

Topology-switching Soft Open Point Assisted Real-time Cooperative Operation of Active Distribution Networks Considering Voltage Stability

Binjie Wang, Wu Tu, Xiaodong Yang, *Senior Member, IEEE*, Hui Fang, Lijian Ding, *Member, IEEE*, Wei Lou, Qiuwei Wu, *Senior Member, IEEE*, and Jinyu Wen, *Senior Member, IEEE*

Abstract—The large-scale integration of photovoltaic (PV) generation poses serious challenges of voltage violations and instability risks to active distribution networks (ADNs). To address these issues, this paper proposes a topology-switching soft open point (TS-SOP) assisted real-time cooperative operation framework of ADNs. The proposed framework ensures voltage stability by explicitly incorporating voltage stability margin (VSM) constraints. A TS-SOP model is developed that enhances power flow controllability and reduces standby losses through switchable feeder interconnections and flexible converter operating modes. The proposed framework coordinates day-ahead pre-dispatch, intra-day dynamic correction, and real-time voltage regulation, featuring a two-stage volt/var control. The first stage employs model predictive control to mitigate PV-driven fluctuations, while the second stage embeds VSM constraints into droop curve parameter optimization, ensuring that local droop control actions respect the system stability limits. Simulations on a modified IEEE 33-node system demonstrates that, compared with conventional fixed-topology SOP and droop-only schemes, the proposed framework not only reduces network losses and voltage violations, but also ensures VSM compliance, effectively coordinating fast voltage regulation with system-wide stability while improving the operating economy.

Index Terms—Active distribution network, soft open point, cooperative operation, voltage stability, voltage security, volt/var control.

I. INTRODUCTION

THE deployment of photovoltaic (PV) generation has expanded rapidly, with an increasing number of PV units integrated into active distribution networks (ADNs). Owing to the stochastic and intermittent nature of PV generation, feeder power flows can vary continuously, which may induce voltage fluctuations and violations. These issues can compromise the reliable operation of ADNs, and increase the risk of voltage instability [1]–[3]. Therefore, effective voltage regulation is essential for ADNs with high PV penetration.

Voltage regulation aims to maintain nodal voltages within allowable limits by coordinating controllable resources in ADNs [4]. Traditionally, voltage regulation has relied on discrete devices such as capacitor banks (CBs) and on-load tap changers (OLTCs). However, due to their discrete control steps, limited operation counts, and slow mechanical response, these devices are often inadequate for mitigating rapid voltage fluctuations caused by PV and load variability [5]. With advances in power-electronic technologies, PV inverters can provide fast reactive power support, making them well suited for voltage regulation in ADNs [6]–[9].

Extensive research has been conducted on volt/var control in ADNs with high PV penetration. Centralized optimization-based strategies enable system-wide coordination; however, they typically depend on comprehensive measurement and communication infrastructures and impose significant computational burdens. Consequently, the achievable control update rate may be insufficient to track rapid voltage fluctuations. Local inverter control allows for rapid response with the minimal communication [10], [11], but it often lacks system-wide coordination and may compromise voltage security at the system level. To balance global coordination with fast local response, hierarchical coordination frameworks are proposed, combining slower centralized setpoint optimization with fast local droop control [12], [13]. Nevertheless, these strategies primarily regulate reactive power and may be inad-

Manuscript received: May 15, 2025; revised: October 9, 2025; accepted: December 29, 2025. Date of CrossCheck: December 29, 2025. Date of online publication: February 2, 2026.

This work was supported in part by the National Natural Science Foundation of China (No. 52377090) and in part by the Fundamental Research Funds for the Central Universities (No. JZ2025HGTG0270).

This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>).

B. Wang, W. Tu, X. Yang (corresponding author), and L. Ding are with the State Key Laboratory of High Efficiency and High Quality Electric Energy Conversion, Hefei University of Technology, Hefei 230009, China (e-mail: dr_wangbinjie@163.com; W_tu224@163.com; yang_xd90@163.com; ljding@hfut.edu.cn).

H. Fang is with the State Grid Anhui Ultra High Voltage Company, Hefei 230022, China (e-mail: fangh_2017@163.com).

W. Lou is with the Electric Power Research Institute of State Grid Anhui Electric Power Co., Ltd., Hefei 230601, China (e-mail: louwei_sgce@foxmail.com).

Q. Wu is with the School of Electronics, Electrical Engineering, and Computer Science, Queen's University Belfast, Belfast, BT8 5BN, U.K. (e-mail: qiuwei.wu@qub.ac.uk).

J. Wen is with the State Key Laboratory of Advanced Electromagnetic Engineering and Technology, School of Electrical and Electronic Engineering, Huazhong University of Science and Technology, Wuhan 430074, China (e-mail: jinyu.wen@hust.edu.cn).

DOI: 10.35833/MPCE.2025.000431



equate in high- R/X feeders, where nodal voltages are strongly influenced by active power injections with high PV penetration [14]. This observation underscores the need to coordinate both active and reactive power resources.

Accordingly, coordinated control of active power curtailment of PV and reactive power support is investigated to mitigate voltage issues [15]. For instance, a decentralized volt/var control strategy with equitable curtailment allocation is proposed in [16]. To enhance the PV energy utilization, [17] and [18] formulate optimization problems aimed at minimizing the active power curtailment of PV while satisfying voltage constraints, demonstrating improved voltage profiles. However, active power curtailment directly reduces the PV energy utilization and may compromise economic benefits. In this context, the soft open point (SOP), a controllable power-electronic interface that enables power exchange between feeders, has garnered significant attention. By regulating both active and reactive power flows, SOPs can reduce dependence on PV curtailment, improve the performance of voltage regulation, and decrease losses [19]–[21].

Recent studies have advanced SOP-assisted operations for loss minimization and voltage regulation [22], uncertainty-aware management [23], and combined centralized-local control to reduce communication and computational burdens [24]. Extensions have also considered distribution network reconfiguration [25], [26] and multiplexed converter designs to improve asset utilization [27]. Building on these developments, hierarchical multi-timescale volt/var control strategies that coordinate SOPs and PV inverters via droop control have been proposed to enhance the performance of voltage regulation [28]. Despite this progress, several challenges remain for SOP-based voltage regulation in ADNs. First, conventional SOPs are capital-intensive and typically deployed with fixed feeder connections [22], which limits the operating flexibility and hinders widespread adoption. Second, converter loss models in many studies are often simplified (e.g., linearized) and neglect standby or no-load losses [24], [28]–[31], which may bias the evaluation of operating cost. Third, limited attention has been given to incorporating voltage stability margin (VSM) constraints, although such constraints are increasingly important as ADNs operate closer to technical limits [32].

To address these issues, a topology-switching soft open point (TS-SOP) assisted real-time cooperative operation (RTCO) framework of ADNs is proposed with explicit VSM constraints. Within this framework, a TS-SOP model with switchable feeder connections and converter operating modes is developed to enhance the power-flow controllability and reduce the no-load losses of converter. The VSM is explicitly incorporated into the multi-timescale optimization, and local droop control is implemented based on optimized volt/var droop curves. The main contributions are summarized as follows.

1) A TS-SOP assisted RTCO framework of ADNs is proposed with explicit VSM constraints. The proposed framework coordinates day-ahead pre-dispatch, intra-day dynamic correction, and real-time voltage regulation, featuring a two-stage volt/var control, thereby improving voltage regulation

capability, satisfying explicit VSM constraints, and enhancing operating economy.

2) A TS-SOP model with switchable feeder connections and converter operating modes is developed, improving the operating flexibility and power-flow controllability while reducing the no-load losses of converter compared with conventional fixed-topology SOPs.

3) Droop curve parameter optimization is deployed for the volt/var droop curves of PV inverters and TS-SOP, where critical operating points are identified and the VSM is imposed as an explicit constraint. The optimized volt/var droop curves are then used for local droop control, improving both voltage security and stability in ADNs.

The remainder of this paper is organized as follows. Section II presents the problem formulation and the proposed framework. Section III details the TS-SOP design, illustrating the TS-SOP model in ADN and the optimization formulation. Section IV presents case studies for model validation. Finally, Section V concludes this paper.

II. PROBLEM FORMULATION AND PROPOSED FRAMEWORK

A. Problem Formulation

This paper considers the real-time cooperative operation of ADNs with high PV penetration. The objective is to improve operating economy while maintaining voltage security and stability. In addition to conventional discrete devices such as OLTCs and CBs, power-electronic resources including the TS-SOP and PV inverters are utilized to provide real-time active and reactive power support for nodal-voltage regulation. Voltage security, voltage stability, and operating cost, mainly driven by network loss cost, are critical operating concerns in ADNs [33], [34], and are adopted as the primary performance metrics in this paper.

Although conventional SOPs have been widely investigated for mitigating voltage fluctuations and reducing losses, fixed feeder connections constrain the power flow flexibility, and no-load losses of converter persist when the transferred power is low. With increasing PV penetration, these limitations may reduce the voltage regulation headroom, driving the ADNs closer to voltage-security limits, and increasing the risk of voltage instability. Meanwhile, the growing portfolio of flexible resources such as PV inverters, energy storage systems, and SOPs, enhances controllability but increases operating complexity. Therefore, the central challenge is to coordinate heterogeneous resources to improve operating economy while maintaining voltage security and stability, which motivates the proposed framework in this paper.

B. Illustration of Proposed Framework

The proposed framework of ADNs with explicit VSM constraints to coordinate flexible resources is illustrated in Fig. 1. The proposed framework coordinates operations across multiple timescales, consisting of a day-ahead pre-dispatch stage (DPS) and a real-time scheduling stage (RTSS). The RTSS is further divided into an intra-day dynamic correction stage (IDCS) and a real-time voltage regulation stage (RVRS). Both the DPS and IDCS are formulated as central-

ized optimization problems, while the RVRS is implemented through real-time volt/var control. In Fig. 1, h is the current time index; T_{MPC} is the horizon length; and Δt , $\Delta \tau$, and $\Delta \varsigma$ will be explained in the following text.

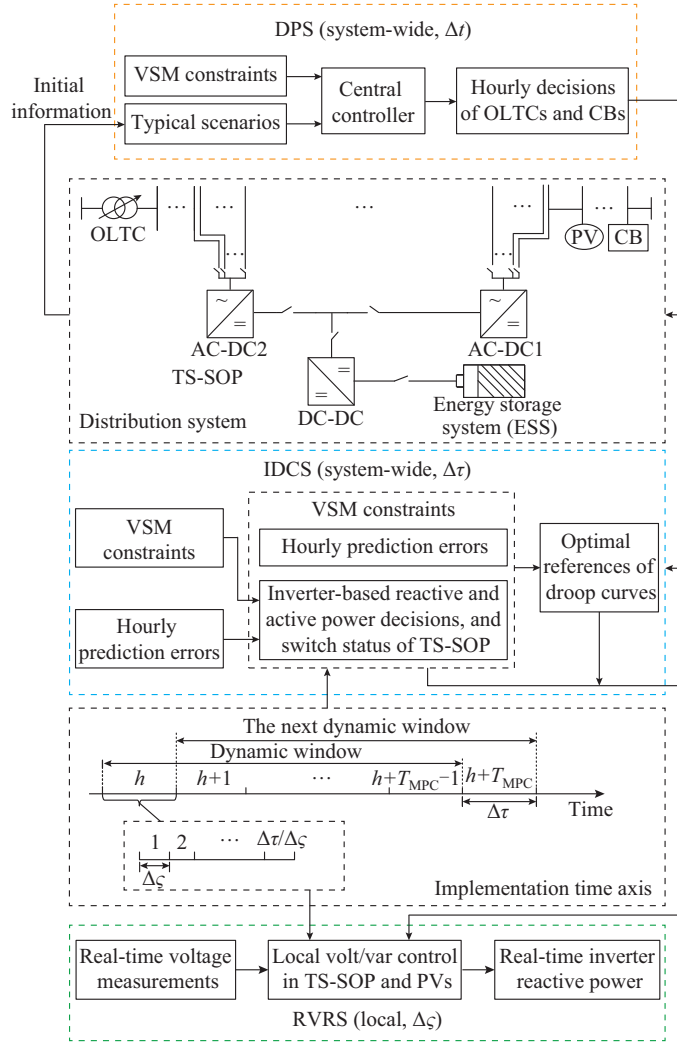


Fig. 1. Proposed framework of ADNs with explicit VSM constraints.

In the DPS, operating schedules for discrete devices such as OLTCs and CBs, as well as the TS-SOP, are determined by solving an operating-cost minimization problem with explicit VSM constraints over the scheduling interval Δt . The TS-SOP model incorporates switchable feeder connections and operating modes of converter, and it is embedded into the network power flow equations to improve the operating flexibility and economy. Within the RTSS, the IDCS applies rolling-horizon model predictive control (MPC) with a time step $\Delta \tau$, where a prescribed VSM requirement is enforced as a stability constraint in each prediction horizon. First, the IDCS determines the active power setpoints of TS-SOP and feeder-connection modes together with reactive power setpoints for all controllable devices. Next, droop curve parameters are optimized for the TS-SOP and PV inverters to support the RVRS. In the RVRS, the TS-SOP and PV inverters regulate reactive power outputs at a fine time resolution of

$\Delta \varsigma$ based on local voltage measurements and the optimized volt/var droop curves. This local regulation tracks the upper-layer setpoints, thereby maintaining voltage security and stability while improving the operating economy.

III. TS-SOP DESIGN

SOPs have been extensively studied for power flow optimization, service restoration, and voltage regulation, as they enable controllable exchange of active and reactive power between feeders. In most applications, however, feeder connections are fixed [22], and loss models are commonly linearized [24], [28]–[30]. These assumptions constrain the operating flexibility and may bias the evaluation of operating cost. To address these limitations, a TS-SOP is designed with an integrated ESS, as shown in Fig. 2.

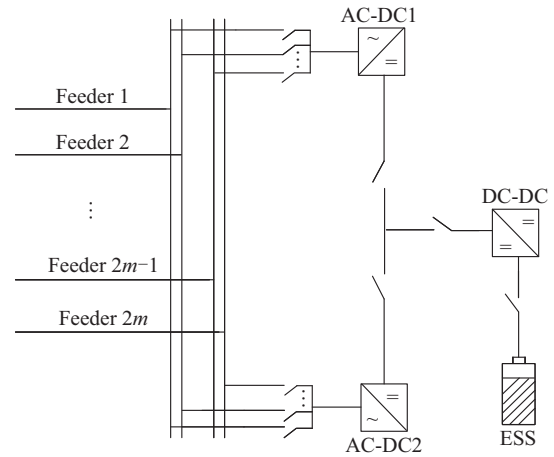


Fig. 2. Configuration of TS-SOP with integrated ESS.

The TS-SOP consists of two bidirectional AC-DC converters (i.e., AC-DC1 and AC-DC2), a bidirectional DC-DC converter, and an ESS. Each AC-DC converter can be connected to a selected feeder from a predefined set, and the DC-DC converter interfaces the ESS with the common DC bus. Four operating states of TS-SOP are shown in Fig. 3. During periods of low power transmission, the TS-SOP can switch to Mode 3 to shut down converters, thereby eliminating no-load losses. The TS-SOP interfaces with the ADN through m feeder pairs, corresponding to $2M$ feeders. The configuration with $M=2$ is studied in this paper. The TS-SOP operates in three modes. In Mode 1, it transfers active power between two selected feeders and provides reactive power support. Compared with a fixed-topology SOP, the TS-SOP improves flexibility by reconfiguring AC-side connections, as shown in Fig. 3(a) and (b). In Mode 2, it exchanges power with the ESS through a single feeder, as shown in Fig. 3(c), thereby enabling feeder-level support and temporal power shifting. When the inter-feeder transfer power is insufficient to offset the no-load losses of converters, the TS-SOP avoids Mode 1 and either operates in Mode 2 or switches to Mode 3 to shut down converters, which improves the TS-SOP utilization and reduces the no-load losses of converters.

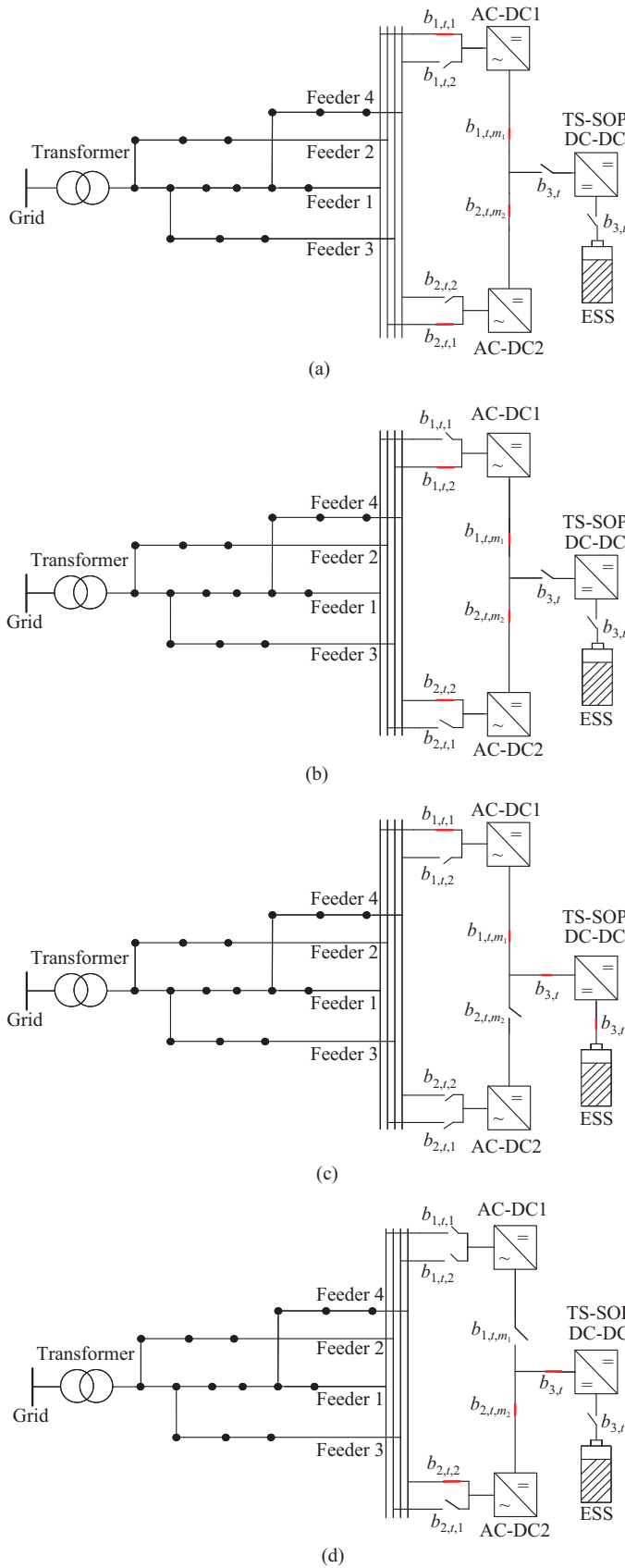


Fig. 3. Operating modes of TS-SOP. (a) Mode 1: connecting Feeder 1 and Feeder 2. (b) Mode 1: connecting Feeder 3 and Feeder 4. (c) Mode 2: only AC-DC1 converter is connected. (d) Mode 3: both AC-DC1 and AC-DC2 converters are de-energized.

Binary variables $b_{1,t,m}$, $b_{2,t,m}$, and $b_{3,t}$ are introduced to describe the converter conduction states, where t is the time index, and m is the feeder index. Specifically, $b_{1,t,m}=1$ (or $b_{2,t,m}=1$) indicates that AC-DC1 (or AC-DC2) is connected and conducting via feeder m , and $b_{3,t}=1$ indicates that the DC-DC converter is energized.

The TS-SOP operates in three modes described as follows.

1) Mode 1 requires $b_{1,t,m_1}=b_{2,t,m_2}=1$ for two distinct feeders ($m_1 \neq m_2$). The state of the DC-DC converter, $b_{3,t} \in \{0, 1\}$, remains an independent decision variable, permitting optional ESS use. Active power and reactive power are exchanged between the two connected feeders.

2) Mode 2 is activated when exactly one AC-DC converter is connected to a feeder m ($b_{i,t,m}=1$ for $i=1$ or 2), the opposite converter is idle ($b_{j,t,n}=0, \forall n$), and the DC-DC converter is on ($b_{3,t}=1$). The TS-SOP supports the single feeder using that AC-DC converter and the ESS.

3) Mode 3 ensures that all converters are de-energized, i.e., $b_{1,t,m}=b_{2,t,m}=0$ for every feeder m , and $b_{3,t}=0$. No-load losses are eliminated.

Compared with conventional fixed-topology SOPs, the TS-SOP enhances power transfer flexibility and exploits the capability to de-energize converters to reduce no-load losses, thereby improving the operating economy of ADN.

A. TS-SOP Model in ADN

This subsection develops an integrated mathematical TS-SOP model in ADN, which includes loss models that account for the no-load losses of converter, TS-SOP operating constraints, and LinDistFlow-based network power flow equations.

1) Loss Models

The total losses of the TS-SOP include the losses of the two AC-DC converters, the DC-DC converter, and the ESS.

1) AC-DC converters

The losses of the two AC-DC converters are modeled in (1) and (2). Each converter is connected to the common DC bus on the DC side and to a selected feeder on the AC side. To enable dynamic feeder-connection reconfiguration, the feeder connection of each AC-DC converter is modeled using binary selection variables associated with the AC-side switch matrix.

$$P_{1,t,m}^{\text{TS-SOP,L}} = b_{1,t,m} (c_{a0} + c_{a1} S_{1,t,m}^{\text{TS-SOP}} + c_{a2} (S_{1,t,m}^{\text{TS-SOP}})^2) \quad (1)$$

$$P_{2,t,m}^{\text{TS-SOP,L}} = b_{2,t,m} (c_{a0} + c_{a1} S_{2,t,m}^{\text{TS-SOP}} + c_{a2} (S_{2,t,m}^{\text{TS-SOP}})^2) \quad (2)$$

where $P_{1,t,m}^{\text{TS-SOP,L}}$ and $P_{2,t,m}^{\text{TS-SOP,L}}$ are the active power losses of AC-DC1 and AC-DC2 connected to the m^{th} feeder at time t , respectively; $S_{1,t,m}^{\text{TS-SOP}}$ and $S_{2,t,m}^{\text{TS-SOP}}$ are the corresponding apparent power magnitudes of AC-DC1 and AC-DC2 converters connected to the m^{th} feeder at time t , respectively; and c_{a0} , c_{a1} , and c_{a2} are the no-load loss coefficient, constant voltage loss coefficient, and ohmic loss coefficient of AC-DC converters, respectively.

2) DC-DC converter

The loss of the DC-DC converter, which connects the ESS device to the DC bus, is modeled as:

$$P_{3,t}^{\text{TS-SOP,L}} = b_{3,t} (c_{d0} + c_{d1} |P_{3,t}^{\text{TS-SOP}}| + c_{d2} (P_{3,t}^{\text{TS-SOP}})^2) \quad (3)$$

where $P_{3,t}^{\text{TS-SOP,L}}$ and $P_{3,t}^{\text{TS-SOP}}$ are the loss and active power of

the DC-DC converter at time t , respectively; $b_{3,t}$ is the binary conduction-state variable of the DC-DC converter; c_{d0} , c_{d1} , and c_{d2} are the no-load loss coefficient, constant voltage loss coefficient, and ohmic loss coefficient of DC-DC converters, respectively.

3) ESS

The loss associated with the ESS is expressed as:

$$P_{4,t}^{\text{TS-SOP,L}} = c_{\text{aux}} + c_{\text{ESS}} (P_{4,t}^{\text{TS-SOP}})^2 \quad (4)$$

where $P_{4,t}^{\text{TS-SOP,L}}$ and $P_{4,t}^{\text{TS-SOP}}$ are the loss and active power of the ESS at time t ; c_{aux} is the loss coefficient of auxiliary and management systems; and c_{ESS} is the internal resistance loss coefficient of the ESS.

2) TS-SOP Operating Constraints

Along with the loss models in (1)-(4), the TS-SOP is subject to the following operating constraints.

1) Active power constraints

The active power balance between the ESS and the DC-DC converter is given by:

$$P_{3,t}^{\text{TS-SOP}} = -(P_{4,t}^{\text{TS-SOP}} - P_{4,t}^{\text{TS-SOP,L}}) \quad (5)$$

And the active power balance of the TS-SOP at the DC bus is enforced by:

$$\sum_{m \in N_m} P_{1,t,m}^{\text{TS-SOP}} + \sum_{m \in N_m} P_{2,t,m}^{\text{TS-SOP}} + P_{3,t}^{\text{TS-SOP}} + \sum_{m \in N_m} P_{1,t,m}^{\text{TS-SOP,L}} + \sum_{m \in N_m} P_{2,t,m}^{\text{TS-SOP,L}} + P_{3,t}^{\text{TS-SOP,L}} = 0 \quad (6)$$

where $P_{1,t,m}^{\text{TS-SOP}}$ and $P_{2,t,m}^{\text{TS-SOP}}$ are the active power outputs of AC-DC1 and AC-DC2 converters connected to the m^{th} feeder at time t , respectively; and N_m is the set of feeder positions associated with the TS-SOP.

2) Capacity constraints

The apparent power ratings of the two AC-DC converters and the active power rating of the DC-DC converter are imposed as:

$$\sqrt{(P_{1,t,m}^{\text{TS-SOP}})^2 + (Q_{1,t,m}^{\text{TS-SOP}})^2} \leq S_{\text{max}}^{\text{AC-DC1}} \quad (7)$$

$$\sqrt{(P_{2,t,m}^{\text{TS-SOP}})^2 + (Q_{2,t,m}^{\text{TS-SOP}})^2} \leq S_{\text{max}}^{\text{AC-DC2}} \quad (8)$$

$$-S_{\text{max}}^{\text{DC-DC}} \leq P_{3,t}^{\text{TS-SOP}} \leq S_{\text{max}}^{\text{DC-DC}} \quad (9)$$

where $Q_{1,t,m}^{\text{TS-SOP}}$ and $Q_{2,t,m}^{\text{TS-SOP}}$ are the reactive power outputs of AC-DC1 and AC-DC2 converters connected to the m^{th} feeder at time t , respectively; $S_{\text{max}}^{\text{AC-DC1}}$ and $S_{\text{max}}^{\text{AC-DC2}}$ are the apparent power ratings of AC-DC1 and AC-DC2 converters, respectively; and $S_{\text{max}}^{\text{DC-DC}}$ is the active power rating of the DC-DC converter.

3) ESS constraints

The state-of-charge (SoC) dynamics of the ESS are modeled as:

$$SoC_{t+1} = SoC_t - \frac{(P_{4,t}^{\text{TS-SOP}} + P_{4,t}^{\text{TS-SOP,L}}) \Delta t}{E_{\text{rated}}} \quad (10)$$

where SoC_t and SoC_{t+1} are the SoC values of the ESS at time intervals t and $t+1$, respectively; Δt is the length of one time interval; and E_{rated} is the rated energy capacity of the ESS. The charging/discharging power limits and SoC limits are imposed by (11) and (12), respectively. The cyclic SoC condition over the scheduling horizon is enforced as (13).

$$-P_{\text{max}}^{\text{ESS}} \leq P_{4,t}^{\text{TS-SOP}} \leq P_{\text{max}}^{\text{ESS}} \quad (11)$$

$$SoC_{\text{min}} \leq SoC_t \leq SoC_{\text{max}} \quad (12)$$

$$SoC_1 = SoC_T \quad (13)$$

where $P_{\text{max}}^{\text{ESS}}$ is the maximum charging/discharging power of the ESS; SoC_{min} and SoC_{max} are the lower and upper limits of SoC, respectively; and SoC_1 and SoC_T are the initial and final SoC values, respectively.

4) Feeder selection constraints

To ensure that each AC-DC converter is connected to at most one feeder at each time and that the feeder connections are consistent with the TS-SOP topology shown in Fig. 3, the feeder selection constraints are imposed as:

$$\begin{cases} \sum_{p \in N_m} b_{1,t,p} \leq 1 \\ \sum_{q \in N_m} b_{2,t,q} \leq 1 \end{cases} \quad (14)$$

$$b_{1,t,p} + b_{2,t,q} \leq 1 \quad \forall p, q \in N_m, p \neq q \quad (15)$$

where $b_{1,t,p}$ and $b_{2,t,q}$ are the binary variables indicating the connection of AC-DC1 converter to feeder p and AC-DC2 converter to feeder q at time t , respectively. Equation (15) enforces coordinated switching such that the two AC-DC converters do not connect to different feeders that would violate the predefined TS-SOP topology, thereby supporting flexible and secure power exchange among multiple feeders.

3) LinDistFlow-based Network Power Flow Equations

The LinDistFlow-based network power flow equations represent the ADN with PVs, CBs, and TS-SOP [13], offering a good trade-off between computational efficiency and accuracy for radial distribution networks under normal operating conditions. The corresponding power flow equations are given as:

$$\sum_{ji \in \Omega_b} P_{t,ji} + P_{t,i} = \sum_{ik \in \Omega_b} P_{t,ik} \quad (16)$$

$$\sum_{ji \in \Omega_b} Q_{t,ji} + Q_{t,i} = \sum_{ik \in \Omega_b} Q_{t,ik} \quad (17)$$

$$V_{t,i} - V_{t,j} - \frac{r_{ij} P_{t,ij} + x_{ij} Q_{t,ij}}{V_1} = 0 \quad (18)$$

$$P_{t,i} = P_{t,i}^{\text{PV}} - P_{t,i}^{\text{L}} + P_{t,i}^{\text{TS-SOP}} \quad (19)$$

$$Q_{t,i} = Q_{t,i}^{\text{PV}} - Q_{t,i}^{\text{L}} + Q_{t,i}^{\text{TS-SOP}} + Q_{t,i}^{\text{CB}} \quad (20)$$

where $P_{t,ij}$ and $P_{t,ik}$ are the active power flows entering node i from upstream node j and leaving node i towards downstream node k at time t , respectively; $Q_{t,ij}$ and $Q_{t,ik}$ are the corresponding reactive power flows; $P_{t,i}$ and $Q_{t,i}$ are the net active and reactive power injections at node i at time t , respectively; $V_{t,i}$ is the voltage magnitude at node i at time t ; Ω_b is the set of branches; r_{ij} and x_{ij} are the resistance and reactance of branch ij , respectively; V_1 is the reference (slack-bus) voltage magnitude; $P_{t,i}^{\text{PV}}$ and $Q_{t,i}^{\text{PV}}$ are the active and reactive power outputs of the PV inverter at node i at time t , respectively; $P_{t,i}^{\text{L}}$ and $Q_{t,i}^{\text{L}}$ are the active and reactive power demands at node i at time t , respectively; $Q_{t,i}^{\text{CB}}$ is the reactive power injection of the CB at node i at time t ; and $P_{t,i}^{\text{TS-SOP}}$ and $Q_{t,i}^{\text{TS-SOP}}$ are the active and reactive power injections of

the TS-SOP connected to the m^{th} feeder at node i at time t , respectively.

The nodal voltage security constraints of the ADN are formulated as:

$$\underline{V} \leq V_{t,i} \leq \bar{V} \quad \forall t, i \quad (21)$$

where $\underline{V}$ and $\bar{V}$ are the lower and upper voltage limits, respectively.

B. Cooperative Operation Considering Voltage Stability

Stochastic PV generation can aggravate the power flow imbalance among feeders, increase network losses, and raise the likelihood of voltage violations, thereby driving ADNs closer to voltage-security limits. Therefore, the VSM is imposed as an explicit operating constraint.

The VSM-constrained DPS problem of the ADN is formulated as:

$$\min f_{\text{PRE}} = \sum_{c \in C} \sum_{t \in \mathcal{T}} (f_{\text{loss},t}^{(c)} + f_{\text{switch},t}^{(c)} + f_{\text{cut},t}^{(c)}) \Delta t \quad (22)$$

$$\begin{cases} f_{\text{loss},t}^{(c)} = C_{\text{loss}} \left[\sum_{ij \in \mathcal{Q}_b} P_{t,ij}^{\text{loss}(c)} + P_{3,t}^{\text{TS-SOP,L}(c)} + P_{4,t}^{\text{TS-SOP,L}(c)} + \sum_{m \in \mathcal{N}_m} (P_{1,t,m}^{\text{TS-SOP,L}(c)} + P_{2,t,m}^{\text{TS-SOP,L}(c)}) \right] \Delta t \\ f_{\text{switch},t}^{(c)} = \sum_{ij \in \mathcal{Q}_0} C_{\text{tap}} |K_{t,ij}^{(c)} - K_{t-1,ij}^{(c)}| + \sum_{i \in \mathcal{Q}_s} C_{\text{cap}} |N_{t,i}^{\text{CB}(c)} - N_{t-1,i}^{\text{CB}(c)}| \\ f_{\text{cut},t}^{(c)} = C_{\text{cut}} \sum_{i \in \mathcal{Q}_s} P_{t,i}^{\text{PV,cut}(c)} \Delta t \end{cases} \quad (23)$$

where f_{PRE} is the total operating cost in the pre-dispatch stage; $f_{\text{loss},t}^{(c)}$, $f_{\text{switch},t}^{(c)}$, and $f_{\text{cut},t}^{(c)}$ are the costs associated with active power losses, switching actions, and active power curtailment of PV under operating condition c at time t , respectively; C_{loss} , C_{tap} , C_{cap} , and C_{cut} are the corresponding cost coefficients; $P_{t,ij}^{\text{loss}(c)}$ is the active power loss of branch ij under operating condition c at time t ; $P_{1,t,m}^{\text{TS-SOP,L}(c)}$ and $P_{2,t,m}^{\text{TS-SOP,L}(c)}$ are the active power losses of AC-DC1 and AC-DC2 converters connected to the m^{th} feeder under operating condition c at time t , respectively; $P_{3,t}^{\text{TS-SOP,L}(c)}$ and $P_{4,t}^{\text{TS-SOP,L}(c)}$ are the losses of the DC-DC converter and the ESS under operating condition c at time t , respectively; $K_{t,ij}^{(c)}$ is the OLTC tap position on branch ij under operating condition c at time t ; $N_{t,i}^{\text{CB}(c)}$ and $P_{t,i}^{\text{PV,cut}(c)}$ are the number of switched CB units and the curtailed PV active power at node i under operating condition c at time t , respectively; $\mathcal{Q}_0$, $\mathcal{Q}_T$, and $\mathcal{Q}_s$ are the sets of OLTC branches, time periods, and nodes, respectively; and $C = \{C_{\text{BN}}, C_{\text{BL}}\}$ is the set of the base operating condition C_{BN} and the critical loading condition C_{BL} .

Objective (22) is subject to various constraints, which are described as:

1) Network Operating Constraints

Under C_{BN} and C_{BL} , the ADN satisfies the LinDistFlow-based network power flow model with the TS-SOP in (16)-(20). Under C_{BN} , the nodal voltage security constraints in (21) are also enforced.

2) TS-SOP Operating Constraints

Under C_{BN} and C_{BL} , the TS-SOP satisfies the loss models and operating constraints in (1)-(15).

3) PV Operating Constraints

The PV active power is modeled as (24) and (25). Meanwhile, the PV capacity limit is given as (26).

$$P_{t,i}^{\text{PV}(c)} = P_{t,i}^{\text{PV,av}(c)} - P_{t,i}^{\text{PV,cut}(c)} \quad (24)$$

$$0 \leq P_{t,i}^{\text{PV,cut}(c)} \leq P_{t,i}^{\text{PV,av}(c)} \quad (25)$$

$$(P_{t,i}^{\text{PV}(c)})^2 + (Q_{t,i}^{\text{PV}(c)})^2 \leq (S_i^{\text{PV,max}})^2 \quad (26)$$

where $P_{t,i}^{\text{PV}(c)}$, $Q_{t,i}^{\text{PV}(c)}$, $P_{t,i}^{\text{PV,av}(c)}$, and $P_{t,i}^{\text{PV,cut}(c)}$ are the active power output, reactive power output, available active power, and active power reduction of the PV inverter at node i under operating condition c at time t , respectively; and $S_i^{\text{PV,max}}$ is the capacity of PV at node i .

4) OLTC and CB Operating Constraints

Discrete operating constraints for OLTCs and CBs are imposed under C_{BN} and C_{BL} using the models in [23].

5) VSM Constraints

To account for voltage stability explicitly, a critical loading condition C_{BL} is constructed by proportionally scaling the base-load condition C_{BN} . Accordingly, the active and reactive load demands at node i at time t under C_{BL} are defined by scaling those under C_{BN} , as given in (27).

$$P_{t,i}^{\text{L}(C_{\text{BL}})} = \beta P_{t,i}^{\text{L}(C_{\text{BN}})} \quad (27)$$

$$\frac{P_{t,i}^{\text{L}(c)}}{Q_{t,i}^{\text{L}(c)}} = H_i \quad (28)$$

where $P_{t,i}^{\text{L}(c)}$ and $Q_{t,i}^{\text{L}(c)}$ are the active and reactive power demands at node i under operating condition c at time t , respectively, and in particular, $P_{t,i}^{\text{L}(C_{\text{BN}})}$ and $P_{t,i}^{\text{L}(C_{\text{BL}})}$ are the active power demands under C_{BN} and C_{BL} , respectively; H_i is the constant power factor at node i ; and β is the loading factor bounded to satisfy the target VSM requirement as follows:

$$\beta \geq \frac{1}{1 - \text{VSM}_{\text{target}}} \quad (29)$$

where $\text{VSM}_{\text{target}}$ is the minimum required VSM. In evaluating the VSM constraints under C_{BL} , reactive power control settings of PV inverters and the TS-SOP, together with the discrete settings of OLTCs and CBs, are fixed at their values under the base operating condition C_{BN} .

Therefore, the pre-scheduling model is formulated as Problem P1:

$$\begin{cases} \min f_{\text{PRE}} = \sum_{c \in C} \sum_{t \in \mathcal{T}} (f_{\text{loss},t}^{(c)} + f_{\text{switch},t}^{(c)} + f_{\text{cut},t}^{(c)}) \Delta t \\ \text{s.t. TS-SOP operating constraints (1)-(15)} \\ \text{Power flow constraints (16)-(21)} \\ \text{VSM constraints (27)-(29)} \\ \text{Operating constraint of other flexible resources} \end{cases} \quad (30)$$

C. Real-time Operation Under Uncertainties

The RTSS mitigates prediction errors and stochastic variations in PV generation and load demand via the IDCS implemented using rolling-horizon MPC and the RVRS.

1) IDCS

In the IDCS, a two-step rolling-horizon MPC scheme is established with explicit VSM constraints. The IDCS uses a control time interval Δt and a prediction horizon $\mathcal{T}_h = \{h, h +$

$1, \dots, h + T_{\text{MPC}} - 1\}$, where h is the current time index; and T_{MPC} is the horizon length. In this paper, T_{MPC} is set to be 8. Within each horizon, the updated predictions of PV generation and load demand are acquired, and a two-step optimization problem is solved in a receding horizon manner. First, the MPC problem determines active power setpoints and TS-SOP operating modes. The resulting setpoints are then used to optimize volt/var droop curve parameters for the TS-SOP and PV inverters. Only the decisions for the first control interval are implemented, after which the horizon advances and the procedure is repeated using updated measurements and predictions.

1) Optimization of setpoints of TS-SOP and PV inverters

In the first step, the objective is to minimize the sum of operating cost and voltage deviation over the prediction horizon. Decision variables include TS-SOP configuration and operating modes, active and reactive power setpoints of the TS-SOP, and active power curtailment and reactive power setpoints of PV inverters. The optimized reactive power setpoints of the TS-SOP and PV inverters serve as target operating points for droop curve parameter optimization in the second step, while the remaining setpoints and switching decisions are implemented in the subsequent RVRS. Through this receding-horizon feedback mechanism, short-term voltage fluctuations can be mitigated, and the likelihood of voltage violations can be reduced, while VSM constraints are enforced within each prediction horizon.

The optimization model in this step is formulated as:

$$\min f_{\text{MPC}} = \sum_{c \in C} \sum_{h \in T_p} [(f_{\text{loss},h}^{(c)} + f_{\text{cut},h}^{(c)}) \Delta \tau + \alpha f_{v,h}^{(c)}] \quad (31)$$

where f_{MPC} is the weighted sum of total operating cost and voltage deviation over the MPC prediction horizon; $f_{\text{loss},h}^{(c)}$ and $f_{\text{cut},h}^{(c)}$ are defined as in (23) with $t=h$ under operating condition c ; and α is a positive parameter that weights the operating cost and voltage deviation terms.

The voltage deviation term $f_{v,h}^{(c)}$ is given by:

$$f_{v,h}^{(c)} = \sum_{i \in \Omega_s} |V_{h,i}^{(c)} - \tilde{V}| \quad (32)$$

where $V_{h,i}^{(c)}$ is the voltage magnitude at node i under operating condition c at time h ; $\tilde{V}$ is the nominal voltage (typically 1.0 p.u.); and Ω_s is the set of nodes equipped with PV inverters and CBs. In practical implementation, $f_{v,h}^{(c)}$ is used to penalize voltage deviations, with an emphasis on violations outside the acceptable voltage band $[\underline{V}_{\text{thr}}, \bar{V}_{\text{thr}}]$, where $\underline{V}_{\text{thr}}$ and $\bar{V}_{\text{thr}}$ are the lower and upper voltage thresholds, respectively.

To preserve the reactive power headroom of the TS-SOP and PV inverters for real-time volt/var control, the apparent power limits in (7), (8), and (26) are tightened using a reservation factor σ , where $0 \leq \sigma < 1$, yielding:

$$\sqrt{(P_{1,h,m}^{\text{TS-SOP}(c)})^2 + (Q_{1,h,m}^{\text{TS-SOP}(c)})^2} \leq (1 - \sigma) S_{\text{max}}^{\text{AC-DC1}} \quad (33)$$

$$\sqrt{(P_{2,h,m}^{\text{TS-SOP}(c)})^2 + (Q_{2,h,m}^{\text{TS-SOP}(c)})^2} \leq (1 - \sigma) S_{\text{max}}^{\text{AC-DC2}} \quad (34)$$

$$(P_{h,i}^{\text{PV}(c)})^2 + (Q_{h,i}^{\text{PV}(c)})^2 \leq (1 - \sigma) (S_i^{\text{PV,max}})^2 \quad (35)$$

where $P_{k,h,m}^{\text{TS-SOP}(c)}$ and $Q_{k,h,m}^{\text{TS-SOP}(c)}$ are the active power and reactive power of AC-DC converter k in the TS-SOP connected

to feeder m under operating condition c at time h , respectively, with $k \in \{1, 2\}$; $S_{\text{max}}^{\text{AC-DC1}}$ and $S_{\text{max}}^{\text{AC-DC2}}$ are the corresponding apparent power ratings; $P_{h,i}^{\text{PV}(c)}$ and $Q_{h,i}^{\text{PV}(c)}$ are the active and reactive power of the PV inverter at node i under operating condition c , respectively; $S_i^{\text{PV,max}}$ is the apparent power rating of the PV inverter at node i ; and σ is a parameter that represents the fraction of apparent power reserved for the RVRS.

Accordingly, the dynamic correction optimization problem can be expressed as problem P2:

$$\begin{cases} \min f_{\text{MPC}} = \sum_{c \in C} \sum_{h \in T_p} [(f_{\text{loss},h}^{(c)} + f_{\text{cut},h}^{(c)}) \Delta \tau + \alpha f_{v,h}^{(c)}] \\ \text{s.t. TS-SOP operating constraints (1)-(15), (32), and (33)} \\ \text{Power flow constraints (16)-(21)} \\ \text{VSM constraints (27)-(29)} \\ \text{PV operating constraints (24)-(26), and (34)} \end{cases} \quad (36)$$

2) Optimization of volt/var droop curve parameters of TS-SOP and PV inverters

In the second step, volt/var droop curve parameters of the TS-SOP and PV inverters are optimized under the base operating condition C_{BN} by minimizing the objective in (31). The optimized volt/var droop curves enable the TS-SOP and PV inverters to adjust reactive power output in the RVRS based on local voltage measurements, thereby improving the voltage regulation capability. Decision variables are the break-point voltages of the volt/var droop curves under C_{BN} , including $V_{\min}$, $V_{h,i}^L$, $V_{h,i}^U$, and $V_{\max}$, as illustrated in Fig. 4, where $V_{\min}$ and $V_{\max}$ are the lower and upper voltage limits of droop curve, respectively; and $V_{h,i}^L$ and $V_{h,i}^U$ are the decision variables that define the lower and upper bounds of the voltage deadband for the inverter at node i at time h , respectively. In Fig. 4, $V_{h,i}$ and $Q_{h,i}^{\text{TS-SOP/PV}}$ are the local voltage measurement and the actual reactive power output of the TS-SOP or PV inverter at node i at time h , respectively; $Q_{h,i,\text{ref}}^{\text{TS-SOP/PV}}$ is the corresponding reactive power reference determined by the upper-layer scheduling; and $Q_{h,i,\text{max}}^{\text{TS-SOP/PV}}$ is the corresponding maximum reactive power capacity of the device.

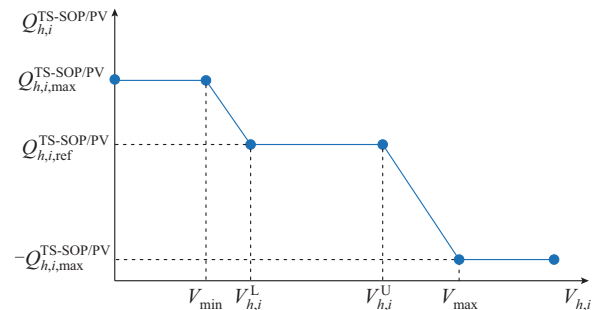


Fig. 4. Volt/var droop curves of TS-SOP and PV inverters under normal operating conditions.

The maximum achievable reactive power of the TS-SOP and PV inverters is given by:

$$Q_{h,i,\text{max}}^{\text{TS-SOP}(c)} = \sqrt{(S_{\text{max}}^{\text{AC-DC1/2}})^2 - (P_{h,i,\text{ref}}^{\text{TS-SOP}(c)})^2} \quad (37)$$

$$Q_{h,i,\text{max}}^{\text{PV}(c)} = 0.44 S_i^{\text{PV,max}} \quad (38)$$

where $Q_{h,i,\max}^{\text{TS-SOP}(c)}$ and $Q_{h,i,\max}^{\text{PV}(c)}$ are the maximum reactive power injections or absorptions of the TS-SOP and PV inverter at node i at time h , respectively; $P_{h,i,\text{ref}}^{\text{TS-SOP}(c)}$ is the active power reference of the TS-SOP obtained from Problem P1; and $S_{\max}^{\text{AC-DC1/2}}$ is the apparent power rating of the corresponding AC-DC converters. Coefficient 0.44 reflects the maximum reactive power capability at power factor 0.9.

The reactive power output under the base operating condition C_{BN} is then expressed as:

$$Q_{h,i}^{\text{TS-SOP/PV}(C_{\text{BN}})} = g_i(V_{h,i}^{(C_{\text{BN}})}) Q_{h,i,\max}^{\text{TS-SOP/PV}(C_{\text{BN}})} \quad (39)$$

where $Q_{h,i}^{\text{TS-SOP/PV}(C_{\text{BN}})}$ and $Q_{h,i,\max}^{\text{TS-SOP/PV}(C_{\text{BN}})}$ are the reactive power and the maximum reactive power of the TS-SOP or PV inverter at node i under operating condition C_{BN} at time h , respectively; and $g_i(\cdot)$ is the normalized droop function that maps the nodal voltage $V_{h,i}^{(C_{\text{BN}})}$ to the per-unit reactive power request.

The piecewise function $g_i(\cdot)$ is defined as:

$$g_i(V_{h,i}^{(C_{\text{BN}})}) = \begin{cases} 1 & V_{h,i}^{(C_{\text{BN}})} \leq V_{\min} \\ 1 + \frac{(1 - w_{h,i})(V_{h,i}^{(C_{\text{BN}})} - V_{\min})}{V_{\min} - V_{h,i}^L} & V_{\min} < V_{h,i}^{(C_{\text{BN}})} < V_{h,i}^L \\ w_{h,i} & V_{h,i}^L \leq V_{h,i}^{(C_{\text{BN}})} \leq V_{h,i}^U \\ \frac{(1 + w_{h,i})(V_{h,i}^{(C_{\text{BN}})} - V_{\max})}{V_{h,i}^U - V_{\max}} - 1 & V_{h,i}^U < V_{h,i}^{(C_{\text{BN}})} \leq V_{\max} \\ -1 & V_{h,i}^{(C_{\text{BN}})} > V_{\max} \end{cases} \quad (40)$$

$$w_{h,i} = \frac{Q_{h,i,\text{ref}}^{\text{TS-SOP/PV}(C_{\text{BN}})}}{Q_{h,i,\max}^{\text{TS-SOP/PV}(C_{\text{BN}})}} \quad (41)$$

where $Q_{h,i,\text{ref}}^{\text{TS-SOP/PV}(C_{\text{BN}})}$ is the reference reactive power of the TS-SOP or PV inverter at node i under operating condition C_{BN} at time h , obtained from Problem P1; and $w_{h,i}$ is the normalized reactive power reference defined as the ratio of the reference reactive power from Problem P1 to the maximum achievable reactive power at node i at time h .

The volt/var droop curve parameters of the TS-SOP and PV inverters are optimized jointly with the scheduling decisions by minimizing (31). The resulting active power schedules and droop curves are updated during the control time interval $\Delta\tau$.

2) RVRS

Although the scheduling decisions obtained from Problem P2 can mitigate the impact of uncertainties, strong PV variability may still lead to voltage violations in real-time operation. Therefore, an RVRS is implemented at a finer time resolution with step size $\Delta\zeta$ to enforce voltage security at each control interval. In this paper, $\Delta\zeta = 1$ min. In this stage, reactive power outputs of the TS-SOP and PV inverters are adjusted based on local voltage measurements and the volt/var droop curves obtained from Problem P2.

1) Real-time volt/var control of PV inverters

The PV inverter at node i follows a piecewise volt/var droop curve defined by (37)-(41). Let $V_{\zeta,i}$ denote the instantaneous voltage at node i at real-time step ζ , and $Q_{\zeta,i,\max}^{\text{PV}}$ and $Q_{\zeta,i,\text{ref}}^{\text{PV}}$ denote the maximum reactive power capability and re-

active power setpoint at node i at real-time ζ , respectively. The reactive power updating rule is given in (SA1) in Supplementary Material A.

2) Real-time volt/var control of TS-SOP

The real-time volt/var control of the TS-SOP follows the same volt/var droop principle as that of PV inverters. Let $Q_{\zeta,i,\max}^{\text{TS-SOP}}$ and $Q_{\zeta,i,\text{ref}}^{\text{TS-SOP}}$ denote the maximum reactive power capability and reactive power setpoint of the TS-SOP at node i at real-time step ζ , respectively. The corresponding updating rule is provided in (SA2) in Supplementary Material A.

In summary, the proposed framework enables coordinated operation to improve the operating economy while maintaining voltage security and stability in ADNs with high PV penetration. Through tight multi-timescale coordination, the proposed framework seamlessly integrates slower centralized scheduling with fast local droop control. In addition, the TS-SOP with switchable feeder connections and flexible converter operating modes (including a shutdown mode) reduces no-load losses of converter and strengthens the voltage regulation capability.

D. Model Transformation

To solve the proposed framework efficiently, problem P1 and problem P2 are reformulated as mixed-integer second-order cone programming (MISOCP) problems by combining linearization with second-order cone reformulations. This reformulation handles nonlinear and nonsmooth constraints arising from quadratic terms, piecewise-defined functions, and absolute-value expressions.

The absolute value terms in the switching cost functions in (23) and in the OLTC and CB operating constraints are linearized using the method in [22]. The quadratic apparent power limit for PV inverters in (26) is recast in SoC form as (42), which enables an MISOCP reformulation.

$$(P_{t,i}^{\text{PV}(c)})^2 + (Q_{t,i}^{\text{PV}(c)})^2 \leq 2 \left(\frac{S_i^{\text{PV},\max}}{\sqrt{2}} \right)^2 \quad (42)$$

Similarly, the voltage deviation penalty term in (32) is linearized using the method in [22]. The tightened capacity constraints in (33) - (35) are expressed directly as SoC constraints. The remaining nonlinearities mainly arise from the piecewise volt/var droop curve relationship in (40) and its alignment with the reference term in (41).

To linearize the droop function $g_i(V_{h,i}^{(C_{\text{BN}})})$, continuous auxiliary variables $a_{h,i,k}$ are introduced for $k \in \{1, 2, \dots, 6\}$, satisfying:

$$V_{h,i}^{(C_{\text{BN}})} = 0.9a_{h,i,1} + V_{\min}a_{h,i,2} + V_{h,i}^La_{h,i,3} + V_{h,i}^Ua_{h,i,4} + V_{\max}a_{h,i,5} + 1.1a_{h,i,6} \quad (43)$$

$$g_i(V_{h,i}^{(C_{\text{BN}})}) = a_{h,i,1} + a_{h,i,2} + w_{h,i}a_{h,i,3} + w_{h,i}a_{h,i,4} - a_{h,i,5} - a_{h,i,6} \quad (44)$$

To enforce the selection of a single segment in the piecewise function, continuous variables $a_{h,i,k}$ are modeled as convex-combination weights. Binary variables $s_{h,i,e}$ select the active segment, where $s_{h,i,e} \in \{0, 1\}$. The segment selection is enforced through the constraints as:

$$\begin{cases} a_{h,i,1} \leq s_{h,i,1} \\ a_{h,i,6} \leq s_{h,i,5} \end{cases} \quad (45)$$

$$a_{h,i,e} \leq s_{h,i,e} + s_{h,i,e-1} \quad e \in \{2, 3, 4, 5\} \quad (46)$$

$$\begin{cases} \sum_{k=1}^6 a_{h,i,k} = 1 \\ \sum_{e=1}^5 s_{h,i,e} = 1 \end{cases} \quad (47)$$

$$a_{h,i,k} \geq 0 \quad k \in \{1, 2, \dots, 6\} \quad (48)$$

The bilinear terms $V_{h,i}^L a_{h,i,3}$ and $V_{h,i}^U a_{h,i,4}$ in (43) are linearized using binary expansion. Taking $V_{h,i}^L a_{h,i,3}$ as an example, $V_{h,i}^L$ is represented as $V_{\min} + 0.01 \sum_{d=0}^3 2^d l_{h,i,d}$, where $l_{h,i,d} \in \{0, 1\}$.

Continuous variables $x_{h,i,d} = l_{h,i,d} a_{h,i,3}$ are introduced, and an auxiliary variable $\beta_{h,i}$ is defined. Consequently, the original bilinear term $V_{h,i}^L a_{h,i,3}$ is equivalently replaced by the linear expression $V_{\min} a_{h,i,3} + 0.01 \beta_{h,i}$. With a big- M constant M selected based on valid bounds of $a_{h,i,3}$, the product term $x_{h,i,d}$ can be linearized using the constraints as:

$$\beta_{h,i} = \sum_{d=0}^3 2^d x_{h,i,d} \quad (49)$$

$$a_{h,i,3} - M(1 - l_{h,i,d}) \leq x_{h,i,d} \leq a_{h,i,3} \quad (50)$$

$$0 \leq x_{h,i,d} \leq M l_{h,i,d} \quad d \in \{0, 1, 2, 3\} \quad (51)$$

IV. CASE STUDIES FOR MODEL VALIDATION

A. Test System Setups

Simulation is conducted on a modified IEEE 33-node system equipped with a four-feeder TS-SOP, corresponding to four candidate feeder-connection points, as shown in Fig. 5. The nominal line-to-line voltage is 12.66 kV. Six PV units are installed at selected nodes, and their locations and rated apparent power are provided in Supplementary Material A Table SAI. To emulate renewable and load prediction errors, multiplicative perturbations of $\pm 20\%$ in PV output and $\pm 2\%$ in load demand are imposed on the corresponding daily profiles.

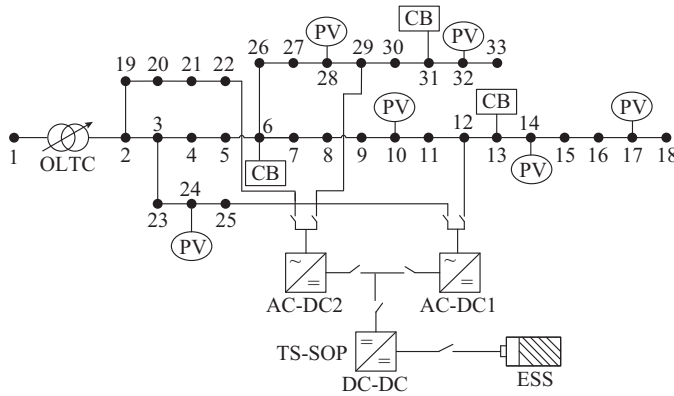


Fig. 5. Modified IEEE 33-node system equipped with a four-feeder TS-SOP.

The TS-SOP comprises two AC-DC converters, one DC-DC converter, an ESS, and an AC-side switch matrix. In the modified IEEE 33-node system, AC-DC1 converter can be connected to node 22 or node 25, and AC-DC2 converter can be connected to node 12 or node 29, through the AC-side switch matrix. Each AC-DC converter is rated at 2 MVA, and the DC-DC converter is rated at 0.6 MW. The ESS is modeled as a battery with an initial stored energy of

0.5 MWh, an energy range of 0.2-1 MWh, and a rated active power of 0.6 MW. In the IDCS, the terminal SoC constraint in (13) is relaxed to prevent an overly restrictive short-horizon correction problem. The auxiliary loss coefficient c_{aux} is set to be zero, and the remaining loss-model parameters for TS-SOP subsystems are adopted from [31]. Parameters for the OLTC and CB models are adopted from [23]. The main system parameters are summarized in Supplementary Material A Table SAII.

B. Results and Analyses

1) Scheduling Strategies and System-level Performance

The optimal multi-timescale scheduling strategies in the modified IEEE 33-node system are illustrated in Fig. 6. Quantitative performances under pre-scheduling and real-time operation are summarized in Supplementary Material A Table SAIII.

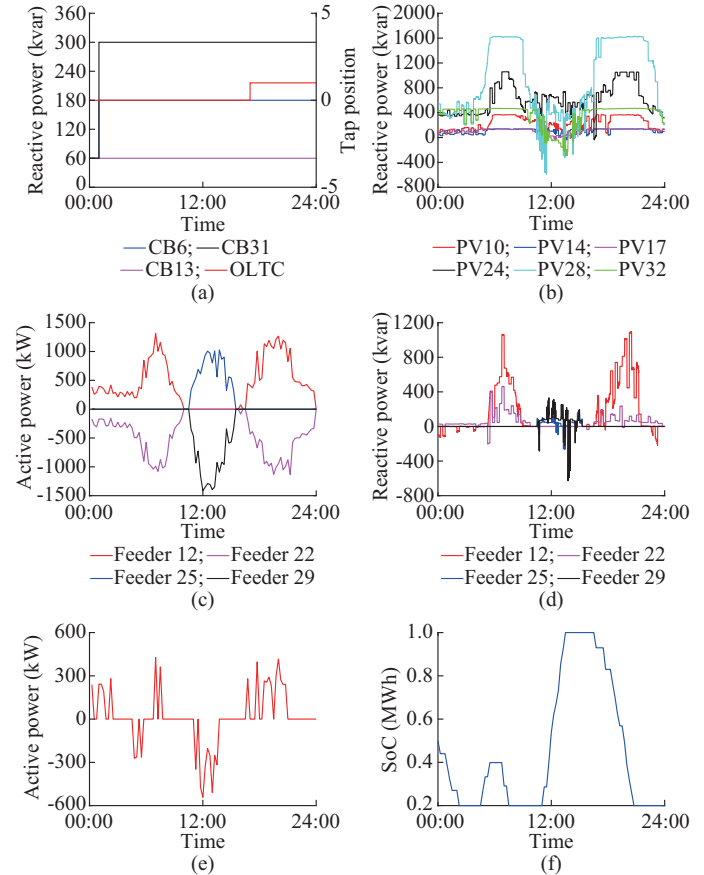


Fig. 6. Optimal multi-timescale scheduling strategies in modified IEEE 33-node system. (a) Tap position of OLTC and reactive power injected by CBs. (b) Reactive power of PV inverters. (c) Active power of TS-SOP. (d) Reactive power of TS-SOP. (e) Active power of ESS. (f) SoC of ESS.

Two observations follow from the active power schedules in Fig. 6(c), (e), and (f). From a temporal scheduling perspective, PV generation is insufficient to meet the load demand during 01:00-09:00 and 17:00-24:00. During these periods, the TS-SOP selects feeder pair 12-22, and the ESS discharges to support the load. During 11:00-15:00, PV generation increases, and the TS-SOP switches to feeder pair 25-29. The ESS then charges to improve the utilization of PV power.

In addition, during 10:00-11:00 and 15:00-16:00, the PV load mismatch is small and the transferred power is insufficient to offset no-load losses of converter. To avoid low-power transfer, the TS-SOP de-energizes both AC-DC converters. These results indicate that the TS-SOP increases power flow regulation flexibility through topology switching, and reduces the no-load losses of converter by exploiting energized and de-energized capabilities of converter, thereby improving the operating economy.

The reactive power schedules in Fig. 6(a), (b), and (d) indicate coordinated operation of controllable devices across multiple timescales. Owing to limited switching operations, the OLTC and CBs follow the day-ahead schedule. By contrast, the TS-SOP and PV inverters provide fast voltage support by regulating reactive power in real time through optimized volt/var droop curves. This coordination maintains all nodal voltages within 0.95-1.05 p.u., as shown in Supplementary Material A Fig. SA1.

2) Analysis of Real-time Volt/var Control

The performances of reactive power control of the TS-SOP and PV inverters at nodes 28 and 29 under real-time volt/var control are illustrated in Fig. 7.

Figure 7(a) and (b) shows that without real-time volt/var control, nodal voltages exceed the upper voltage limit during 13:00-15:00, as indicated by the purple areas. With real-time control enabled, voltages remain within the allowable range.

Figure 7(c)-(f) presents the corresponding reactive power responses. During 13:15-13:30, the optimized deadbands are 0.97-1.05 p.u. at node 28 and 0.99-1.05 p.u. at node 29, respectively. The reactive power references are 266.7 kvar for the PV inverter and 81.8 kvar for the TS-SOP. When the measured voltage remains within the deadband, each device tracks its reactive power reference. When the voltage exceeds the upper deadband limit, reactive power injection is reduced and may transition to reactive power absorption to mitigate the overvoltage. When the voltage falls below the lower deadband limit, reactive power injection increases to support voltage recovery, as observed during 13:00-13:15.

These results indicate that the real-time volt/var control enables the TS-SOP and PV inverters to respond to voltage fluctuations at a minute-level timescale and maintain real-time voltage security in ADNs.

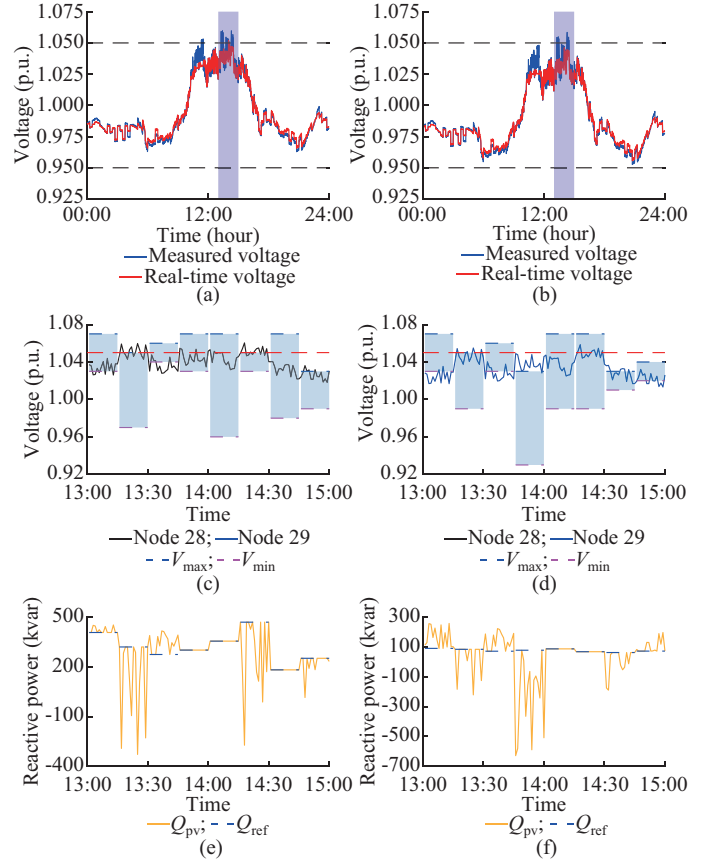


Fig. 7. Performances of reactive power control of TS-SOP and PV inverters at nodes 28 and 29 under real-time volt/var control. (a) 24-hour voltage response at node 28. (b) 24-hour voltage response at node 29. (c) Detailed voltage profile at node 28 during 13:00-15:00. (d) Detailed voltage profile at node 29 during 13:00-15:00. (e) Reactive power of TS-SOP at node 28. (f) Reactive power of PV at node 29.

C. Superiority Validation of TS-SOP Model

The introduction of feeder-switching capability enhances the power transfer flexibility of TS-SOP and improves the economic efficiency of distribution system operation. To validate the superiority of the TS-SOP model, four comparative scenarios are designed based on the pre-scheduling model, and the corresponding results are summarized in Table I.

TABLE I
COMPARISON OF OPERATING COSTS OF FOUR SCENARIOS IN DPS

Scenario	C_{BN}			C_{BL}			Total cost (\$)
	Network loss cost (\$)	ESS/TS-SOP loss cost (\$)	OLTC and CB operating cost (\$)	Network loss cost (\$)	ESS/TS-SOP loss cost (\$)	OLTC and CB operating cost (\$)	
1	239.65	39.01	2.60	340.87	42.30	2.60	667.03
2	264.78	25.81	6.60	387.37	19.92	6.60	711.08
3	220.07	48.58	1.44	318.33	52.66	1.44	642.52
4	233.57	40.01	2.84	332.38	44.38	2.84	656.02

- 1) Scenario 1: an ESS-SOP is installed on branch 12-22.
- 2) Scenario 2: an ESS-SOP is installed on branch 25-29.
- 3) Scenario 3: two ESS-SOPs are installed on branches 12-22 and 25-29, respectively.
- 4) Scenario 4: the TS-SOP model with four connecting lines is installed and connected to nodes 12, 22, 25, and 29.

As shown in Fig. 8, feeder pairs of the TS-SOP change over time. When PV generation exceeds the load demand, which is approximately during 11:00-15:00, the TS-SOP connects nodes 25 and 29 to promote local PV utilization. When PV generation is substantially lower than load demand, which is approximately during 01:00-09:00 and 17:00-

24:00, the TS-SOP switches to connect nodes 12 and 22 to support upstream power exchange.

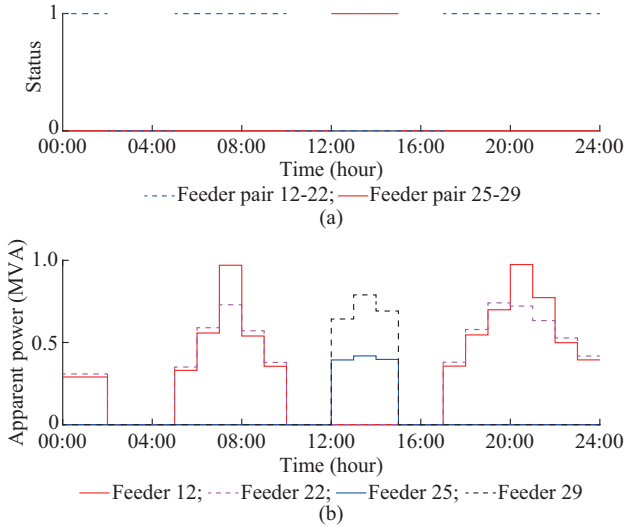


Fig. 8. Topology switching behavior of TS-SOP in Scenario 4. (a) Conducting status of predefined feeder pairs. (b) Real-time apparent power of feeder ports.

As shown in Table I, the TS-SOP with switchable feeder connections slightly increases the ESS-related loss cost, whereas the improved power-transfer capability reduces network loss cost under both the base operating condition C_{BN} and the critical loading condition C_{BL} . As a result, the total operating cost is reduced. Compared with Scenario 2, Scenario 4 reduces network loss cost by 11.8% under C_{BN} and by 14.2% under C_{BL} , which yields a 7.74% reduction in total operating cost. Although Scenario 3 attains the lowest total operating cost, which is 2.06% lower than Scenario 4, it relies on two ESS-assisted SOPs and would typically imply

higher capital investment than the single TS-SOP in Scenario 4.

Overall, the proposed TS-SOP model employs a switch-based topology switching mechanism to reconfigure feeder connections in response to operating conditions. This capability improves the power flow control flexibility and operating economy in the tested scenarios, particularly when multiple fixed-topology SOPs would otherwise be required to provide comparable capability.

D. Case Studies

To evaluate the performance of the proposed framework, four representative cases are examined. The settings of each case are described as follows.

1) Case 1: based on the two-stage optimization strategy in [22], [35], a conventional SOP is installed between nodes 25 and 29.

2) Case 2: the SOP in Case 1 is replaced by a TS-SOP, whose connecting feeders are nodes 12, 22, 25 and 29, while all other settings remain the same.

3) Case 3: real-time volt/var control for inverter-based devices is added to Case 2 based on [24].

4) Case 4: the proposed framework is implemented, integrating feeder-switching TS-SOP operation, multi-timescale scheduling, and real-time volt/var control with voltage-stability constraints.

Table II compares the voltage profiles and operating costs among the four cases. Supplementary Material A Fig. SA2 presents voltage magnitudes of all 33 nodes at 17:00. Figure 9 shows the P - V curve of node 18 at 17:00 for steady-state VSM analysis, computed using continuous power flow. The results indicate that the proposed framework improves voltage security and achieves the highest VSM among the compared cases.

TABLE II
COMPARISON OF VOLTAGE PROFILES AND OPERATING COSTS AMONG FOUR CASES

Case	Network loss cost (\$)	SOP/TS-SOP loss cost (\$)	Total loss cost (\$)	Voltage deviation (p.u.)	Voltage violation probability (%)	VSM (%)
1	310.22	41.01	351.23	15.96	0.86	67.07
2	254.68	58.18	312.86	15.05	0.98	67.35
3	259.42	58.86	318.28	14.54	0.21	67.46
4	272.65	73.52	346.17	12.72	0.07	68.49

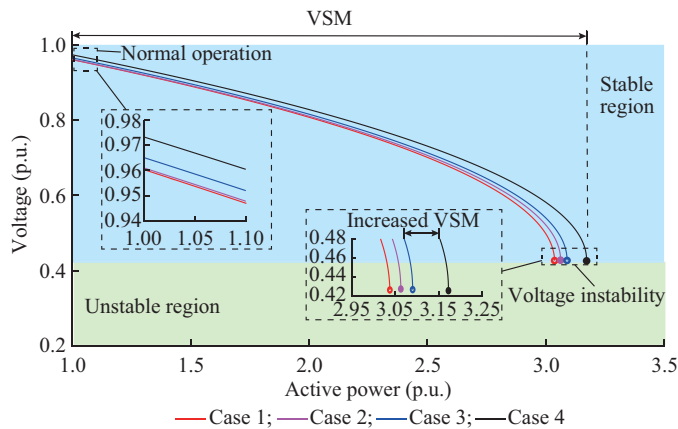


Fig. 9. P - V curve of node 18 at 17:00 for steady-state VSM analysis.

Table II shows that Case 1 exhibits the largest voltage deviation and the highest total loss cost. Although its SOP/TS-SOP loss cost is lower than that of Case 2, its network loss cost is 21.81% higher, which yields a 12.26% increase in total operation cost. These results indicate that, compared with a conventional SOP, the TS-SOP provides stronger power-flow controllability and reduces the overall operation cost.

Cases 1 and 2 consider PV and load uncertainties only in the day-ahead stage. As the optimization interval is relatively long, noticeable voltage violations can still occur. In Cases 3 and 4, real-time volt/var control is introduced so that the reactive power of inverter is continuously adjusted, which reduces the likelihood of voltage violations. Under heavy load, the system may operate closer to the stability boundary, and

a larger voltage stability margin is required. Case 4 therefore embeds VSM constraints into volt/var droop curve parameter optimization. As shown in Table II, the VSM in Case 4 reaches 68.49%, which is the highest among all cases. Compared with Case 3, Case 4 increases the steady-state voltage at node 18 from 0.969 p.u. to 0.978 p.u., thereby satisfying the prescribed VSM_{target} . This additional margin yields a more conservative operating point and increases the total operating cost by 8.76%.

Overall, the proposed TS-SOP model enhances the performance of feeder-level power flow control and suppresses low-power transfers that are inefficient due to the no-load losses by leveraging switchable feeder connections as well as the on-off capability of converter. Combined with VSM-oriented droop curve parameter optimization, the proposed framework improves real-time voltage security and strengthens steady-state voltage stability while maintaining the operating economy.

IV. CONCLUSION

A TS-SOP model is developed, featuring switchable feeder connections and a detailed polynomial loss model. Binary variables represent switchable feeder connections and operating states of converter, while the polynomial loss model captures nonlinear subsystem losses. An AC-side switch matrix is incorporated to enable feeder reconfiguration. Simulation results demonstrate that the TS-SOP improves power flow controllability compared with conventional fixed-topology SOPs. In the tested scenarios, this improvement reduces network loss cost by more than 10% and decreases the total operating cost by more than 7.7%.

In addition, a TS-RTCO framework is proposed to coordinate the scheduling and control of flexible resources in ADNs with high PV penetration. By embedding VSM constraints into the optimization and enabling real-time volt/var control, the proposed framework improves the voltage security and voltage stability. Numerical results show that the proposed framework increases the VSM by 1.03% and reduces voltage deviation by 12.52%, while maintaining nodal voltages within allowable limits under uncertainties.

Overall, the proposed framework provide a systematic solution to improve the economic performance while strengthening voltage security and voltage stability in ADNs with high PV penetration. Future work includes refining subsystem loss modeling, incorporating explicit switching-cost formulations, and exploring joint optimization of short-term operation and long-term capacity-expansion planning.

REFERENCES

- [1] X. Huang, Z. Ding, Z. Liu *et al.*, "Bidding strategy for hybrid PV-BESS plants via knowledge-data-complementary learning," *Journal of Modern Power Systems and Clean Energy*, vol. 13, no. 1, pp. 365-378, Jan. 2025.
- [2] J. Bian, Y. Song, C. Ding *et al.*, "Optimal bidding strategy for PV and BESSs in joint energy and frequency regulation markets considering carbon reduction benefits," *Journal of Modern Power Systems and Clean Energy*, vol. 12, no. 2, pp. 427-439, Mar. 2024.
- [3] Y. Zhao, G. Zhang, W. Hu *et al.*, "Meta-learning based voltage control for renewable energy integrated active distribution network against topology change," *IEEE Transactions on Power Systems*, vol. 38, no. 6, pp. 5937-5940, Nov. 2023.
- [4] S. Wang, J. Duan, D. Shi *et al.*, "A data-driven multi-agent autonomous voltage control framework using deep reinforcement learning," *IEEE Transactions on Power Systems*, vol. 35, no. 6, pp. 4644-4654, Nov. 2020.
- [5] H. An, J. Yi, Y. Xing *et al.*, "A robust V2G voltage control scheme for distribution networks against cyber attacks and customer interruptions," *IEEE Transactions on Smart Grid*, vol. 15, no. 4, pp. 3966-3978, Jul. 2024.
- [6] H. Qiu, V. Veerasamy, C. Ning *et al.*, "Two-stage robust optimization for assessment of PV hosting capacity based on decision-dependent uncertainty," *Journal of Modern Power Systems and Clean Energy*, vol. 12, no. 6, pp. 2091-2096, Nov. 2024.
- [7] C. Zhang, R. Xu, and L. Yang, "Optimization of local voltage control with coordinating droop functions under high PV penetration," *IEEE Transactions on Power Systems*, vol. 39, no. 5, pp. 6776-6779, Sept. 2024.
- [8] S. Talkington, S. Grijalva, M. J. Reno *et al.*, "Solar PV inverter reactive power disaggregation and control setting estimation," *IEEE Transactions on Power Systems*, vol. 37, no. 6, pp. 4773-4784, Nov. 2022.
- [9] R. Xu, C. Zhang, Y. Xu *et al.*, "Multi-objective hierarchically-coordinated Volt/Var control for active distribution networks with droop-controlled PV inverters," *IEEE Transactions on Smart Grid*, vol. 13, no. 2, pp. 998-1011, Mar. 2022.
- [10] X. Sun, J. Qiu, and J. Zhao, "Optimal local volt/var control for photovoltaic inverters in active distribution networks," *IEEE Transactions on Power Systems*, vol. 36, no. 6, pp. 5756-5766, Nov. 2021.
- [11] R. Cheng, N. Shi, S. Maharjan *et al.*, "Automatic self-adaptive local voltage control under limited reactive power," *IEEE Transactions on Smart Grid*, vol. 14, no. 4, pp. 2851-2862, Jul. 2023.
- [12] C. Zhang and Y. Xu, "Hierarchically-coordinated voltage/var control of distribution networks using PV inverters," *IEEE Transactions on Smart Grid*, vol. 11, no. 4, pp. 2942-2953, Jul. 2020.
- [13] C. Zhang, Y. Xu, Y. Wang *et al.*, "Three-stage hierarchically-coordinated voltage/var control based on PV inverters considering distribution network voltage stability," *IEEE Transactions on Sustainable Energy*, vol. 13, no. 2, pp. 868-881, Apr. 2022.
- [14] H. Liu, C. Zhang, Q. Chai *et al.*, "Robust regional coordination of inverter-based volt/var control via multi-agent deep reinforcement learning," *IEEE Transactions on Smart Grid*, vol. 12, no. 6, pp. 5420-5433, Nov. 2021.
- [15] A. Inaolaji, A. Savasci, and S. Paudyal, "Distribution grid optimal power flow in unbalanced multiphase networks with volt-var and volt-watt droop settings of smart inverters," *IEEE Transactions on Industry Applications*, vol. 58, no. 5, pp. 5832-5843, Sept. 2022.
- [16] Y. Z. Gerdoodbari, R. Razzaghi, and F. Shahnia, "Decentralized control strategy to improve fairness in active power curtailment of PV inverters in low-voltage distribution networks," *IEEE Transactions on Sustainable Energy*, vol. 12, no. 4, pp. 2282-2292, Oct. 2021.
- [17] X. Sun, J. Qiu, Y. Tao *et al.*, "Coordinated real-time voltage control in active distribution networks: an incentive-based fairness approach," *IEEE Transactions on Smart Grid*, vol. 13, no. 4, pp. 2650-2663, Jul. 2022.
- [18] Y. Cao, D. Li, Y. Zhang *et al.*, "Optimal energy management for multi-microgrid under a transactive energy framework with distributionally robust optimization," *IEEE Transactions on Smart Grid*, vol. 13, no. 1, pp. 599-612, Jan. 2022.
- [19] W. Huang, C. Gao, R. Li *et al.*, "A model predictive control-based voltage optimization method for highway transportation power supply networks with soft open points," *IEEE Transactions on Industry Applications*, vol. 60, no. 1, pp. 1141-1150, Jan. 2024.
- [20] T. Qian, W. Ming, C. Shao *et al.*, "An edge intelligence-based framework for online scheduling of soft open points with energy storage," *IEEE Transactions on Smart Grid*, vol. 15, no. 3, pp. 2934-2945, May 2024.
- [21] V. B. Pamshetti, S. Singh, A. K. Thakur *et al.*, "Multistage coordination volt/var control with CVR in active distribution network in presence of inverter-based DG units and soft open points," *IEEE Transactions on Industry Applications*, vol. 57, no. 3, pp. 2035-2047, May 2021.
- [22] P. Li, H. Ji, C. Wang *et al.*, "Coordinated control method of voltage and reactive power for active distribution networks based on soft open point," *IEEE Transactions on Sustainable Energy*, vol. 8, no. 4, pp. 1430-1442, Oct. 2017.
- [23] X. Yang, C. Xu, Y. Zhang *et al.*, "Real-time coordinated scheduling for active distribution networks with soft open points and charging stations," *IEEE Transactions on Power Systems*, vol. 36, no. 6, pp. 5486-

- 5499, Nov. 2021.
- [24] P. Li, H. Ji, H. Yu *et al.*, "Combined decentralized and local voltage control strategy of soft open points in active distribution networks," *Applied Energy*, vol. 241, pp. 613-624, May 2019.
 - [25] S. Wang, X. Li, Y. Zhang *et al.*, "Non-iterative sensitivity-based switch exchange method for distribution network reconfiguration," *IEEE Transactions on Smart Grid*, vol. 16, no. 4, pp. 2830-2845, Jul. 2025.
 - [26] A. Azizi, B. Vahidi, and A. F. Nematollahi, "Reconfiguration of active distribution networks equipped with soft open points considering protection constraints," *Journal of Modern Power Systems and Clean Energy*, vol. 11, no. 1, pp. 212-222, Jan. 2023.
 - [27] M. Deakin, "Multiplexing power converters for cost-effective and flexible soft open points," *IEEE Transactions on Smart Grid*, vol. 15, no. 1, pp. 260-271, Jan. 2024.
 - [28] Q. Hou, N. Dai, and Y. Huang, "Voltage regulation enhanced hierarchical coordinated volt/var and volt/watt control for active distribution networks with soft open points," *IEEE Transactions on Sustainable Energy*, vol. 15, no. 3, pp. 2021-2037, Jul. 2024.
 - [29] X. Yang, C. Xu, J. Wen *et al.*, "Cooperative repair scheduling and service restoration for distribution systems with soft open points," *IEEE Transactions on Smart Grid*, vol. 14, no. 3, pp. 1827-1842, May 2023.
 - [30] X. Yang, Z. Hu, H. Fang *et al.*, "Voltage control embedded resilient distribution system restoration considering uncertainties," *IEEE Transactions on Smart Grid*, vol. 60, no. 4, pp. 5609-5623, Jul. 2024.
 - [31] I. Sarantakos, M. Pekar, N.-M. Zografou-Barredo *et al.*, "A robust mixed-integer convex model for optimal scheduling of integrated energy storage-soft open point devices," *IEEE Transactions on Smart Grid*, vol. 13, no. 5, pp. 4072-4087, Sept. 2022.
 - [32] R. Zafar, J. Ravishanker, J. E. Fletcher *et al.*, "Multi-timescale voltage stability-constrained volt/var optimization with battery storage system in distribution grids," *IEEE Transactions on Sustainable Energy*, vol. 11, no. 2, pp. 868-878, Apr. 2020.
 - [33] X. Yang, C. Xu, H. He *et al.*, "Flexibility provisions in active distribution networks with uncertainties," *IEEE Transactions on Sustainable Energy*, vol. 12, no. 1, pp. 553-567, Jan. 2021.
 - [34] Y. Zheng, Y. Song, A. Huang *et al.*, "Hierarchical optimal allocation of battery energy storage systems for multiple services in distribution systems," *IEEE Transactions on Sustainable Energy*, vol. 11, no. 3, pp. 1911-1921, Jul. 2020.
 - [35] R. Zafar, J. Ravishanker, J. E. Fletcher *et al.*, "Multi-timescale model predictive control of battery energy storage system using conic relaxation in smart distribution grids," *IEEE Transactions on Power Systems*, vol. 33, no. 6, pp. 7152-7161, Nov. 2018.
- Binjie Wang** received the M.S. degree in control science and engineering from Zhejiang University of Technology, Hangzhou, China, in 2024. He is currently pursuing the Ph.D. degree in electrical engineering at Hefei University of Technology, Hefei, China. His research interests include three-phase unbalanced voltage security and control in distribution networks.
- Wu Tu** received the M.S. degree in electrical engineering from Hefei University of Technology, Hefei, China, in 2025. His research interest includes voltage regulation in power systems.
- Xiaodong Yang** received the Ph.D. degree in control science and engineering from Zhejiang University of Technology, Hangzhou, China, in 2019. He is currently an Associate Professor with the School of Electrical Engineering and Automation, Hefei University of Technology, Hefei, China. His research interests include resilient distribution system, multi-microgrid, and power system control and optimization.
- Hui Fang** received the B.S. and M.S. degrees in electrical engineering from Hefei University of Technology, Hefei, China, in 2016 and 2019, respectively, where she is currently pursuing the Ph.D. degree in electrical engineering. She is also an Engineer with the State Grid Anhui Ultra High Voltage Company, Hefei, China. Her research interests include planning and operation optimization of distribution networks with high renewable penetration.
- Lijian Ding** received the Ph.D. degree from North China Electric Power University, Beijing, China, in 2000. He is a Full Professor and the Dean with the School of Electrical Engineering and Automation, Hefei University of Technology, Hefei, China. His research interests include high-voltage technology, fault identification, electromagnetic compatibility, and reliability of wide bandgap power electronic system.
- Wei Lou** received the Ph.D. degree from Dalian University of Technology, Dalian, China, and is currently a Senior Engineer with the Electric Power Research Institute of State Grid Anhui Electric Power Co., Ltd., Hefei, China. His research interests include intelligent operation and control of new distribution system.
- Qiuwei Wu** received the Ph.D. degree from Nanyang Technological University, Singapore, in 2009. His research interests include decentralized and distributed optimal operation and control of power system with high penetration of renewable energy.
- Jinyu Wen** received the Ph.D. degree in electrical engineering from Huazhong University of Science and Technology (HUST), Wuhan, China, in 1998. He is currently a Professor with the School of Electrical and Electronic Engineering, HUST. His research interests include renewable energy integration, energy storage, multi-terminal high-voltage direct current, and power system operation and control.