determining the generation schedules and market prices. However, its inherent nonconvexity and discontinuity pose significant challenges to efficient pricing. The convex hull method addresses this nonconvexity by taking the slope of the convex envelope of cost function for generating units over the convex hull of the feasible set as the price. Nevertheless, due to the nonsmoothness, the convex hull price cannot be computed by using the gradient-based methods, which increases the computational complexity. Smoothing techniques provide an effective way by converting the nonsmooth problem into a smooth one, which enhances the computational efficiency. This paper proposes a real-time pricing scheme for the smart grid that incorporates renewable energy generation and energy storage devices on the demand side and multiple generating units on the supply side. Considering the dynamic demand of users and startup costs of generating units, a social welfare maximization model is formulated. By employing the convex hull method, the convex envelopes of the cost functions for generating units are derived. The smoothing technique is then employed to convert the original model into a smooth one. Furthermore, a distributed iterative algorithm on the basis of gradient projection method is developed by leveraging the separable structure of the variables to solve the model efficiently. Simulation results validate the feasibility and effectiveness of the convex hull method for real-time pricing in smart grid with multiple generating units via smoothing technique.

Index Terms—Smart grid, real-time pricing, generating unit, convex hull method, smoothing technique, renewable energy generation, energy storage.

I. INTRODUCTION

THE growing and diversifying demand for electricity challenges the smart grid to balance the supply and demand while scheduling generation efficiently. The complex

characteristics of cost for generating units further hinder the economical and stable operation of the power system. The unit commitment (UC) problem aims to minimize the total system cost by determining the optimal commitment and dispatch of multiple generating units for a given demand [1], [2]. In the UC problem, the cost of a generating unit includes commitment cost (e.g., startup cost) and generation cost, while decision variables encompass both discrete variables (e.g., commitment status) and continuous variables (e.g., power generation) [3]–[5]. Crucially, the inclusion of startup cost renders the cost functions of generating units discontinuous and nonconvex, making the UC problem inherently difficult to solve with conventional optimization methods [6].

In the UC problem, the discrete commitment decisions and nonconvex cost functions may lead to decentralized unit decisions based on marginal prices to deviate from the centralized optimal schedule, which results in scheduling inconsistencies [7]. To resolve this, uplift payments are essential to compensate market participants for losses, mitigate scheduling inconsistency risks, and maintain system balance. Convex hull pricing is a key method for minimizing such uplift payments and enhancing market transparency [8]–[10]. Specifically, this method proceeds by replacing the original UC problem with its convex hull and taking the slope of the convex envelope of the total cost function as the convex hull price [11]–[13]. Reference [14] first proposes the convex hull method and establishes its connection to dual theory, showing that the convex hull price equals the dual solution (i.e., the Lagrange multiplier) of the primal formulation. Given this equivalence, the calculation of the convex hull price has attracted considerable attention. Several methods have been developed to solve the dual problem of the UC problem efficiently, including subgradient simplex cutting plane method [15], extreme-point subdifferential method [16], and extreme-point enumeration method [17]. For both quadratic and piecewise linear cost functions, [18] characterizes the convex hull of the UC problem using a polyhedral method, reformulating the convexified problem as a second-order cone program and a linear program, respectively. Using the Dantzig-Wolfe decomposition method, [19] iteratively constructs the convex hull of the UC problem and solves the resulting linear program via column generation. Further extensions address the market design and practical complexities. Reference [20] establishes a UC model with non-discriminatory

Manuscript received: July 28, 2025; revised: November 4, 2025; accepted: December 21, 2025. Date of CrossCheck: December 21, 2025. Date of online publication: April 2, 2026.

This work was supported by National Natural Science Foundation of China (No. 72071130).

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DOI: 10.35833/MPCE.2025.000688



and cost-recovery constraints, and derives additional constraints by applying convex relaxation to the Karush-Kuhn-Tucker (KKT) conditions of the self-dispatch problem. To incorporate the ancillary service of frequency regulation, [21] formulates a UC model and solves it using a tractable shrunk convex hull method. A novel method in [22] integrates the machine learning with column generation to compute the convex hull price. For stochastic electricity markets, [23] develops a UC model with chance constraints and employs a linear program to calculate the convex hull price. A common observation is that the existing UC studies primarily focus on unit characteristics, simplifying the demand side by assuming a given aggregate demand to minimize the total system cost. However, the dynamic demand of users, as an essential feature of the electricity market, is not captured in these studies [24]. Consequently, the prices derived from such research do not reflect the behavior of users and thus fail to constitute an effective demand response (DR) mechanism.

DR is an effective mechanism that uses price signals or incentives to guide users in adjusting their consumption patterns, promoting efficient electricity utilization and mitigating energy waste [25]-[27]. Pricing constitutes an essential component of DR by guiding users toward more rational power consumption. The price-based DR primarily encompasses fixed pricing, time-of-use pricing, and real-time pricing. Real-time pricing, which facilitates bidirectional communication between the supply and demand, is recognized as one of the most promising pricing schemes [28], [29]. The social welfare maximization model is prevalent in real-time pricing. It aims to achieve the optimal power consumption and supply by maximizing the aggregate welfare of users and minimizing the total cost of supply side. Reference [30] introduces the social welfare maximization model for the electricity pricing problem. Building on this foundation, subsequent research has focused on adapting this model to real-time pricing and incorporating key features of modern grids [9], [31]-[37]. For instance, [31] addresses the peer-to-peer trading, while [32] explicitly models uncertainty from photovoltaic (PV) generation and user demand. Reference [33] introduces a novel utility function to capture the differences of individual user, and [34] employs a smoothing technique to solve the user risk classification problem. Further extending this research stream, [35] studies a system with users equipped with electric vehicles, establishing a multi-period model. While this body of work broadens the application scope of real-time pricing by developing realistic models, a fundamental simplification persists on the supply side. Specifically, these studies typically rely on the assumption of a single generating unit on the supply side with a quadratic or piecewise linear cost function, neglecting the discrete nature of generation costs [33], [36]. The use of such convex cost functions within electricity pricing models fails to reflect the actual characteristics of power systems, creating obstacles to the direct application of theoretical research in practical electricity markets.

To address the challenge of nonconvex costs and dynamic demand in power systems, [9] and [37] propose different social welfare maximization models and solve these models

via convex hull methods. Nevertheless, the resulting convexified problems are nonsmooth, and the prices are calculated using subgradient methods. Consequently, the electricity market may fail to provide participants with exact prices, resulting in suboptimality of social welfare.

Nonsmooth optimization problems, due to their nondifferentiable nature, render traditional gradient methods inapplicable [38]. Smoothing techniques address this by constructing a smooth approximation of the original problem, which enhances the computational tractability and efficiency [39], [40]. This approximation simplifies the computation of gradients and Hessian matrices, enabling the use of efficient algorithms to ensure the reliable global convergence and numerical stability [41], [42]. Additionally, smoothing techniques provide flexibility for algorithmic adaptation under diverse conditions. Several smoothing techniques have been applied to piecewise linear cost functions [36] and nonsmooth KKT systems [34], [43] in the context of real-time pricing, demonstrating notable numerical performance. However, being tailored to specific nonsmooth structures, these techniques cannot be directly applied to handle the nonsmoothness arising from the convex hull method in the UC problem. More fundamentally, the underlying studies overlook the inherent nonconvexity of generation costs.

The smart grid, as a modern system, requires a flexible and efficient management framework to respond to the demand of users, determine the optimal dispatch schedules, and balance the supply and demand. To effectively meet these complex system operational needs, this paper develops the convex hull method for a real-time pricing scheme in the smart grid with multiple generating units via a smoothing technique. The real-time pricing scheme aims to maximize the total welfare of users and minimize the total cost of multiple generating units, explicitly accounting for the startup costs of these units. Specifically, we employ the Moreau proximal smoothing technique [44], a well-established technique for constructing smooth approximations of nonsmooth convex functions. Notably, we apply this smoothing technique to the original problem (i.e., the convex envelope of the cost function for generating units) rather than to the dual problem, as the former ensures explicit economic implications of the solution.

The main contributions of this study are as follows.

- 1) A social welfare maximization model for real-time pricing considering the startup costs of generating units is presented for a power system that contains renewable energy generation and energy storage devices.

- 2) The convex hull of the feasible set and the convex envelope of the cost function for generating unit with startup cost are constructed. By employing the smoothing technique, the original social welfare maximization model is converted into a smooth one. Subsequently, the computation of the convex hull price is given through dual theory.

- 3) A distributed iterative algorithm on the basis of gradient projection method is presented to solve the social welfare maximization model.

II. SYSTEM MODEL

Smart grids comprising multiple generating units and sev-

eral users are considered in system modeling. Let $\mathcal{N} = \{1, 2, \dots, N\}$ denote the set of N users. The time cycle for the operation is one day, which is divided into K time slots, denoted as $\mathcal{K} = \{1, 2, \dots, K\}$. Based on the operational characteristics, the generating units are categorized into M_1 base load units $\mathcal{M}_1 = \{1, 2, \dots, M_1\}$ and M_2 peaking units $\mathcal{M}_2 = \{M_1 + 1, M_1 + 2, \dots, M_1 + M_2\}$. Base load units operate continuously to provide a stable power supply, whereas peaking units are dispatchable to supplement generation during peak periods. The set of all generating units is $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$. The power market scheduling center (PMSC) is responsible for managing the generating units and planning the power supply. Additionally, each user is equipped with the renewable energy generation and energy storage devices.

A. Demand Side

Users maximize their welfare by adjusting the power consumption and managing the renewable energy generation and energy storage devices.

1) Utility function: according to microeconomic theory, utility characterizes the satisfaction that users derive from commodity consumption, and utility functions are commonly assumed to be concave and nondecreasing [25]. In the smart grid, the utility function represents the satisfaction that users obtain from power consumption. The quadratic utility function $U(x, \omega)$ is normally adopted [33], [34]:

$$U(x, \omega) = \begin{cases} \omega x - \frac{\alpha}{2} x^2 & 0 \leq x \leq \frac{\omega}{\alpha} \\ \frac{\omega^2}{2\alpha} & x > \frac{\omega}{\alpha} \end{cases} \quad (1)$$

where x is the power consumption of users; $\omega > 0$ is an elastic coefficient that represents the consumption preference of users; and α ($0 < \alpha \leq 1$) is a parameter that reflects the sensitivity of users to consumption changes, which is predefined. The value of ω is time-varying and may differ among users in different time slots. For a fixed α , a higher value of ω indicates a higher willingness to pay, which increases the value of the utility function $U(x, \omega)$ [28].

The logarithmic utility function $U(x, \omega)$ is also adopted [43]:

$$U(x, \omega) = \begin{cases} \omega \ln x + d & x > 0 \\ 0 & x = 0 \end{cases} \quad (2)$$

where d is a constant.

2) Renewable energy generation: renewable energy generation devices installed on the demand side such as PV panels and wind turbines (WTs) are usually distributed on a small scale, which are used to satisfy the demand of users. Due to the capacity limits, the renewable energy is only consumed by users and can neither be sold to the grid nor stored in the energy storage device. The output of renewable energy ranges between the minimum and maximum generation capacities. Suppose that the output of renewable energy for user i in time slot k is given, denoted as $x_{Re,i}^k$, satisfying:

$$0 \leq x_{Re,i}^k \leq x_{Re,i}^{\max} \quad (3)$$

where $x_{Re,i}^{\max}$ is the maximum generation capacity of the renewable energy of user i .

Suppose that for the renewable energy generation devices,

the generation costs can be neglected and the costs of maintenance and depreciation need to be taken into account. Generally, the cost function of renewable energy generation is convex and increasing, which is commonly expressed as a quadratic function [29], [45]:

$$C_{Re}(x_{Re,i}^k) = \zeta_{Re,i}(x_{Re,i}^k)^2 + \gamma_{Re,i}x_{Re,i}^k \quad (4)$$

where $\zeta_{Re,i}$ and $\gamma_{Re,i}$ are the preset parameters, with $\zeta_{Re,i}$ reflecting the increasing marginal cost of renewable energy generation and $\gamma_{Re,i}$ representing the average cost per unit of renewable energy generation.

3) Energy storage: an energy storage system consists of various energy storage devices such as electric vehicles and batteries, which store power generated from the grid for later use. The operation of each device is restricted by its energy storage capacity. That is, the stored energy must be maintained between the minimum and maximum energy storage capacities. Denote b_i^k as the energy available at the end of time slot k , which is described as:

$$b_i^k = \sum_{t=1}^k x_{E,i}^t, \quad 0 \leq b_i^k \leq S_i \quad (5)$$

where S_i is the maximum energy storage capacity for user i ; and $x_{E,i}^k$ is the amount of power charged from ($x_{E,i}^k > 0$) or discharged to the grid ($x_{E,i}^k \leq 0$) by user i in time slot k .

The operational cost of an energy storage system depends on the amount and rate of charging and discharging. It is assumed that the cost function of energy storage system is convex [37], [46], which is commonly expressed as:

$$C_E(x_{E,i}^k) = \zeta_{E,i}(x_{E,i}^k)^2 + \gamma_{E,i}x_{E,i}^k \quad (6)$$

where $\zeta_{E,i}$ and $\gamma_{E,i}$ are the predetermined parameters.

Taking into account the installation of renewable energy generation and energy storage devices by users, the demand of users may fluctuate to a certain extent. Let $x_i^{k,\min}$ and $x_i^{k,\max}$ denote the minimum and maximum power consumptions of user i , respectively. The power consumption x_i^k of user i in time slot k satisfies $x_i^{k,\min} \leq x_i^k \leq x_i^{k,\max}$. In fact, the renewable energy generation is prioritized to satisfy the demand of users. Users purchase power from the PMSC when the renewable energy generation is unable to satisfy their demands. During the operation of energy storage devices, they are charged when electricity prices are low and discharged when electricity prices are high. It is assumed that the charging and discharging are not performed simultaneously. Let p^k denote the electricity price in time slot k . The total welfare of users in a time cycle is given by:

$$W_1 = \sum_{k=1}^K \sum_{i=1}^N [U(x_i^k, \omega_i^k) - C_{Re}(x_{Re,i}^k) - C_E(x_{E,i}^k) - p^k(x_i^k + x_{E,i}^k - x_{Re,i}^k)] \quad (7)$$

where the first term $U(x_i^k, \omega_i^k)$ is the utility function of user i ; the second term $C_{Re}(x_{Re,i}^k)$ and third term $C_E(x_{E,i}^k)$ are the cost functions for renewable energy generation and energy storage devices, respectively; and the last term $p^k(x_i^k + x_{E,i}^k - x_{Re,i}^k)$ is the cost of user i for power purchased from the PMSC.

B. Supply Side

The PMSC aims to maximize the total profit of multiple

generating units by determining the optimal electricity price and power supply scheme.

1) Total cost: the total cost incorporates the costs of both base load units and peaking units. The costs of base load units vary with the amount of power generation, while the costs of peaking units consist of variable costs and startup costs. The base load unit $j \in \mathcal{M}_1$ is committed within the operating cycle. For each peaking unit $j \in \mathcal{M}_2$, a binary variable v_j^k is used to indicate its commitment status: $v_j^k = 1$ if the unit is committed in time slot k , and 0 otherwise. Let L_j^k denote the output of unit j in time slot k , and we have the following constraints:

$$L_j^{\min} \leq L_j^k \leq L_j^{\max} \quad \forall j \in \mathcal{M}_1 \quad (8)$$

$$v_j^k L_j^{\min} \leq L_j^k \leq v_j^k L_j^{\max} \quad \forall j \in \mathcal{M}_2 \quad (9)$$

where $L_j^{\min}$ and $L_j^{\max}$ are the minimum and maximum outputs of unit j , respectively.

The total cost function of generating units in time slot k is expressed as:

$$C(\mathbf{L}^k, \mathbf{v}^k) = \sum_{j=1}^{M_1} C_j^1(L_j^k) + \sum_{j=M_1+1}^{M_1+M_2} C_j^2(L_j^k, v_j^k) \quad (10)$$

where $\mathbf{L}^k = [L_1^k, L_2^k, \dots, L_{M_1}^k, L_{M_1+1}^k, \dots, L_{M_1+M_2}^k]$ is the vector of outputs for generating units in time slot k ; and $\mathbf{v}^k = [v_{M_1+1}^k, v_{M_1+2}^k, \dots, v_{M_1+M_2}^k]$ is the vector of commitment statuses for peaking units in time slot k ; and $C_j^1(L_j^k)$ and $C_j^2(L_j^k, v_j^k)$ are the cost functions of base load units and peaking units, respectively.

The cost function of base load unit is given by [33]:

$$C_j^1(L_j^k) = a_j(L_j^k)^2 + b_j L_j^k + c_j \quad (11)$$

where $a_j > 0$, $b_j > 0$, and $c_j > 0$ are the cost coefficients.

The cost function of peaking unit is represented as [37]:

$$C_j^2(L_j^k, v_j^k) = \begin{cases} 0 & v_j^k = 0 \\ a_j(L_j^k)^2 + b_j L_j^k + h_j v_j^k & v_j^k = 1 \end{cases} \quad (12)$$

where $h_j > 0$ is the startup cost for peaking unit $j \in \mathcal{M}_2$. The cost function $C_j^2(L_j^k, v_j^k)$ is nonconvex and nonsmooth due to the binary variables v_j^k .

2) Total profit: the PMSC maximizes the total profit of generating units through pricing and dispatch schemes based on the demand of users. For a given price vector $\mathbf{p} = [p^1, p^2, \dots, p^K]$, the total profit is expressed as:

$$W_2 = \sum_{k=1}^K \left[\sum_{i=1}^N p^k (x_i^k + x_{E,i}^k - x_{Re,i}^k) - \left(\sum_{j=1}^{M_1} C_j^1(L_j^k) + \sum_{j=M_1+1}^{M_1+M_2} C_j^2(L_j^k, v_j^k) \right) \right] \quad (13)$$

where the first term on the right hand is the profit from selling power to users; and the second term is the total cost of generating units.

C. Social Welfare Maximization Model

We consider the supply and demand sides as a social system, with the total social welfare defined as the sum of the aggregate welfare of users and the total profit of multiple generating units. Therefore, we formulate a social welfare maximization model as:

P1:

$$\max_{x_i^k, x_{E,i}^k, L_j^k, v_j^k} \sum_{k=1}^K \left[\sum_{i=1}^N (U(x_i^k, \omega_i^k) - C_{Re}(x_{Re,i}^k) - C_E(x_{E,i}^k)) - \left(\sum_{j=1}^{M_1} C_j^1(L_j^k) + \sum_{j=M_1+1}^{M_1+M_2} C_j^2(L_j^k, v_j^k) \right) \right] \quad (14)$$

s.t.

$$\sum_{i=1}^N (x_i^k + x_{E,i}^k - x_{Re,i}^k) \leq \sum_{j=1}^{M_1+M_2} L_j^k \quad \forall k \in \mathcal{K} \quad (15)$$

$$x_i^{\min} \leq x_i^k \leq x_i^{\max} \quad \forall i \in \mathcal{N}, \forall k \in \mathcal{K} \quad (16)$$

$$0 \leq \sum_{k=1}^K x_{E,i}^k \leq S_i \quad \forall i \in \mathcal{N} \quad (17)$$

$$L_j^{\min} \leq L_j^k \leq L_j^{\max} \quad \forall j \in \mathcal{M}_1, \forall k \in \mathcal{K} \quad (18)$$

$$v_j^k L_j^{\min} \leq L_j^k \leq v_j^k L_j^{\max} \quad \forall j \in \mathcal{M}_2, \forall k \in \mathcal{K} \quad (19)$$

$$v_j^k \in \{0, 1\} \quad \forall j \in \mathcal{M}_2, \forall k \in \mathcal{K} \quad (20)$$

Constraint (15) guarantees that the power supply should cover the aggregate demand. Constraint (16) bounds the consumption of each user between its minimum and maximum values. Under constraint (17), the stored energy of the battery over all time slots is between 0 and S_i . In constraints (18)-(20), the output of each generating unit is bounded by its minimum and maximum outputs.

Let X , Y_1 , and Y_2 denote the feasible sets for the decision variables. Specifically, X is the set for power consumptions of users, Y_1 is the set for outputs of base load units, and Y_2 is the set for outputs and commitment decisions of peaking units. These sets are defined as:

$$\begin{cases} X = \{(x_i^k, x_{E,i}^k) | (16), (17)\} \\ Y_1 = \{L_j^k | (18)\} \\ Y_2 = \{(L_j^k, v_j^k) | (19), (20)\} \end{cases} \quad (21)$$

The cost function $C_j^2(L_j^k, v_j^k)$ of the peaking unit $j \in \mathcal{M}_2$ is lower semicontinuous and nonconvex due to the binary variable v_j^k , rendering the problem P1 nonconvex and nonsmooth.

III. PROBLEM TRANSFORMATION

To solve the problem P1 and determine the optimal price, we employ the convex hull method. This method relaxes the problem P1 into a convex problem by constructing the convex hull of the feasible set and the convex envelope of the cost function for each generating unit. Since the cost functions of the base load units ($j \in \mathcal{M}_1$) are convex and do not contain startup costs, the convex hull construction is applied only to the peaking units ($j \in \mathcal{M}_2$). In the implementation process, the feasible set for each peaking unit is replaced with its convex hull, and each cost function is replaced by its convex envelope. However, the resulting convex envelope for each peaking unit is nonsmooth, which precludes the calculation of an exact electricity price. To address this, we apply a smoothing technique to obtain a smooth approximation of the constructed convex envelope.

A. Convex Hull Method

For a clear exposition, we first introduce the convex hull

method. Let $\text{conv}(Y_2)$ denote the convex hull of Y_2 , and let $C_{j,Y_2}^{2,**}(L_j^k, v_j^k)$ denote the convex envelope of the cost function $C_j^2(L_j^k, v_j^k)$ over Y_2 . This convex envelope is the largest convex function on $\text{conv}(Y_2)$ that underestimates $C_j^2(L_j^k, v_j^k)$ over Y_2 , and is equal to the biconjugate of $C_j^2(L_j^k, v_j^k)$.

According to [18], we have the following theorem.

Theorem 1 The convex envelope of the cost function $C_j^2(L_j^k, v_j^k)$ over Y_2 is the function $C_{j,Y_2}^{2,**}(L_j^k, v_j^k): \text{conv}(Y_2) \rightarrow \mathbb{R}$, defined by:

$$C_{j,Y_2}^{2,**}(L_j^k, v_j^k) = \begin{cases} 0 & v_j^k = 0 \\ a_j \frac{(L_j^k)^2}{v_j^k} + b_j L_j^k + h_j v_j^k & v_j^k > 0 \end{cases} \quad (22)$$

where $\text{conv}(Y_2) = \{(L_j^k, v_j^k) | (20), v_j^k \in \{0, 1\}, \forall j \in \mathcal{M}_2\}$.

Theorem 1 provides an analytical expression for the function $C_{j,Y_2}^{2,**}(L_j^k, v_j^k)$. The graph of the function $C_j^2(L_j^k, v_j^k)$ comprises a point $(0, 0, 0)$ and a curve $a_j(L_j^k)^2 + b_j L_j^k + h_j v_j^k$ on the plane $v_j^k = 1$. By relaxing the binary variables v_j^k and applying the interpolation formula from Theorem 1, we preserve the values of the cost function $C_j^2(L_j^k, v_j^k)$ at feasible integer points and provide a convex surrogate function when the variable v_j^k is a fraction.

B. Smoothing Technique

Using the convex hull method, we obtain the convex envelope of the cost function $C_j^2(L_j^k, v_j^k)$, i.e., $C_{j,Y_2}^{2,**}(L_j^k, v_j^k)$. However, this convex envelope is nonsmooth. To address this, we apply a smoothing technique to transform it into a smooth function. Note that the function $C_{j,Y_2}^{2,**}(L_j^k, v_j^k)$ is a bivariate function. The established smoothing techniques for such functions include Nesterov's smoothing technique and the Moreau proximal smoothing technique. Nesterov's smoothing technique, which is applicable when a function can be represented as a specific "max" type function, cannot be applied to the constructed convex envelope [47]. In contrast, the Moreau proximal smoothing technique is appropriate for any nonsmooth convex function while preserving convexity, and it yields a globally smooth approximation with a computable gradient [39].

Definition 1 For a function $f(y): Q \rightarrow \mathbb{R}$, the Moreau proximal smoothing approximation of $f(y)$ with the parameter $\mu > 0$ is defined as:

$$f_\mu(y) = \inf_{r \in Q} \left\{ f(r) + \frac{\|y - r\|^2}{2\mu} \right\} \quad (23)$$

where $Q \subseteq \mathbb{R}^d$.

Equation (23) defines the Moreau proximal smoothing approximation of $f(y)$ via a minimization problem with a quadratic regularization term. For a convex function $f(y)$, $f_\mu(y)$ is parameterized by a positive scalar μ and approximates $f(y)$ from below [42].

For a continuous function $f(y)$ on a closed and bounded set Q , the Moreau proximal smoothing approximation $f_\mu(y)$ is given by:

$$f_\mu(y) = \min_{r \in Q} \left\{ f(r) + \frac{\|y - r\|^2}{2\mu} \right\} \quad (24)$$

Proposition 1 If $f(y)$ is convex, continuous, and bounded

from below, then the minimization problem defined in (23) is strongly convex. According to the convex optimization theory, this problem has a unique solution $q(y)$ that is an approximation point to y . Then, for every $\mu > 0$, the function $f_\mu(y)$ has the following properties.

1) The function $f_\mu(y)$ is convex and continuously differentiable:

$$\nabla f_\mu(y) = \frac{1}{\mu} (y - q(y)) \quad (25)$$

2) The function $f_\mu(y)$ also satisfies:

$$\lim_{\mu \rightarrow 0} f_\mu(y) = f(y) \quad (26)$$

Definition 1 and Proposition 1 indicate that $C_{j,Y_2}^{2,**}(L_j^k, v_j^k)$ can be approximated by a continuously differentiable convex function with the following form:

$$C_{j,Y_2,\mu_j}^{2,**}(L_j^k, v_j^k) = \min_{r \in \text{conv}(Y_2)} \left\{ C_{j,Y_2}^{2,**}(r) + \frac{\|(L_j^k, v_j^k) - r\|^2}{2\mu_j} \right\} \quad (27)$$

where the parameter $\mu_j > 0$.

The accuracy of approximation between $C_{j,Y_2,\mu_j}^{2,**}(L_j^k, v_j^k)$ and $C_{j,Y_2}^{2,**}(L_j^k, v_j^k)$ depends on the parameter μ_j . As the value of μ_j decreases, $|C_{j,Y_2}^{2,**}(L_j^k, v_j^k) - C_{j,Y_2,\mu_j}^{2,**}(L_j^k, v_j^k)|$ decreases. For a specific implementation, we select the appropriate parameter μ_j according to the required computational accuracy.

The Moreau proximal smoothing technique is employed to construct a smooth function with the same domain as the original function. The constructed smooth function serves as a lower bound that sufficiently approximates the original function, thereby ensuring that the optimal solution of the smooth problem closely approximates that of the original problem. Since the function $C_{j,Y_2}^{2,**}(L_j^k, v_j^k) \geq 0$ is convex and continuous, Proposition 1 guarantees that $\lim_{\mu_j \rightarrow 0} C_{j,Y_2,\mu_j}^{2,**}(L_j^k, v_j^k) = C_{j,Y_2}^{2,**}(L_j^k, v_j^k)$ for $\mu_j > 0$. The relevant theoretical proofs are provided in [40].

C. Smooth Social Welfare Maximization Model

Based on the above discussion, we replace the cost function $C_j^2(L_j^k, v_j^k)$ of peaking unit $j \in \mathcal{M}_2$ with the smooth function $C_{j,Y_2,\mu_j}^{2,**}(L_j^k, v_j^k)$ and convert the original problem P1 into the following form:

P2:

$$\begin{cases} \max_{x_i^k, x_{E,i}^k, L_j^k, v_j^k} \sum_{k=1}^K \left[\sum_{i=1}^N (U(x_i^k, \omega_i^k) - C_{Re}(x_{Re,i}^k) - C_E(x_{E,i}^k)) - \left(\sum_{j=1}^{M_1} C_j^1(L_j^k) + \sum_{j=M_1+1}^{M_1+M_2} C_{j,Y_2,\mu_j}^{2,**}(L_j^k, v_j^k) \right) \right] \\ \text{s.t.} \quad \sum_{i=1}^N (x_i^k + x_{E,i}^k - x_{Re,i}^k) \leq \sum_{j=1}^{M_1+M_2} L_j^k \\ (x_i^k, x_{E,i}^k) \in X \\ L_j^k \in Y_1 \quad \forall j \in \mathcal{M}_1 \\ (L_j^k, v_j^k) \in \text{conv}(Y_2) \quad \forall j \in \mathcal{M}_2 \end{cases} \quad (28)$$

By Proposition 1, the problem P2 is a convex and smooth problem that can be solved to obtain an approximate solu-

tion to the problem P1.

IV. SOLUTION METHOD

The problem P2 is convex and can be solved using classical optimization methods. However, it does not explicitly include a variable related to the electricity price, meaning that the electricity price cannot be obtained by solving the problem P2 directly. To resolve the problem P2 in a distributed manner and obtain the optimal electricity price, we adopt the Lagrange dual gradient projection (LDGP) method, in which the Lagrange multiplier corresponding to the system supply and demand constraint serves as the electricity price.

The Lagrange function of the problem P2 is:

$$L(\mathbf{z}, \boldsymbol{\lambda}) = \sum_{k=1}^K \left\{ \sum_{i=1}^N (U(x_i^k, \omega_i^k) - C_{Re}(x_{Re,i}^k) - C_E(x_{E,i}^k)) - \left(\sum_{j=1}^{M_1} C_j^1(L_j^k) + \sum_{j=M_1+1}^{M_1+M_2} C_{j,Y_2,\mu_j}^{2,**}(L_j^k, v_j^k) \right) - \lambda^k \left[\sum_{i=1}^N (x_i^k + x_{E,i}^k - x_{Re,i}^k) - \sum_{j=1}^{M_1+M_2} L_j^k \right] \right\} \quad (29)$$

where $\mathbf{z} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N, \mathbf{x}_{E,1}, \mathbf{x}_{E,2}, \dots, \mathbf{x}_{E,N}, \mathbf{L}_1, \mathbf{L}_2, \dots, \mathbf{L}_{M_1+M_2}, \mathbf{v}_{M_1+1}, \mathbf{v}_{M_1+2}, \dots, \mathbf{v}_{M_1+M_2}]$ and $\boldsymbol{\lambda} = [\lambda^1, \lambda^2, \dots, \lambda^K]$ represent the primal and dual variables, respectively, $\mathbf{x}_i = [x_i^1, x_i^2, \dots, x_i^K]$, $\mathbf{x}_{E,i} = [x_{E,i}^1, x_{E,i}^2, \dots, x_{E,i}^K]$, $\mathbf{L}_j = [L_j^1, L_j^2, \dots, L_j^K]$, $\mathbf{v}_j = [v_j^1, v_j^2, \dots, v_j^K]$ and λ^k is the Lagrange multiplier corresponding to system supply and demand constraint in time slot k . Additionally, the dual problem for the problem P2 is given by:

$$\min_{\lambda \geq 0} H(\lambda) \quad (30)$$

where $H(\lambda)$ can be expressed as:

$$H(\lambda) = \max_{\mathbf{z}} L(\mathbf{z}, \boldsymbol{\lambda}) \quad (31)$$

The process of solving problem (31) is given as:

$$(\mathbf{x}_i^*, \mathbf{x}_{E,i}^*) = \arg \max_{\mathbf{x}_i, \mathbf{x}_{E,i}} \sum_{k=1}^K \sum_{i=1}^N [U(x_i^k, \omega_i^k) - C_{Re}(x_{Re,i}^k) - C_E(x_{E,i}^k) - \lambda^k (x_i^k + x_{E,i}^k - x_{Re,i}^k)] \quad (32)$$

$$(\mathbf{L}_j^*, \mathbf{v}_j^*) =$$

$$\arg \max_{\mathbf{L}_j, \mathbf{v}_j} \sum_{k=1}^K \left[\sum_{j=1}^{M_1+M_2} \lambda^k L_j^k - \left(\sum_{j=1}^{M_1} C_j^1(L_j^k) + \sum_{j=M_1+1}^{M_1+M_2} C_{j,Y_2,\mu_j}^{2,**}(L_j^k, v_j^k) \right) \right] \quad (33)$$

where * represents the optimal solution. Equations (32) and (33) represent the subproblems of the demand side and supply side, respectively.

The problem P2 is a convex optimization problem with inequality constraints. It possesses a nonempty feasible set and satisfies the Slater constraint qualification, which ensures that the strong duality holds between the problem P2 and its dual problem (30). To solve this dual problem, we employ the gradient projection method as:

$$\lambda_{t+1} = [\lambda_t - \rho \nabla H(\lambda)]^+ = \left[\lambda_t - \rho \left(\sum_{j=1}^{M_1+M_2} \mathbf{L}_j^*(\lambda_t) - \sum_{i=1}^N (\mathbf{x}_i^*(\lambda_t) + \mathbf{x}_{E,i}^*(\lambda_t) - \mathbf{x}_{Re,i}) \right) \right]^+ \quad (34)$$

where $t \in \tau$ is the iteration number, and τ is the set of iteration numbers; $\rho > 0$ is the step size of the gradient projection method; $[\cdot]^+ = \max\{\cdot, 0\}$; $\mathbf{L}_j^*(\lambda_t) = [L_j^{1,*}(\lambda_t^1), L_j^{2,*}(\lambda_t^2), \dots, L_j^{K,*}(\lambda_t^K)]$; $\mathbf{x}_i^*(\lambda_t) = [x_i^{1,*}(\lambda_t^1), x_i^{2,*}(\lambda_t^2), \dots, x_i^{K,*}(\lambda_t^K)]$; $\mathbf{x}_{Re,i} = [x_{Re,i}^1, x_{Re,i}^2, \dots, x_{Re,i}^K]$; and $\mathbf{x}_{E,i}^*(\lambda_t) = [x_{E,i}^{1,*}(\lambda_t^1), x_{E,i}^{2,*}(\lambda_t^2), \dots, x_{E,i}^{K,*}(\lambda_t^K)]$. By solving the dual problem (30), we obtain the solution λ_p , i.e., the electricity price.

The real-time pricing scheme reflects the dynamic interaction between the supply and demand. When the price is updated, the users and PMSC maximize their benefits by utilizing the price to determine the power consumption and generation, respectively. Crucially, the distributed iterative algorithm is performed without requiring the PMSC to collect or estimate the private parameters of users such as the coefficients of utility functions, the capacities and charging/discharging rates of energy storage devices, and the output of renewable energy generation. The specific execution process of the distributed iterative algorithm is described in Algorithm 1 and Algorithm 2.

Algorithm 1: distributed iterative algorithm performed by each user i

Input: electricity price λ_i

Output: power consumptions $\mathbf{x}_i^*(\lambda_t)$ and $\mathbf{x}_{E,i}^*(\lambda_t)$

Step 1: measure $\mathbf{x}_{Re,i}$ (initialization)

Step 2: **for** each $t \in \tau$ **do**

Step 3: Update $\mathbf{x}_i^*(\lambda_t)$ and $\mathbf{x}_{E,i}^*(\lambda_t)$ by solving (32) for the given λ_t

Step 4: Send the updated $\mathbf{x}_i^*(\lambda_t) + \mathbf{x}_{E,i}^*(\lambda_t) - \mathbf{x}_{Re,i}$ to PMSC

Step 5: **end for**

Algorithm 2: distributed iterative algorithm performed by PMSC

Input: $\mathbf{x}_i^*(\lambda_t) + \mathbf{x}_{E,i}^*(\lambda_t) - \mathbf{x}_{Re,i}$

Output: updated electricity price λ_{t+1}

Step 1: input initial electricity price λ_0 , ρ , and tolerable error ε (initialization)

Step 2: **for** each $t \in \tau$ **do**

Step 3: Update $\mathbf{L}_j^*(\lambda_t)$ by solving (33)

Step 4: Calculate λ_{t+1} via (34)

Step 5: If $\|\lambda_{t+1} - \lambda_t\| < \varepsilon$, terminate; else, send the updated electricity price λ_{t+1} to all users

Step 6: **end for**

The problem P2 has a smooth objective and linear inequality constraints, ensuring that the dual problem (30) is continuously differentiable. Moreover, the gradient of objective function of the dual problem is Lipschitz continuous with constant $S > 0$, and the sequence $\{\lambda_t\}$ is bounded. According to [48], the sequence $\{\lambda_t\}$ converges to the optimal point λ^* with a constant step size ρ satisfying $0 < \rho < 2/S$. At the same time, the optimal solutions $(\mathbf{x}_i^*, \mathbf{x}_{E,i}^*) = (\mathbf{x}_i^*(\lambda^*), \mathbf{x}_{E,i}^*(\lambda^*))$, and $(\mathbf{L}_j^*, \mathbf{v}_j^*) = (\mathbf{L}_j^*(\lambda^*), \mathbf{v}_j^*(\lambda^*))$ are obtained on the demand side and supply side by solving subproblems (32) and (33), respectively.

V. SIMULATION EXPERIMENTS

Simulation experiments are conducted to evaluate the performance of the real-time pricing scheme. All simulations are performed in MATLAB R2023b environment running on

an Intel Core i5 processor (3.6 GHz) and 32 GB memory.

A. Simulation Setup

We consider a simulation system comprising 10 generating units and 100 users ($N=100$). The time cycle is one day that is divided into 48 time slots, each with a duration of 30 min, i.e., $K=48$. The generating units are categorized into six ($\mathcal{M}_1=\{1,2,3,4,5,6\}$) base load units and four ($\mathcal{M}_2=\{7,8,9,10\}$) peaking units. We adopt the modification example of the units in [3]. The cost functions of the generating units are given by (11) and (12) with the relevant cost parameters listed in Table I. The utility function is given in

TABLE I
RELEVANT COST PARAMETERS

Unit	The maximum output (kW)	The minimum output (kW)	h_j (CNY)	a (CNY)	b (CNY)	c (CNY)
Unit 1	126	25		0.0022	0.28	3
Unit 2	105	15		0.0025	0.21	2
Unit 3	100	20		0.0036	0.32	3
Unit 4	100	20		0.0040	0.23	3
Unit 5	85	15		0.0042	0.35	2
Unit 6	90	10		0.0038	0.34	2
Unit 7	100	10	3	0.0052	0.58	
Unit 8	90	15	4	0.0051	0.64	
Unit 9	80	0	4	0.0063	0.84	
Unit 10	70	0	3	0.0078	0.88	

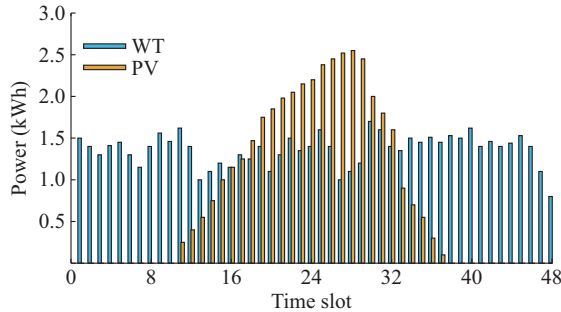


Fig. 1. Power outputs of PV and WT for user 10 during a day.

B. Results Analysis

Given an initial electricity price λ_0 with each component λ_0^k ($k=1,2,\dots,K$) randomly chosen from $[0.5,1.8]$, we iteratively solve the dual problem (30) to compute λ^* , $\mathbf{x}_i^*$, $\mathbf{x}_{E,i}^*$, and $\mathbf{L}_j^*$.

Figure 2 illustrates the optimal electricity price derived through the distributed iterative algorithm. Under this optimal price, the power system achieves supply and demand balance, as illustrated in Fig. 3, indicating the avoidance of energy waste. The corresponding optimal dispatch scheme is presented in Fig. 4, which shows the schedules for power consumption, power supply, renewable energy generation, and charging and discharging power of energy storage device. Notably, the total power consumption is higher than the power supply, implying that users utilize renewable energy and energy storage to meet the demand. For a more detailed view of the optimal dispatch scheme for users, Fig. 5

(1), where $\alpha=0.25$ and ω is generated randomly from the interval $[1,10]$. The demand of each user is between 5 and 15 kWh. Each user is equipped with an energy storage device (10 kW capacity), a solar PV (3 kW capacity), and a WT (3 kW capacity). Based on the statistical features [49], Fig. 1 illustrates the power outputs of the PV and WT for user 10 during a day. The cost parameters of the renewable energy generation and energy storage devices are $\zeta_{Re,i}=0.1$, $\gamma_{Re,i}=0$, $\zeta_{E,i}=0.1$, and $\gamma_{E,i}=0$ [46]. The smoothing parameters are $\mu_7=\mu_8=\mu_9=\mu_{10}=0.001$. For the gradient projection method, the step size is $\rho=0.001$ and the tolerable error is $\varepsilon=0.001$.

takes user 10 as an example to present the power consumption, renewable energy generation, charging and discharging power of energy storage device during an operation cycle. Additionally, the optimal dispatch scheme of generating units is displayed in Fig. 6.

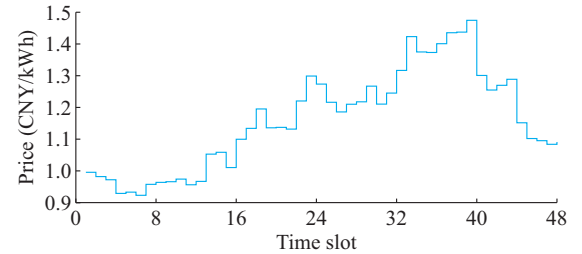


Fig. 2. Optimal electricity price derived through distributed iterative algorithm.

Figures 2-6 demonstrate that the variations in electricity price are closely correlated with consumption preferences of users, renewable energy generation, the charging and discharging power of energy storage device, and the total cost of units. During the low consumption periods, e.g., time slots 1-15 and 45-48, the electricity prices are lower and the energy storage devices are charged. Meanwhile, the generating units operate at a low power output. Conversely, during the peak consumption periods, e.g., time slots 22-25, 29, and 31-43, the electricity prices are higher, and the energy storage devices are discharged. These results indicate that users respond to price changes and actively participate in electricity market.

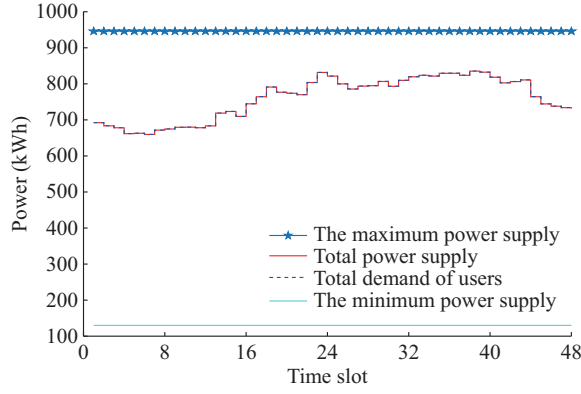


Fig. 3. Illustration of supply and demand balance.

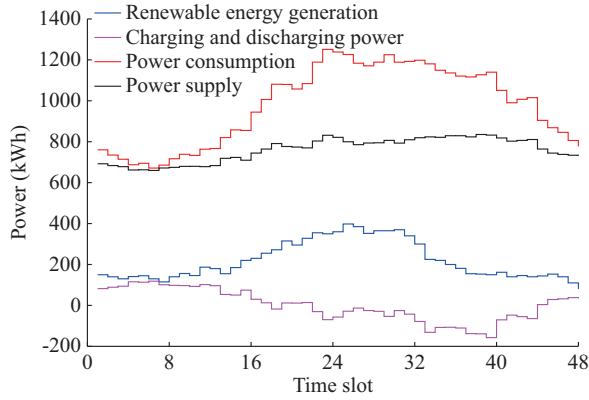


Fig. 4. Schedules for power consumption, power supply, renewable energy generation, and charging and discharging power of energy storage device.

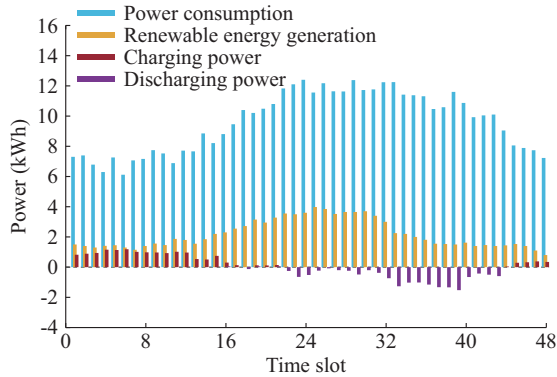


Fig. 5. Optimal dispatch scheme for user 10.

C. Comparison of Different Pricing Schemes

To evaluate the performance of different pricing schemes, we compare the peak shaving and valley filling effects and social welfare under three schemes: real-time pricing (proposed), time-of-use pricing, and fixed pricing. For the time-of-use pricing scheme, one day is divided into on-peak (time slots 32-40), shoulder-peak (time slots 16-31 and 41-45), and off-peak (time slots 1-15 and 46-48) periods [27]. For the fixed pricing scheme, the total power consumption remains below the total power supply, and the price p_{fixed}^k is determined as a stable point based on the demand of users [37], [50]:

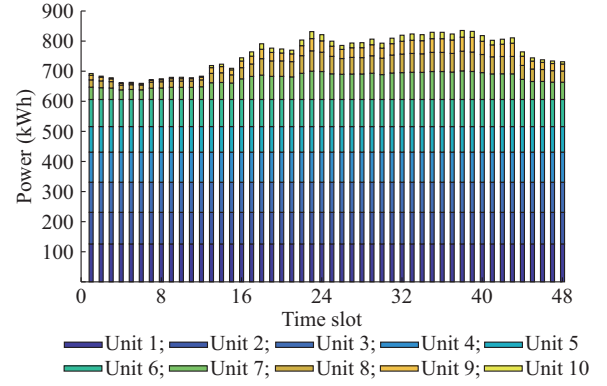


Fig. 6. Optimal dispatch scheme for generating units.

$$p_{fixed}^k = \omega_{i,max}^k - \frac{\alpha L_{fixed}^k}{N} \quad (35)$$

where $\omega_{i,max}^k$ is the maximum value of ω_i^k ; and L_{fixed}^k is the power supply in time slot k under the fixed pricing scheme.

Figure 7 compares the grid load among three pricing schemes. It shows that under the real-time pricing scheme, the grid load is the lowest during the peak periods and exhibits the smallest fluctuations, indicating that the real-time pricing scheme effectively guides users to adjust the consumption and participate in market operations.

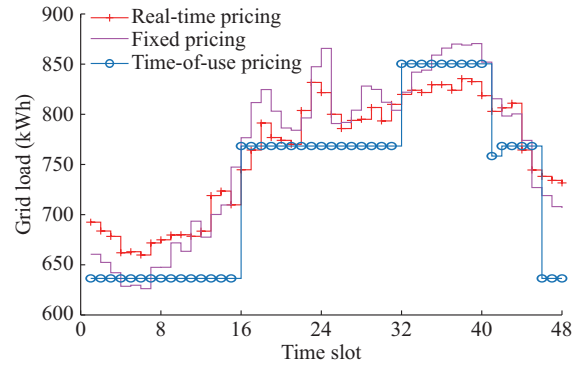


Fig. 7. Comparison of grid load among three pricing schemes.

Table II compares the operational and economic performance of the three pricing schemes using five metrics: grid standard deviation, peak-to-valley difference, the profit of supply side, the welfare of demand side, and social welfare. The grid standard deviation quantifies load fluctuation, with a smaller value indicating a more stable load profile. The peak-to-valley difference of grid load is defined as the gap between the maximum and minimum load values within a time cycle. In Table II, the real-time pricing scheme achieves the lowest grid standard deviation and peak-to-valley difference among the three pricing schemes. This indicates the flattest load profile under the real-time pricing scheme, demonstrating its greatest effectiveness in peak shaving and valley filling. Furthermore, the real-time pricing scheme yields the highest profit of the supply side, the highest welfare of the demand side, and the highest social welfare. Figure 8 depicts the social welfare under three pricing schemes. These results demonstrate the superior performance of real-time pricing scheme in flattening the load profile and improving the social welfare.

TABLE II
OPERATIONAL AND ECONOMIC PERFORMANCE UNDER THREE PRICING SCHEMES

Pricing scheme	Grid standard deviation	Peak-to-valley difference	Profit of supply side (CNY)	Welfare of demand side (CNY)	Social welfare (CNY)
Real-time	59.00	175.81	16175.70	71985.03	88160.73
Fixed	78.86	244.26	12847.76	68065.76	80913.52
Time-of-use	82.16	213.88	14704.52	49437.02	64141.54

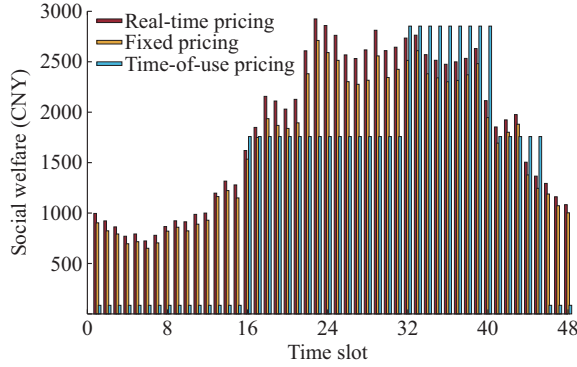


Fig. 8. Comparison of social welfare among three pricing schemes.

Compared with the fixed pricing and time-of-use pricing schemes, the real-time pricing scheme reduces the grid standard deviation by 25.18% and 28.18%, decreases the peak-to-valley difference by 28.02% and 17.79%, and increases the social welfare by 8.22% and 27.24%, respectively. These results demonstrate that the real-time pricing scheme is more effective in ensuring the grid operational stability and enhancing the economic efficiency.

D. Comparison of Smoothing Technique and Nonsmoothing Technique

In order to demonstrate the advantages of the LDGP method for solving the problem P2, we compare the results with those by solving the following nonsmooth social welfare maximization model:

$\hat{P}2$:

$$\begin{cases} \max \sum_{k=1}^K \left[\sum_{i=1}^N (U(x_i^k, \omega_i^k) - C_{Re}(x_{Re,i}^k) - C_E(x_{E,i}^k)) - \left(\sum_{j=1}^{M_1} C_j^1(L_j^k) + \sum_{j=M_1+1}^{M_1+M_2} C_{j,Y_2}^{2,**}(L_j^k, v_j^k) \right) \right] \\ \text{s.t. } \sum_{i=1}^N (x_i^k + x_{E,i}^k - x_{Re,i}^k) \leq \sum_{j=1}^{M_1+M_2} L_j^k \\ (x_i^k, x_{E,i}^k) \in X \\ L_j^k \in Y_1 \quad \forall j \in \mathcal{M}_1 \\ (L_j^k, v_j^k) \in \text{conv}(Y_2) \quad \forall j \in \mathcal{M}_2 \end{cases} \quad (36)$$

The problem $\hat{P}2$ is addressed using the Lagrange dual sub-gradient projection (LDSPG) method. The problems $\hat{P}2$ and P2 are solved with the same termination criterion $\|\lambda_{t+1} - \lambda_t\| < \varepsilon$.

Figures 9-11 compare the iterations of electricity price, to-

tal demand of users, and total power supply with the LDGP and LDSGP methods in time slot 36. These figures and Table III indicate that the solutions to problem P2 converge faster than those of problem $\hat{P}2$. Furthermore, Fig. 12 demonstrates that the problem P2 achieves higher social welfare. These results illustrate that the smoothing technique effectively improves the computational efficiency and social welfare.

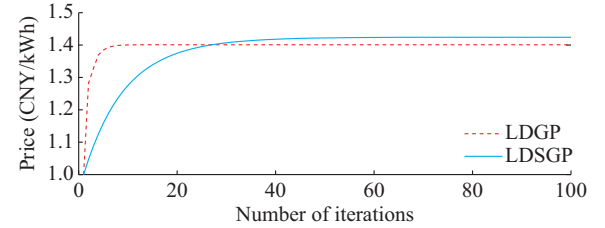


Fig. 9. Iterations of electricity price with LDGP and LDSGP methods.

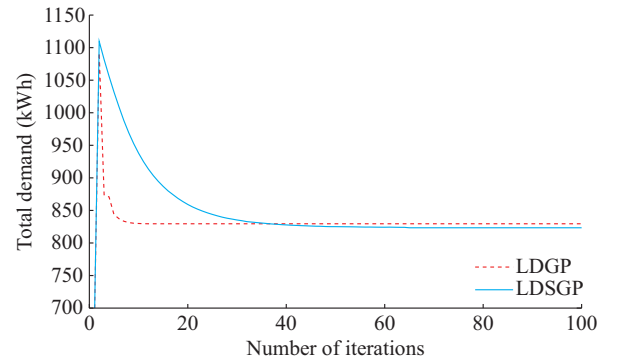


Fig. 10. Iterations of total demand of users with LDGP and LDSGP methods.

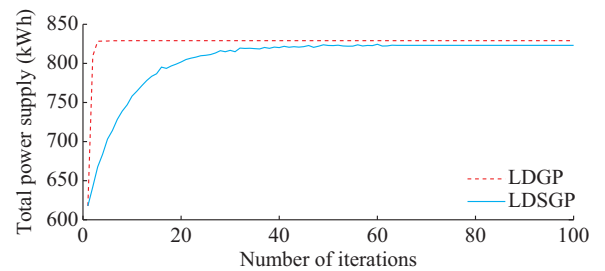


Fig. 11. Iterations of total power supply with LDGP and LDSGP methods.

TABLE III
COMPARISON OF PERFORMANCE BETWEEN TWO SOLUTION METHODS

Method	Number of iterations	Social welfare (CNY)
LDGP	13	88160.73
LDSGP	66	85947.53

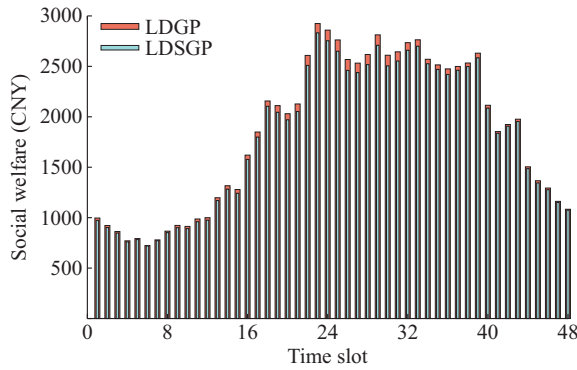


Fig. 12. Comparison of social welfare between LDGP and LDSGP methods.

E. Comparison by Solving Different Problems

To further evaluate the quality of the solution, we compare the social welfare achieved by the problems P1 and P2. The problem P1 is solved using the BONMIN solver (v1.8.6).

Figure 13 compares the social welfare achieved by solving the problems P1 and P2. The values of social welfare are 84745.54 CNY for the problem P1 and 88160.73 CNY for the problem P2. The results show that the problem P2 achieves a 3.87% higher social welfare, demonstrating that its solution closely approximates that of the original nonconvex problem. This indicates that the convex hull method for real-time pricing via the smoothing technique provides a favorable approximation to the original problem.

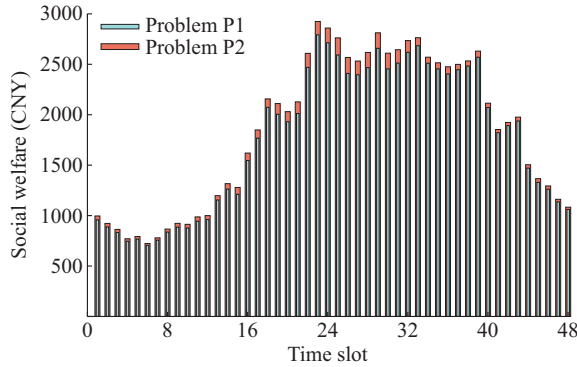


Fig. 13. Comparison of social welfare by solving problem P1 and P2.

VI. CONCLUSION

This paper presents a real-time pricing scheme to maximize the social welfare in the smart grid that consists of multiple generating units and several users, with each user equipped with renewable energy generation and energy storage devices. To address the discontinuity and nonconvexity of cost functions, the convex hull method is employed to construct the convex hull of the feasible set and the convex envelope of the cost function for each generating unit with startup cost. Subsequently, the original problem is transformed into a smooth problem using the Moreau proximal smoothing technique. Based on the dual theory, a distributed iterative algorithm is developed to compute the optimal price. Numerical simulations demonstrate the feasibility and

effectiveness of the convex hull method. The results show that the real-time pricing scheme yields an optimal energy dispatch while maximizing the social welfare. Compared with the fixed pricing and time-of-use pricing schemes, the real-time pricing scheme achieves better performance in peak shaving and valley filling and yields higher social welfare, effectively ensuring the grid stability. The comparisons with the nonsmoothing technique indicate that the smoothing technique improves the computational efficiency and the resulting smooth problem offers higher social welfare. Furthermore, the comparison with the original nonconvex problem shows that the convex hull method for real-time pricing via the smoothing technique provides a close approximation to the original problem.

The real-time pricing scheme and solution method demonstrate significant potential for the application in existing electricity markets. This pricing scheme dynamically balances the supply and demand through prices and provides a scalable scheme for the real-time electricity market to integrate a high penetration of distributed renewable energy. The distributed iterative algorithm allows users equipped with renewable energy generation and energy storage devices to respond to price signals while preserving their privacy. This feature directly supports current demand response policies that aggregate distributed resources into virtual power plants for system regulation. Furthermore, the convex hull method for real-time pricing via the smoothing technique yields accurate convex hull price, enabling a precise economic evaluation of the UC problem in the day-ahead market and consequently leading to market clearing outcomes with higher social welfare.

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