

Integral Quadratic Constraint Based Framework for Robust Control Synthesis and Analysis in Microgrid

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Abstract—The transition to sustainable energy systems demands robust and reliable control strategies to ensure stability and performance under dynamic and uncertain conditions. This paper presents an integral quadratic constraint (IQC) based framework for robust control synthesis and analysis in a microgrid, with guarantees of robust stability and performance. The IQC theory offers a powerful mathematical framework for analyzing the impact of uncertainties and nonlinearities in energy systems, providing more generalized and flexible tools compared to the widely used structured singular value i.e., μ -synthesis. The application in the microgrid control system synthesis demonstrates the superior performance and robustness of the proposed IQC-based framework in the presence of parameter variations and exogenous disturbances. This paper bridges the gap between advanced control theory and its practical applications in microgrids, offering a new perspective for researchers and stakeholders seeking innovative solutions for the control system design and analysis.

Index Terms—Integral quadratic constraint (IQC), robust stability, robust control synthesis, performance assessment, microgrid, control system design, dynamic system analysis.

I. INTRODUCTION

DUE to the development of renewable energy sources (RESs), a new era of environmentally friendly and sustainable power generation has emerged on the global energy landscape. These unlimited natural resources are transforming traditional power grids from fossil-based energies to RESs such as wind and solar energy [1]. However, the true potential of RESs lies not only in their ability to generate electricity but also in their intelligent integration into localized grids known as microgrids (MGs). Grid-connected converters (GCCs), as the key block of the MGs, provide unique resilience and flexibility. The GCCs enable MGs to operate correctly by seamlessly combining RESs with traditional power grids and customers, thereby protecting communities from the risks of centralized power grids. The conver-

gence of RESs, MG architecture, and sophisticated GCCs will enable universal access to clean and reliable electricity, empowering societies to progress in harmony with the natural world [2].

With the increasing application of MGs and their crucial role in the power generation revolution, it is important to note that MGs require appropriate control systems. Indeed, stability, power quality, and continuous power supply in the power grid depend on proper control methods that are well-versed in the complexities of grid dynamics and RES behaviors. MGs include converters that are working in the grid-forming (GFM) operation mode and are equipped with effective control strategies, accurately managing the energy flow to ensure reliable and efficient system operation [2]. Thus, MGs can properly respond to variations, disturbances, and uncertainties. A resilient and up-to-date modern power grid could operate with the development of reliable control strategies for MGs, which is a major step in the future power grid revolution [1].

The development of effective controllers to ensure the MG stability and performance has gained significant attention. Robust control frameworks are widely used for designing such controllers, especially in GFM converters, to manage uncertainties and disturbances [3], [4]. In [5], a sliding mode control scheme is proposed for direct-current (DC) MGs with nonlinear constant power loads, achieving global stability and decentralized power sharing. It employs a high-order finite-time observer to mitigate disturbances and integrates robust sliding mode with droop control for voltage regulation and power sharing.

While MGs enable efficient energy usage and RES integration, their reliability is challenged by unpredictable sources and loads. A robust fault-ride-through (FRT) method based on reactive power synchronization is proposed in [6], ensuring transient stability after faults. In [7], a single-phase community MG model highlights the difficulty in maintaining stability under fluctuating conditions. Indeed, a controller combining predictive current control and μ -synthesis is proposed, addressing model uncertainties and load variations. Reference [8] presents an event-triggered sliding mode controller for wind energy systems, which synchronizes the static synchronous compensator (STATCOM) and rotor-side converter, reduces chattering, and eases communication load.

To address stability in droop-controlled isolated MGs, [9]

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proposes a two-stage robust coordination strategy. The first stage involves day-ahead unit commitment planning under uncertainty; and the second adjusts hourly dispatch and droop gains to reduce costs and enhance small-signal stability. Reference [10] introduces a voltage-current droop controller that manages harmonic voltages and feeder impedance uncertainties. Additionally, [11] proposes a fault-tolerant control method for the GFM converters with inductor-capacitor (LC) filters, using a robust observer-based digital-twin model to enhance resilience and accuracy under multiplicative faults.

Reference [12] presents a nested-loop control for isolated MGs, using sliding mode control in the inner current loop and mixed H_2/H_∞ in the outer voltage loop, offering low harmonic distortion, consistent switching, and transient robustness. Finally, [13] introduces a GFM converter control technique tolerant to varying short-circuit ratios, using an active susceptance loop for damping and standard vector control without software changes.

Evaluating robust stability and performance in the MG control architectures is essential for reliable RES integration. Assessment frameworks validate control strategies and highlight improvement areas, forming a foundation for modern power grids. In [14], robust stability of a GFM converter connected via a line is analyzed by modeling arbitrary perturbations and constructing a switched system. The stability region of the switched system lies within the perturbed system. Reference [15] applies the μ -synthesis shown to a voltage-current control scheme for a distributed generation (DG) unit, using a self-modeling current controller and a predictive voltage controller to meet transient stability requirements.

In [16], the bifurcation theory assesses parameter stability in droop-controlled alternating-current (AC) MGs with induction motor loads. The study models system dynamics, identifies instability boundaries, and employs singular perturbation for model reduction. Reference [17] explores interactions between the GFM converters using the component connection method and evaluates control robustness via μ -synthesis. Similarly, [18] investigates the impacts of control parameters and grid condition on the stability, presenting an iterative design and comprehensive robustness analysis. Finally, [19] studies how synchronization units affect the MG stability, showing that parallel operation reduces synchronverter robustness. Moreover, virtual impedance is proposed as a mitigation strategy.

Many MG control studies have relied on linear time-invariant (LTI) models and classical μ -synthesis [20], [21]. However, these studies fail to capture time-dependent behavior, leading to a validation gap between theory and real-world performance. The MGs often exhibit linear time-varying (LTV) dynamics due to goal-driven operation, which cannot be accurately modelled by static and infinite-horizon methods.

The integral quadratic constraint (IQC) based framework addresses these limitations by accommodating a broader range of uncertainties, including time-varying parameters, nonlinearities, and delays. This allows for tighter modeling

and less conservative controller design, improving system performance and operation range [22]. The IQC theory offers an efficient method for stability and performance analysis in complex systems. The introduction of recurrent equilibrium networks (RENs) has further extended the IQC applications to nonlinear control and machine learning tasks, ensuring robustness and behavioral stability [23].

Unlike traditional μ -synthesis, phase-based robustness metrics offer less conservative stability guarantees for structured perturbations by linking phase measures with the IQC theory [24]. The discrete stochastic IQC (DIQC) based framework has also been used in networked control systems with noise and delay, ensuring uniform exponential stability [25]. In the MGs, the IQC theory has been applied to mitigate actuator faults and uncertainties in secondary control [26], and to coordinate battery energy storage via dissipative analysis [27]. Dispersive IQC formulations further enhance scalability for large interconnected systems without sacrificing accuracy [28]. The application of the IQC theory to the load frequency control (LFC) with sampling delays ensures exponential stability in sustainable power grids [29]. Recently, the IQC-based framework also supports neural network control design and performance analysis under line failures, highlighting their robustness and computational efficiency [30].

However, the adoption of the IQC-based framework in practical industrial settings such as MGs has been limited, as control engineers often default to more conventional techniques like single-input single-output (SISO) proportional-integral-derivative (PID) tuning, Bode and Nyquist analysis, μ -synthesis, and time-domain simulations. While these have worked well in many applications, the increasing complexity of modern power grids calls for advanced capabilities enabled by the IQC-based framework.

Bridging this gap requires not only further algorithmic and computational advancements in the IQC analysis but also clearer demonstrations of the benefits it can provide over traditional methods, particularly for challenging applications like the MG systems and the GFM converters. By expanding the use of the IQC theory to systematically control synthesis and analysis for a wider range of system uncertainties, the control community can unlock more reliable and high-performing solutions for complex, mission-critical systems [22].

Robust frequency regulation in islanded and renewable-rich MGs has been actively investigated using a variety of control paradigms. For instance, hybrid optimal and robust structures that combine classical proportional-integral (PI) structure with optimal state-feedback have been proposed to improve transient response and robustness in the islanded AC MGs (e.g., linear quadratic regulator (LQR)-PI integration in [31]). Robust-control design based on μ -synthesis has also received significant attention, including decentralized fixed-structure μ -synthesis for isolated MGs with comparative studies against PID controllers [32], as well as multi-objective optimization strategies to tune μ -synthesis weighting functions and improve robustness and performance trade-offs [33].

In parallel, optimization-based tuning of filtered PI controllers has been explored for robust LFC in renewable-inte-

grated multi-area power grids [34]. Beyond the classical robust control, robust model predictive control (MPC) has emerged as a promising direction for converter-based MGs, particularly due to its ability to explicitly handle constraints and forecasts. Comprehensive reviews highlight robust MPC variants, e.g., tube-based, stochastic, adaptive, and cyber-resilient MPC, as important enablers for practical MG deployment [35]. More recently, emerging computational paradigms such as quantum-accelerated optimization have been discussed as potential methods to alleviate computational burdens in constrained optimal control and MPC formulations [36].

While these recent studies demonstrate substantial progress, they largely rely on either fixed-structure robust design (μ -synthesis and PI tuning controllers) or constraint-handling optimal control (robust MPC). In contrast, the IQC theory provides a mathematically unified framework that can capture structured uncertainties and broad classes of nonlinearities under certified robust stability and performance conditions. Recent IQC advances have also appeared in data-driven robust synthesis, including the use of non-parametric IQC multipliers in non-power-grid applications [37]. Nevertheless, despite this modeling flexibility, the IQC-based framework for robust control synthesis and analysis has not been widely adopted in energy systems and the MG control. This motivates the present work, which demonstrates an IQC-based robust control synthesis and analysis workflow for the MG frequency regulation and benchmarks it directly against established robust methods under a common testbed.

As mentioned above, despite the significant advantages of the IQC-based framework in modeling and analyzing a broader class of system uncertainties, its adoption has been limited in the specific domain of robust control synthesis and analysis in an MG. This issue presents an opportunity to demonstrate the power and applicability of the IQC-based framework for the GFM converter control in an MG.

In this paper, we aim to introduce the ability of the IQC-based framework for robust control synthesis and analysis in an MG. By comparing the IQC-based framework against the well-established μ -synthesis and H_∞ , we highlight the unique advantages that the IQC-based framework can bring in terms of modeling flexibility, reduced conservatism, and overall effectiveness in evaluating the stability and performance of an MG in the presence of complex uncertainties. Additionally, we aim to provide more knowledge of when and how to use the IQC-based framework to achieve superior outcomes for mission-critical sustainable energy system applications by comparing the three mentioned powerful analysis methods directly.

Additionally, the IQC-based framework is adopted because it provides a unified way to model and certify robust stability and performance for systems subject to a broader class of uncertainties and non-idealities (including time-varying parameters, nonlinearities, and delays) than what is typically handled by classical LTI-based robust tools. In MG applications, where such effects can meaningfully influence frequency dynamics, the IQC-based framework can reduce conservatism and enable improved disturbance rejection under

broader operation and uncertainty conditions.

Motivated by the limited adoption of the IQC theory in the robust control synthesis and analysis despite the modeling flexibility, this paper provides the IQC-based framework for robust control synthesis and analysis in an MG. The main contributions are summarized as follows:

- 1) We formulate and implement a synthesis workflow for IQC-based controller using general dynamic multipliers (as supported by the IQClab solver) to guarantee robust stability and performance under modeled uncertainties, without relying on multiplier factorization.

- 2) We introduce side-by-side benchmarking against standard robust and practical baselines. In identical scenarios and across uncertainty ranges, we systematically compare the IQC-based control method with μ -synthesis, H_∞ , and multiple PI tuning control methods, highlighting the resulting differences in disturbance rejection and robustness margins.

- 3) We perform robustness assessment beyond parametric uncertainty. We assess the robustness with respect to sector-bounded and slope-restricted nonlinearities representing practical non-ideal behaviors, e.g., nonlinear characteristics of converter, load, and RES, and quantify worst-case L_2 -gain (i.e., energy-to-energy gain) as a function of nonlinearity bounds and multiplier basis length l .

- 4) We conduct regional robustness characterization using invariant and reachable sets. We provide a regional analysis showing that increasing the multiplier basis length l yields progressively tighter invariant ellipsoids and converges toward the tight reachable set, illustrating how dynamic multipliers reduce conservatism in practice.

In the rest of the paper, Section II presents the IQC theory. Section III presents the small-signal modelling. Section IV describes the IQC-based framework for robust control synthesis and analysis. Section V presents the results and discussion. Limitations and further work are presented in Section VI, and Section VII concludes the paper.

II. IQC THEORY

A. Feedback Loop and Uncertainty

Evaluating stability and performance characteristics of feedback loop is essential in numerous control applications. It is generally accepted to base the design of control algorithms on mathematical models that describe the dynamics of the system under investigation. However, it is not possible to obtain ideal models of these systems. Therefore, any given mathematical model will naturally differ from its matching reality. Even in data-driven methods, the presence of unknown disturbances and uncertainties should still be carefully considered. To account for this mismatch, known as uncertainty, a robust control framework is employed. Uncertainties may arise from the following sources:

- 1) Physical parameters are often characterized by a specific range of values.

- 2) Dynamical models generally depict phenomena in the lower frequency range.

- 3) Measurement and actuation delays are frequently omitted in design work.

4) The complex and dynamic nature of nonlinear and/or time-varying behavior is often neglected or excessively simplified.

An essential part of a robust control framework is the analysis of an uncertain dynamical system, as illustrated in Fig. 1. Here, $G \in RH_{\infty}^{n_q \times n_p}$ is a transfer matrix denoting a stable LTI system. RH_{∞} indicates the real rational H_{∞} space, and the superscripts n_q and n_p indicate the numbers of input signal p and output signal q of G , respectively, while $n_q \times n_p$ denotes the dimension of G . Δ stands for the uncertainty operator belonging to the admissible uncertainty set Δ , namely $\Delta \in \Delta$. q_0 is the external disturbance or reference input. This setting defines the structure and characteristics of uncertainties. The feedback loop, i.e., $q \rightarrow \Delta \rightarrow p \rightarrow G \rightarrow q_0$, is illustrated in Fig. 1.

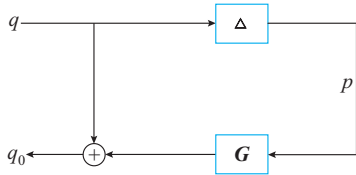


Fig. 1. Uncertain dynamical system.

Note that two assumptions are provided for the uncertainty set Δ .

Assumption 1 Δ is star-shaped with center of zero.

Assumption 2 $\Delta \in \Delta$ indicates that $\tau\Delta \in \Delta$ holds for all $\tau \in [0, 1]$, where τ is a scalar real parameter.

To avoid confusion with the operator and set notations, Δ is used only as a compact notation for the collection of uncertainty set Δ_i ($i=1, 2, \dots, N$) and performance block P_j ($j=1, 2, \dots, M$) in the IQClab implementation (e.g., $\Delta = \{\Delta_1, \Delta_2, \dots, \Delta_N, P_1, P_2, \dots, P_M\}$), and should be interpreted as a block-diagonal or cell-array aggregate rather than the operator Δ itself. Finally, small-signal deviations and perturbations of electrical variables are denoted explicitly by a leading symbol Δ (e.g., Δf for frequency deviation), which are consistent across the equations, figures, and captions.

As illustrated in Fig. 1, a critical problem is addressed as follows: how to systematically verify system stability and guarantee the required performance under all modeled uncertainties $\Delta \in \Delta$?

An extensive study has been conducted in this field, resulting in the development of the known structured single value or μ -synthesis control method [38]. The literature in this field is rich, and the results have demonstrated significant efficacy in numerous real-world control applications using the capabilities of robust control toolbox of MATLAB.

On the other hand, the IQC-based control method, which emerged in the 1990 s in its modern form [39], can be observed as the combination of two early established robust control methods: the scaled-small gain (with μ -synthesis or H_{∞}) and the positivity/passivity methods.

To better understand the relative strengths and limitations of the IQC-based, μ -synthesis, and H_{∞} control methods, a comprehensive comparison is provided. Each of the methods has been extensively used in robust stability and perfor-

mance analysis, but they differ significantly in their modeling capabilities, computational efficiency, and applicability to modern control challenges. The IQC-based control method offers a flexible and generalized framework for handling a wide range of uncertainties and nonlinear dynamics, making them particularly suitable for emerging applications such as MGs and machine learning-based control systems. While μ -synthesis control method excels in analyzing structured uncertainties in the LTI systems, it may struggle with nonlinearity and high-dimensional systems. Meanwhile, H_{∞} control method focuses on minimizing the impact of disturbances on system performance but is often limited to unstructured uncertainties in linear systems.

The IQC-based control method can be used to control synthesis and analysis in various types of systems. It involves substituting each segment of uncertainty with an IQC representation of its inputs and outputs, which can be more detailed. Indeed, the IQC-based control method is noteworthy because of its coherent and integrated structure and its utilization of the frequency domain method. Despite its significant effectiveness, the IQC-based control method has not been widely used in energy systems and MG control.

B. Main Concept and Definitions

The IQC-based control method provides a comprehensive framework for the synthesis and analysis of the stability and performance of a dynamic system in the presence of nonlinearities, time variability, and uncertain parameters. The methodology [40] has been developed by incorporating elements from both Western and Russian control theory traditions.

The relationship between an operator's input and output signals can be described by the IQC. These restrictions can be defined in either the time domain or the frequency domain since the two formulations are equivalent. However, compared with the impulse response for the time-domain representation, stability requirements obtained with frequency-domain constraints are frequently easier to handle, hence they are often selected. Thus, the IQC-based control method is defined in terms of the frequency domain [40]-[41].

Definition 1 Both signals p and q are square integrable over the interval $[0, \infty)$, i.e.:

$$\begin{cases} p \in L_2^{n_p}[0, \infty) \\ q \in L_2^{n_q}[0, \infty) \end{cases} \quad (1)$$

where $L_2^{n_p}[0, \infty)$ and $L_2^{n_q}[0, \infty)$ are the spaces of square-integrable n_p - and n_q -dimensional signals defined on $[0, \infty)$, respectively. The signals p and q satisfy the IQC characterized by the multiplier if and only if:

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{q}(j\omega) \\ \hat{p}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \hat{q}(j\omega) \\ \hat{p}(j\omega) \end{bmatrix} d\omega \geq 0 \quad (2)$$

where $\hat{p}(j\omega)$ and $\hat{q}(j\omega)$ are the Fourier transforms of $p(t)$ and $q(t)$ with t denoting the time interval, respectively; $\Pi(j\omega)$ is the IQC multiplier, and $\Pi(\cdot)$ is a measurable Hermitian matrix function belonging to the space of continuous matrix-valued function $\mathcal{C}^{(n_q+n_p) \times (n_q+n_p)}$, and $\omega \in \mathbb{R}$ is the angu-

lar frequency; and $[\cdot]^*$ denotes the complex conjugate transpose. Note that the relationship $p = \Delta(q)$ holds in this system. Typically, it is adequate to utilize rational operators that have limitations on the imaginary axis with $\Pi = \Pi^* \in RL_{\infty}^{(n_q+n_p) \times (n_q+n_p)}$ [42], where Π^* is the conjugate transpose matrix of Π . $RL_{\infty}^{(n_q+n_p) \times (n_q+n_p)}$ denotes the space of real-rational matrix-valued functions with the dimension of n_q+n_p rows and n_q+n_p columns, which are uniformly bounded along the entire imaginary frequency axis $j\omega$ without mandatory stability constraints. Π could be considered as in (3) using Parseval's theorem [42].

$$\Pi = \begin{bmatrix} \Pi_{11}(j\omega) & \Pi_{12}(j\omega) \\ \Pi_{12}^*(j\omega) & \Pi_{22}(j\omega) \end{bmatrix} \quad (3)$$

$$\mathcal{I}(\Pi, q, p) := \left\langle \begin{bmatrix} q \\ p \end{bmatrix}, \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^* & \Pi_{22} \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} \right\rangle \geq 0 \quad (4)$$

where Π_{11} is the $n_q \times n_q$ real-rational stable submatrix multiplier within $RH_{\infty}^{n_q \times n_q}$ for weighting the n_q -dimensional uncertainty output signal; Π_{22} is the $n_p \times n_p$ real-rational stable submatrix multiplier in $RH_{\infty}^{n_p \times n_p}$ for weighting the n_p -dimensional plant input signal; Π_{12} is an $n_q \times n_p$ rectangular real-rational stable cross-coupling submatrix multiplier from $RH_{\infty}^{n_q \times n_p}$ that characterizes the coupling relationship between the uncertainty output and plant input; Π_{12}^* is the conjugate transpose of Π_{12} , which is configured to maintain the global Hermitian symmetry property of Π ; and $\mathcal{I}(\Pi, q, p)$ is the quadratic form of the stacked signal $\begin{bmatrix} q \\ p \end{bmatrix}$ with respect to Π . Π also represents the corresponding operator for frequency-domain multiplication. For $p = \Delta(q)$, the following constraint is required for all $q \in L_2^{n_q}$. Δ is the uncertainty operator mapping signal q to $p = \Delta(q)$.

$$\mathcal{I}(\Pi, q, \Delta(q)) \geq 0 \quad \forall q \in L_2^{n_q} \quad (5)$$

According to Fig. 1, the uncertain dynamical system is defined by [43]:

$$\begin{cases} q = Gp + q_0 \\ p = \Delta(q) \end{cases} \quad (6)$$

Note that $p = \Delta(q)$ forms the feedback interconnection. In robust stability analysis, the primary objective is to determine if the feedback loop shown in Fig. 1 is stable for all possible variations of $\Delta \in \Delta$. To provide precise presentations, the following definitions are necessary [43].

Definition 2 The feedback loop of Fig. 1 is considered well-posed if, for any $q_0 \in L_{2e}^{n_q}$ ($L_{2e}^{n_q}$ is the extended L_2 -signal space of n_q -dimensional vector signals), there is a single response $q \in L_{2e}^{n_q}$ such that $(I - G\Delta)(q) = q - G\Delta(q) = q_0$, where I is the identity matrix and q is causally dependent on q_0 . More simply, this indicates that $I - G\Delta: L_{2e}^{n_q} \rightarrow L_{2e}^{n_q}$ must have a causal inverse.

Definition 3 The stability of the feedback loop in Fig. 1 is ensured if it is well-posed and the mapping from q_0 to q has a bounded L_2 -gain.

Definition 4 The feedback loop in Fig. 1 is considered robustly stable if it remains stable for all possible variations of $\Delta \in \Delta$.

C. Robust Stability Analysis

To illustrate the robust stability using the IQC-based control method, the following fundamental analysis results are used [44].

Theorem 1 For $G \in RH_{\infty}^{n_q \times n_p}$ and $\Delta \in \Delta$, assume that:

- 1) For any $\tau \in [0, 1]$, the feedback loop in Fig. 1 is well-posed while Δ is replaced by $\tau\Delta$.
- 2) There exists $\Pi = \Pi^* \in RL_{\infty}^{(n_q+n_p) \times (n_q+n_p)}$ such that the IQC-based control method holds for $\tau\Delta$ with all $\tau \in [0, 1]$.
- 3) There exists a constant $\varepsilon > 0$ that satisfies:

$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} \leq -\varepsilon I \quad \forall \omega \in \mathbb{R} \quad (7)$$

The third condition can also be expressed as the following frequency domain inequality (FDI):

$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} < 0 \quad \forall \omega \in \mathbb{R} \cup \{\infty\} \quad (8)$$

where < 0 denotes strict negative definiteness; and the frequency range includes $\omega \rightarrow \infty$. The above FDI is abbreviated as $\begin{bmatrix} G \\ I \end{bmatrix}^* \Pi \begin{bmatrix} G \\ I \end{bmatrix} < 0$.

This notation greatly simplifies the presentation and avoids potential misunderstandings. For inequalities involving transfer matrices, the evaluation is performed in the frequency domain, while those concerning regular matrices follow standard matrix inequality rules.

Now, we apply Theorem 1 to analyze the robust stability of the uncertain feedback loop shown in Fig. 1. To this end, a full set of multipliers $\Pi \in RL_{\infty}^{(n_q+n_p) \times (n_q+n_p)}$ will be typically constructed so that (5) holds for all $\pi \in \Pi$ and $\Delta \in \Delta$. We restrict the multiplier set to:

$$\Pi := \left\{ \Pi = \text{diag}(\pi_{11} I_{n_q}, -\pi_{11} I_{n_p}), \pi_{11} \in RL_{\infty}: \pi_{11} > 0 \right\} \quad (9)$$

where $\pi_{11} \in RL_{\infty}$ denotes a real-rational, bounded multiplier with $\pi_{11} > 0$; and I_{n_q} and I_{n_p} are the identity matrices of sizes n_q and n_p , respectively. Considering $\Delta := \{\Delta \in H_{\infty}^{n_q \times n_p}: \|\Delta\|_{\infty} \leq 1\}$, (5) is valid for all $\pi \in \Pi$ and $\Delta \in \Delta$, which is obvious from:

$$\begin{aligned} \int_{-\infty}^{\infty} \begin{bmatrix} \star \\ \star \end{bmatrix}^* \begin{bmatrix} \pi_{11}(j\omega) I & 0 \\ 0 & -\pi_{11}(j\omega) I \end{bmatrix} \begin{bmatrix} \hat{q}(j\omega) \\ \Delta(j\omega) \hat{q}(j\omega) \end{bmatrix} d\omega = \\ \int_{-\infty}^{\infty} \pi_{11}(j\omega) (\hat{q}(j\omega))^* (I - (\Delta(j\omega))^* \Delta(j\omega)) \hat{q}(j\omega) d\omega \geq 0 \end{aligned} \quad (10)$$

where $\Delta(j\omega)$ is the frequency response of Δ when applicable; and $\hat{q}$ is the Fourier transform of q . For all $q \in L_2^{n_q}$, $\|\Delta\|_{\infty} \leq 1$ and $\pi_{11} > 0$. Objects that can be inferred by symmetry are indicated by $\star$. Now, Theorem 1 can be rewritten in a modified version.

Corollary 1 Assume that:

- 1) The feedback loop in Fig. 1 is well-posed for all $\Delta \in \Delta$.
- 2) For any $\Delta \in \Delta$ and $\pi \in \Pi$, (5) is valid.

The feedback loop in Fig. 1 is robustly stable if there is $\pi \in \Pi$ such that (8) is satisfied.

D. Robust Performance Analysis

Frequently, the concern is not only in ensuring robust stability but also in achieving proper performance that is robust to variations. To achieve this objective, the feedback loop shown in Fig. 1 is enhanced by adding an extra performance channel, as illustrated in Fig. 2.

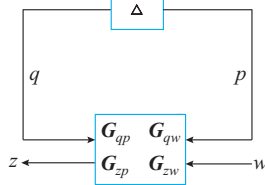


Fig. 2. Typical feedback interconnection for robust performance assessment.

The generalized system is defined using the following linear fractional representation as:

$$\begin{bmatrix} q \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} G_{qp} & G_{qw} \\ G_{zp} & G_{zw} \end{bmatrix}}_G \begin{bmatrix} p \\ w \end{bmatrix} \quad p = \Delta(q) \quad (11)$$

where w is the external disturbance or reference input; z is the performance output; G_{qp} is the transfer matrix mapping p to q ; G_{qw} is the transfer matrix from w to q ; G_{zp} is the transfer matrix that transmits p to z ; and G_{zw} is the direct transfer matrix linking w to z . This interconnection is properly formulated for $\Delta \in \Delta$ if $I - G_{qp}\Delta$ has a causal inverse. If the statement is correct, it can be represented as $z = (\Delta \star G)(w)$ along with (12), and $(I - G_{qp}\Delta)^{-1}$ exists for well-posed interconnection.

$$\Delta \star G := G_{zw} + G_{zp}(I - G_{qp}\Delta)^{-1}G_{qw} \quad (12)$$

Once more, $G \in RH_{\infty}^{(n_q+n_z) \times (n_p+n_w)}$ is a stable LTI system. The uncertainty channel is represented by $p \rightarrow q$, and the performance channel is represented by $w \rightarrow z$. The system performance will be constrained using IQC as:

$$\mathcal{I}_p(\Pi_p, z, w) := \left\langle \begin{bmatrix} z \\ w \end{bmatrix}, \begin{bmatrix} \Pi_{p11} & \Pi_{p12} \\ \Pi_{p12}^* & \Pi_{p22} \end{bmatrix} \begin{bmatrix} z \\ w \end{bmatrix} \right\rangle \leq -\varepsilon \|w\|^2 \quad (13)$$

where Π_p is the Hermitian matrix-valued performance multiplier, and the subscript p indicates that this multiplier is dedicated to evaluating system performance; and $\varepsilon > 0$. $\Pi_p \in RL_{\infty}^{(n_z+n_w) \times (n_z+n_w)}$ is selected as the performance index of the following frequency-based set.

$$\Pi_p \in RL_{\infty}^{(n_z+n_w) \times (n_z+n_w)}: \Pi_{p11} \geq 0 \quad (14)$$

The nonnegativity requirement on Π_{p11} is necessary to simultaneously determine both robust stability and robust performance.

Definition 5 The system in Fig. 1 achieves robust performance with respect to Π_p if the interconnection is robustly stable (i.e., $I - G_{qp}\Delta$ has a causal and bounded inverse for all $\Delta \in \Delta$). For any $\pi_p \in \Pi_p$, there exists $\varepsilon > 0$ such that the IQC

holds for all system trajectories driven by $w \in L_2^{n_w}$.

Corollary 1 is further extended to robust performance analysis as follows.

Corollary 2 Assume that:

- 1) The interconnection in (11) is well-posed for all $\Delta \in \Delta$.
- 2) For any $\Delta \in \Delta$ and $\pi \in \Pi$, (5) is valid.

If there are $\pi \in \Pi$ and $\pi_p \in \Pi_p$ that satisfy the following FDI, then the interconnection in (11) will be robustly stable and have robust performance with respect to Π_p on the channel $w \rightarrow z$. The matrix inequality is evaluated pointwise over frequency. Robust performance can be verified by checking the existence of qualified Π and Π_p that satisfy the FDI.

$$\begin{bmatrix} G_{qp} & G_{qw} \\ I & 0 \\ G_{zp} & G_{zw} \\ 0 & I \end{bmatrix}^* \begin{bmatrix} \Pi & 0 \\ 0 & \Pi_p \end{bmatrix} \begin{bmatrix} G_{qp} & G_{qw} \\ I & 0 \\ G_{zp} & G_{zw} \\ 0 & I \end{bmatrix} \prec 0 \quad (15)$$

III. SMALL-SIGNAL MODELLING

This paper synthesizes and analyzes a robust controller for an isolated MG [38]. The IQC-based framework is used to design the robust controller, then the robust stability and robust performance of the closed-loop system are analyzed, which are constructed based on both H_{∞} and μ -synthesis control methods. Here, an islanded AC MG is considered [38], as shown in Fig. 3, which comprises AC load, a wind turbine generator, a photovoltaic system, a diesel engine generator, a microturbine, a fuel cell, and energy storage devices such as flywheel energy storage and battery energy storage systems. The AC bus is connected to distributed energy resources through power electronic devices, as illustrated in Supplementary Material A Fig. SA1. The corresponding system parameters used in this paper are the same as [38].

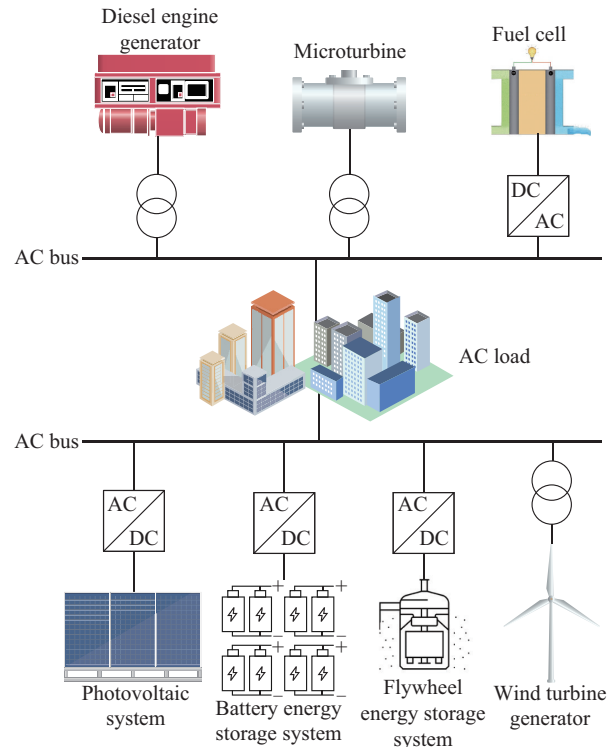


Fig. 3. Simplified schematic of an islanded AC MG.

IV. IQC-BASED FRAMEWORK FOR ROBUST CONTROL SYNTHESIS AND ANALYSIS

A. Robust Control Synthesis

The IQClab solver supports robust control synthesis using general dynamic multipliers, and the obtained controllers can maintain satisfactory performance under predefined uncertainties [22]. The algorithm adopted in this paper follows the formulation in [43], which realizes nominal controller design using unstable weighting functions without poles on the imaginary axis, as further discussed in [43]-[46]. This algorithm is similar to μ -synthesis control method, and is applicable to robust controller design for systems with general linear fractional uncertainties.

Traditional methods typically require the factorization of IQC multipliers, which often introduces numerical ill-conditioning. In contrast, the algorithm bypasses such factorization, resulting in a more numerically stable and significantly streamlined procedure for robust control synthesis using general dynamic IQC multipliers [22]. The synthesis is based on the system interconnection illustrated in Fig. 4. Here, system G is represented as the generalized plant, K is the robust controller to be synthesized. Three types of channels are defined: the performance channel $w_p \rightarrow z_p$, the uncertainty channel $w_\Delta \rightarrow z_\Delta$, and the control channel $u \rightarrow y$.

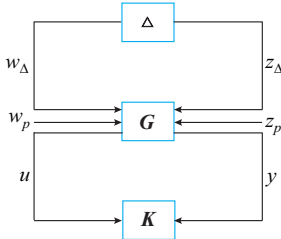


Fig. 4. Conceptual framework of robust control synthesis.

The target controller has the form $u = Ky$. In the studied benchmark, the feedback signal is the frequency deviation, i.e., $y = \Delta f$, and u is the control input that adjusts the frequency-control action. For all $\Delta \in \Delta$, $\Delta \star G$ is stabilized and the induced L_2 -gain from the performance input signal w_p to the output signal z_p remains below a prescribed induced L_2 -gain bound $\gamma > 0$. Δ is structured for compatibility with the IQClab toolbox for the robust control synthesis and analysis. The algorithm enforces the following constraint on the induced L_2 -norm of the performance channel $w_p \rightarrow z_p$:

$$\int_{-\infty}^{\infty} \begin{bmatrix} z_p(t) \\ w_p(t) \end{bmatrix}^T \begin{bmatrix} \frac{1}{\gamma} I & 0 \\ 0 & -\gamma I \end{bmatrix} \begin{bmatrix} z_p(t) \\ w_p(t) \end{bmatrix} dt \geq 0 \quad \forall \gamma \geq 0 \quad (16)$$

The algorithm operates through an iterative procedure that alternates between the nominal (weighted) control synthesis and the robustness control analysis, following a strategy like that of μ -synthesis. A key distinction, however, lies in its ability to incorporate general dynamic IQC multipliers without factorization, which enhances numerical stability and extends applicability to complex uncertainty structures. The de-

tailed algorithm procedure is provided in the supplementary Material A.

B. Comparison to H_∞ and μ -synthesis Control Methods

The most important differences between the IQC-based, H_∞ , and μ -synthesis control methods from a computational workflow point of view are summarized, as presented in the Supplementary Material A Table SAI.

V. RESULTS AND DISCUSSION

A. IQC-based Framework for Robust Control Synthesis

In this paper, a robust controller is designed for the islanded MG system introduced in [38] using the IQC-based framework. The IQC-based framework systematically addresses uncertainties in the MG dynamics and ensures robust stability and performance across a wide range of operation conditions.

Regarding the practical deployment and computational-time considerations, the IQC-based framework consists of an offline design step and an online execution step. The IQC-based framework for robust control synthesis (using IQClab solver, YALMIP solver, and SDP solver) is performed offline to obtain K . During online operation, the controller is implemented in standard state-space (discrete-time) form and computed in real time as $u = Ky$.

Since the case study targets the MG frequency regulation using dispatchable units, the controller is naturally deployed at the MG supervisory or secondary frequency control level. If operation points or grid conditions change significantly, controller redesign can be performed periodically offline, or a set of controllers can be pre-computed offline for representative operation points and selected online (gain scheduling).

Moreover, the performance of the IQC-based controller is systematically compared with several benchmark controllers including: ① a PI controller designed via tuner-based optimization in MATLAB (PI-Tuner controller), ② a PI controller derived from the internal model control (IMC) (PI-IMC controller), ③ a PI controller tuned using the Ziegler-Nichols method (PI-ZN controller), and ④ robust controllers designed using H_∞ and μ -synthesis control methods proposed in [38]. The performance of the controllers under some challenging conditions is thoroughly analyzed and discussed.

In Fig. 5(a), we present the root-mean-square (RMS) frequency deviation of the MG under increasing parameter uncertainty levels, comparing all the mentioned controllers. The IQC-based controller maintains about zero RMS frequency deviation across all uncertainty levels, highlighting its strong robustness. While the PI-IMC controller exhibits the highest RMS frequency deviation, followed by the PI-ZN and PI-Tuner controllers. This demonstrates the superior capability of the IQC-based controller in mitigating disturbances and maintaining frequency stability under uncertainty. In Fig. 5(b), we illustrate the peak frequency deviation for the same range of parameter uncertainties and controller types. The IQC-based controller consistently suppresses peak deviations to near zero, regardless of the uncertainty level.

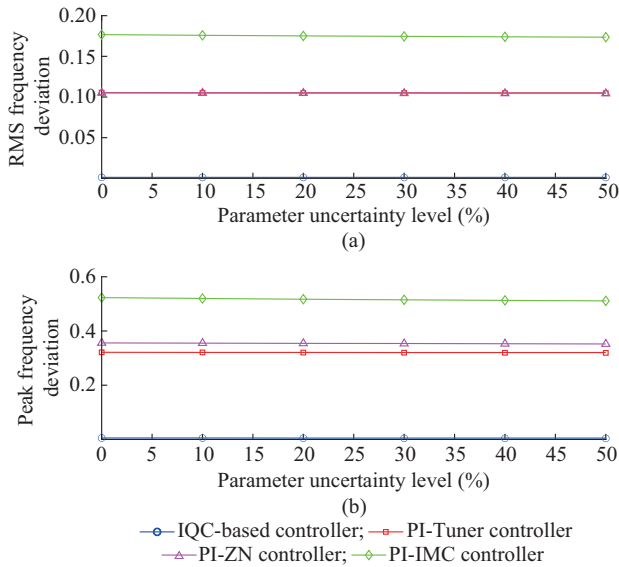


Fig. 5. Frequency deviations of MG under increasing parameter uncertainty levels for different controllers. (a) RMS frequency deviation. (b) Peak frequency deviation.

Among the PI controllers, the PI-IMC controller again performs the worst, showing the highest peak frequency deviations. The PI-ZN and PI-Tuner controllers show moderate but non-negligible sensitivity to system uncertainty. These results confirm that the IQC-based controller offers significantly enhanced robustness and frequency regulation performance in MG applications subject to modeling uncertainties.

Figure 6(a) shows the control signal applied by different controllers over a 30-s interval in response to a disturbance in the MG. The IQC-based controller applies a relatively smooth and bounded control signal with minimal overshoot and settling time. While, the PI-IMC controller produces a highly oscillatory and aggressive control action, particularly during the first 10 s, reflecting poor damping and potential actuator stress. The PI-ZN and PI-Tuner controllers show moderate control effort with noticeable oscillations.

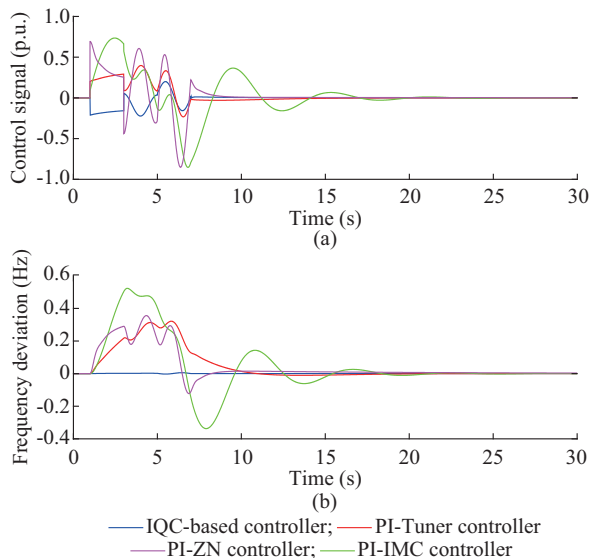


Fig. 6. MG frequency response. (a) Control signal. (b) Frequency deviation.

Figure 6(b) presents the frequency deviation over time. The IQC-based controller achieves the best performance, maintaining frequency deviation near zero throughout the transient and steady-state periods compared with the PI-IMC, PI-ZN, and PI-Tuner controllers.

These results confirm that the IQC-based controller provides superior transient response and frequency regulation, making it highly suitable for robust MG operation under disturbances.

In Fig. 7(a), the profiles of three simultaneous disturbances are applied to the MG within the first 6 s: a step load increase (blue), a wind speed ramp (red), and two square pulses in solar irradiation (green). All disturbances are expressed in per unit and represent typical variations of renewable-integrated MGs. Figure 7(b) illustrates the resulting frequency deviation.

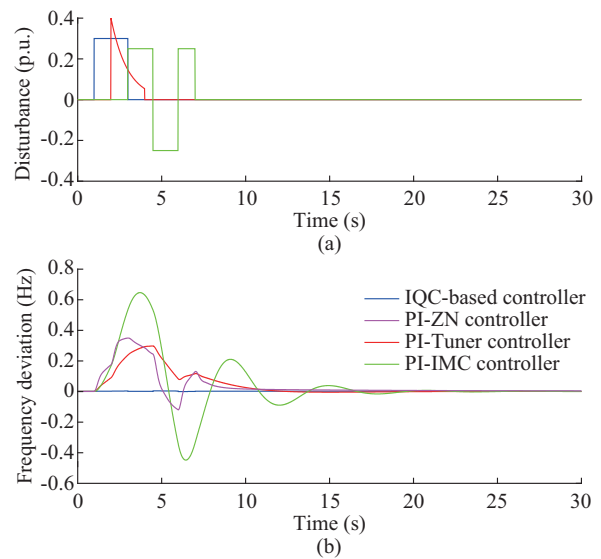


Fig. 7. Frequency response in case of simultaneous disturbances. (a) Disturbance. (b) Frequency deviation.

The IQC-based controller demonstrates superior performance, maintaining minimal frequency deviation and fast recovery despite the complexity of the disturbances. The PI-IMC controller exhibits significant overshoot and sustained oscillations, indicating weak disturbance rejection.

The PI-ZN and PI-Tuner controllers show moderate transient behavior but still underperform the IQC-based controller in terms of peak frequency deviation and settling time.

Compared with PI controllers and traditional robust controllers, the proposed IQC-based controller achieves remarkable improvements in frequency regulation performance. To sum up, the IQC-based controller maintains nearly zero RMS and peak frequency deviations under 0-50% variations of inertia and damping, outputs smooth and bounded control signals with small overshoot and short settling time, and keeps Δf close to zero in both transient and steady states. Its improved disturbance rejection capability is also verified under combined renewable and load disturbances.

Figure 8 shows singular value plots for nominal and perturbed systems.

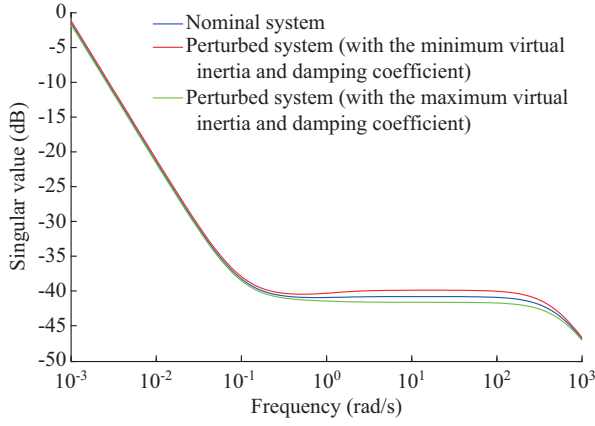


Fig. 8. Singular value plots for nominal and perturbed systems.

Despite variations in inertia and damping, all singular values (responses) remain below 0 dB, indicating robust stability. The largest differences appear in mid-frequency ranges but remain bounded. The figure confirms robust stability of the IQC-based controller under uncertainty.

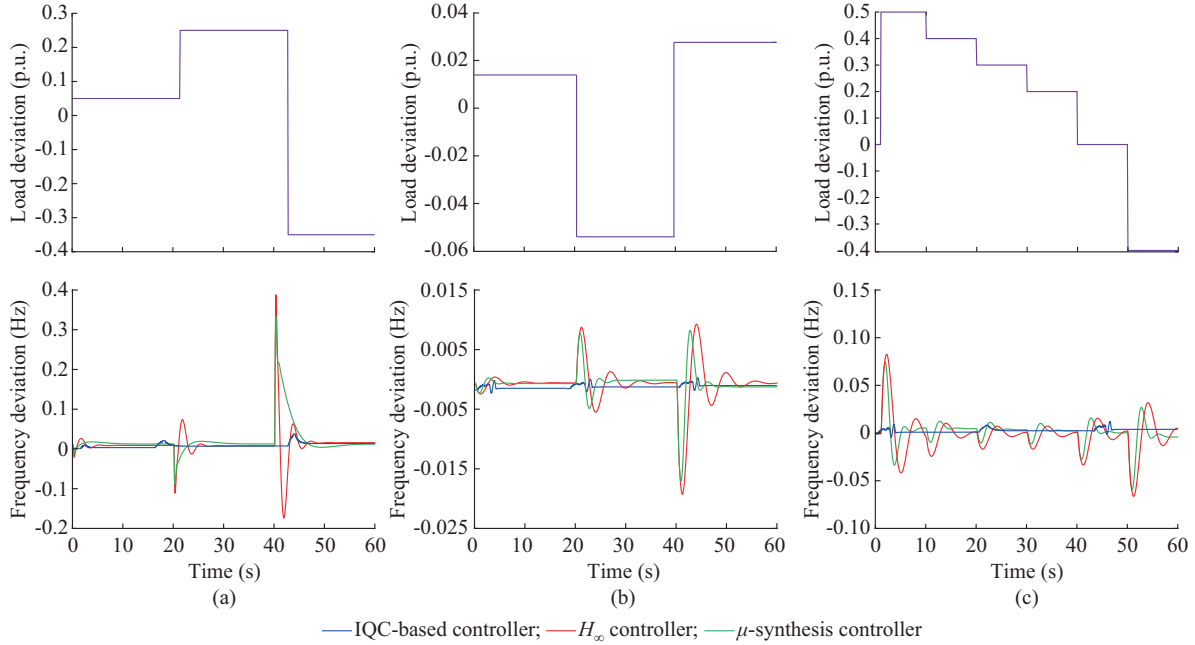
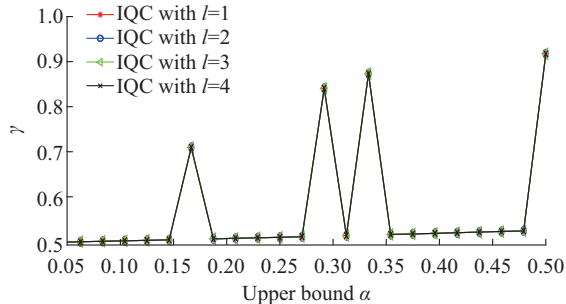

 Fig. 9. Comparison between system responses for IQC-based, μ -synthesis, and H_∞ controllers in presence of three load deviation scenarios. (a) Scenario 1. (b) Scenario 2. (c) Scenario 3.

 Fig. 10. Variation of γ (load deviation scenario 1).

Figure 9 shows a comparison between system response for IQC-based, μ -synthesis, and H_∞ controllers in load deviation scenarios 1-3. All the results show that the IQC-based controller performs much better than μ -synthesis and H_∞ controllers.

Figure 10 illustrates the variation of γ as a function of the sector upper bound α , which corresponds to load deviation scenario 1 in Fig. 9(a). The results are presented for four different basis function lengths $l=1, 2, 3, 4$, which determine the richness of the multiplier used in the IQC. Figure 11 further presents the variation of γ as a function of the nonlinear sector upper bound α , which corresponds to load deviation scenario 2 in Fig. 9(b). When Fig. 11 is generated, every nonlinear element of the system is restricted by both sector and slope limits. This joint constraint accurately reproduces real-world rate-limited nonlinear behaviors such as high-frequency nonlinear damping, saturated virtual inertia of converter-based sources, and slow actuator adjustments from control lags. Such a dual restriction reduces conservatism and delivers more physically meaningful robustness results relative to pure sector constraints.

B. Summary of Quantitative Comparison

To complement the graphical results, Supplementary Material A Table SAII summarizes the key performance indices of the proposed IQC-based controller against the benchmark PI controllers (i.e., PI-Tuner, PI-IMC, and PI-ZN controllers) and robust controllers (i.e., H_∞ and μ -synthesis controllers). This table provides a concise view of the relative performance and robustness advantages.

C. Robustness Analysis with Sector-bounded and Slope-restricted Nonlinearities

In this subsection, some results are presented to evaluate

the robust stability and performance of the IQC-based controller. The MG structure incorporates various nonlinear components, including converter control mechanisms, load characteristics, and RES dynamics. These nonlinearities are modeled as sector-bounded and slope-restricted functions, defined within specific input-output constraints to accurately represent their behavior under varying operation conditions. The detailed explanation is provided in the Supplementary Material A.

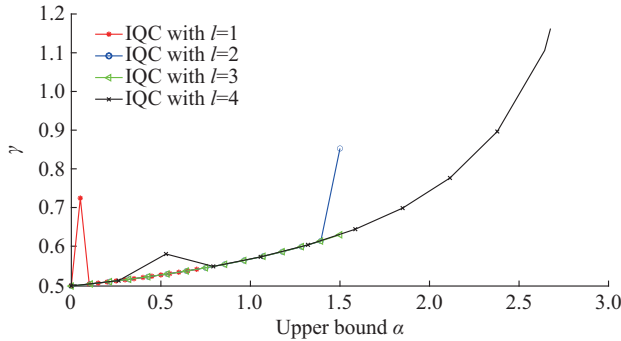


Fig. 11. Variation of γ (load deviation scenario 2).

D. Regional Robustness Analysis Under LTI Parametric Uncertainties

Considering the interconnected system $\Delta \star G$, $G = ss(A, B, C, D)$ denotes the open-loop LTI system and $ss(A, B, C, D)$ denotes the linear time-invariant generalized plant represented in state-space form, where A , B , C , and D correspond to the state, input, output, and feedthrough matrices, respectively. Each scalar nonlinearity δ is constrained to the range $\delta \in [-1, 1]$. Then, the smallest ellipsoidal region captures the set of z of the understudy system in response to $\|w\| \leq 1$, under the influence of $\delta = -1$. This setup reflects a critical robustness scenario where damping and inertia values are most adversarial within their defined uncertainty bounds. Figure 12 presents the results of this regional analysis.

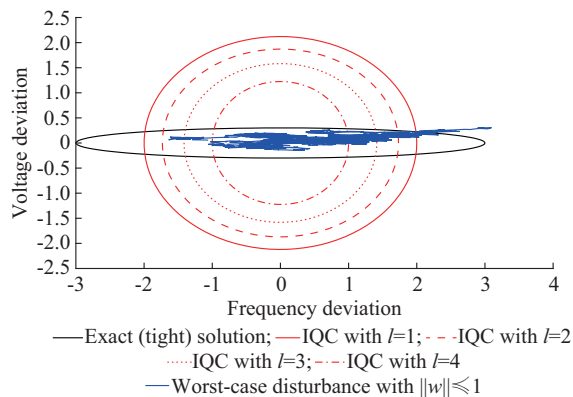


Fig. 12. Results of regional analysis.

For IQC with $l=1$, the resulting ellipsoid significantly over-approximates the true reachable set, yielding a conservative bound. As l increases, the computed ellipsoids become progressively tighter and more aligned with the exact bound-

ary, demonstrating improved accuracy with dynamic multipliers. This convergence toward the tight solution highlights the value of incorporating temporal dynamics into the multiplier design, allowing for a less conservative and more accurate characterization of the system response under uncertainty.

These results highlight the effectiveness of robust control analysis in providing scalable and accurate robustness guarantees, especially when parametric uncertainties like damping and inertia are critical to system behavior.

It is noteworthy to mention that the MG and converter dynamics used in this paper are based on an averaged representation and linearized around a nominal operation point, enabling systematic synthesis and robustness analysis in a small-signal setting. This is widely used for the GFM converter studies because it captures the dominant electromechanical-frequency dynamics and control-loop interactions below the switching frequency while avoiding the computational burden of explicit switching simulation.

All numerical studies in this paper were carried out in MATLAB 2024a on Macintosh HD OS (MacBook Pro M2). The robust control synthesis and analysis were implemented using the IQClab solver, with YALMIP solver used as the modeling interface for the underlying optimization problems. Semidefinite programs were solved using MOSEK and SDPT3. For clarity and accessibility, Supplementary Material A Fig. SA2 summarizes the complete workflow adopted in this paper.

Nevertheless, it is important to recognize the limitations of small-signal model in the design of IQC-based controller. First, switching behavior and high-frequency ripple are not explicitly represented. The model is intended to describe the low-frequency dynamics where the GFM control objectives (voltage and frequency regulations, power sharing, and damping) are formulated. Second, the linearization is locally valid. Under large disturbances (e.g., serious faults, very large load steps, or large setpoint changes), nonlinear effects such as modulation limits, converter current limiting, DC-link constraints, and protection logic can dominate the response. These may alter the effective control action and lead to the behaviors that deviate from linear predictions.

Additionally, while the results in this paper establish stability and performance properties around the considered operation points, large-signal verification should be performed using nonlinear time-domain simulation (e.g., electromagnetic transient and switching models) and/or experimental and hardware-in-the-loop (HIL) testing when assessing severe transients. Incorporating saturation and protection elements and quantifying the deviation between linear predictions and nonlinear responses under increasing disturbance magnitudes constitute a natural extension of this paper.

Although the results presented in this paper are obtained in simulation, the GFM control is intended for real-time implementation on standard digital signal processor (DSP) and field-programmable gate array (FPGA)-based converter controllers. Experimental validation, e.g., HIL and laboratory testing, is therefore the next important step. To support this transition and clarify practical applicability, the main non-

idealities that must be considered in the HIL and prototype realization are summarized in three terms. More explanation is provided in the Supplementary Material A.

VI. LIMITATION AND FUTURE WORK

We investigate the frequency response dynamics of an islanded AC MG and demonstrate the IQC-based framework for robust control synthesis and analysis in MG. Several limitations and new directions for future work should be noted.

1) Scalability to larger and meshed AC/DC MGs: the proposed IQC-based framework is applied to a generalized plant G . Therefore, the extension to meshed AC MGs primarily requires constructing the corresponding averaged and small-signal interconnection model, including network coupling (line impedances and bus dynamics) and multiple converter-interfaced sources. For hybrid AC/DC MGs, the generalized plant can be augmented to include DC-bus dynamics and interlinking-converter dynamics, while uncertainties and nonlinearities, e. g., uncertain line parameters, load variations, and converter limits, are represented through appropriate IQC blocks.

2) Computational and numerical considerations in large-scale systems: the computational burden of the robust control synthesis and analysis is dominated by the YALMIP and SDP solvers, whose size grows with the plant order, uncertainty description, and multiplier complexity. Hence, for large-scale systems, fully centralized synthesis may become computationally demanding and more sensitive to numerical conditioning. Practical pathways to address this include exploiting grid sparsity, adopting modular and compositional subsystem modeling, and employing hierarchical and decentralized control architectures, e. g., local GFM control with higher-level coordination, to reduce dimensionality and improve scalability.

These directions provide a clear route for extending the IQC-based framework beyond the single-MG benchmark considered in this paper.

VII. CONCLUSION

This paper presents an IQC-based framework for robust control synthesis and analysis in MG. An IQC-based controller has been synthesized based on the IQC theory and applied to an islanded MG. The performance and robustness of the IQC-based controller are evaluated under various disturbances and parameter uncertainties.

Comparative analyses with the PI controllers tune via multiple classical methods, as well as μ -synthesis and H_∞ control methods, demonstrating the superior disturbance rejection, frequency regulation, and overall robustness of the IQC-based controller. The results confirm that IQC not only offers a mathematically general and flexible framework for uncertainty modeling but also enables practical implementation with improved performance outcomes in complex energy systems. Its ability to simultaneously certify robust stability and performance across a range of operation conditions highlights its strong potential for real-world applications in MG.

In quantitative terms, the IQC-based controller achieves

the smallest frequency deviations across all evaluated scenarios. Under parameter uncertainty, the RMS frequency deviation remains near zero, and the peak deviation is strongly suppressed, whereas PI-IMC controller exhibits noticeably larger RMS and peak deviations. In the disturbance-profile comparisons against robust benchmark controllers, the IQC-based design consistently provides tighter frequency regulation and faster recovery, with reduced oscillatory behavior relative to the μ -synthesis and H_∞ controllers. These results demonstrate that the IQC-based framework is not only flexible for modeling uncertainties and nonlinear descriptions, but also effective for achieving improved disturbance rejection and robustness in MG frequency control.

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