

Flexible Holomorphic Embedding Solution for Power Flow Calculation in Hybrid AC/DC Active Distribution Networks

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Abstract—Holomorphic embedding (HE) method has gained increasing attention in power system analysis because of its robustness. This paper further extends the HE theories into the power flow calculation in hybrid AC/DC active distribution networks (ADNs). A flexible holomorphic embedding solution (FHES) method is proposed for the power flow calculation in AC/DC ADNs in the grid-connected and islanded modes. This paper also develops the HE models of voltage source converters (VSCs) with active and reactive power control modes. The HE models of loads in AC/DC ADNs are also formulated by taking full account of static voltage and frequency characteristics. Furthermore, the HE models of the distributed generators (DGs) under various control modes are developed. A sequential iterative method is employed to solve the developed HE models of AC/DC ADNs in the grid-connected and islanded modes. The performance of the proposed FHES method is evaluated using the modified IEEE 33-node and IEEE 123-node AC/DC ADNs. Finally, the testing results validate the accuracy and robustness of the proposed FHES method.

Index Terms—Holomorphic embedding (HE), power flow calculation, active distribution network (ADN), distributed generator, voltage source converter (VSC).

NOMENCLATURE

A. Indices and Sets

c	Index for voltage source converters (VSCs)
d, k	Indices for DC nodes
$DROOP_{AC}$	Set of AC droop control nodes
$DROOP_{DC}$	Set of DC droop control nodes
i, j	Indices for AC nodes
n	Order of power series
P_{DC}	Set of constant active power injection nodes

PQ_{AC}	Set of PQ nodes
PV_{AC}	Set of PV nodes
REF	Set of phase reference nodes
SL	Set of slack nodes
U_{DC}	Set of constant voltage nodes
$*$	Conjugate operation

B. Parameters and Constants

φ_c	Power factor angle of VSC c
a_{pi}, b_{pi}, c_{pi}	Percentages in terms of active power of constant impedance load, constant current load, and constant power load at AC node i
a_{qi}, b_{qi}, c_{qi}	Percentages in terms of reactive power of constant impedance load, constant current load, and constant power load at AC node i
a_{pd}, b_{pd}, c_{pd}	Percentages in terms of active power of constant impedance load, constant current load, and constant power load at DC node d
f_0	Rated frequency
f_{max}, f_{min}	Upper and lower limits of system frequency
$G_{dc, dk}$	Conductance of branch between DC nodes d and k
k_d	Droop gain for active power of DC node d
k_{pi}, k_{qi}	Static voltage and frequency characteristic parameters of loads at AC node i
m_i, n_i	Droop gains for active and reactive power of AC node i
M, N	Numbers of DC and AC nodes
M_c, η_c	Modulation ratio and efficiency coefficient of VSC c
$P_{ac, Li}^{(0)}, Q_{ac, Li}^{(0)}$	Active and reactive power of load at node i under nominal voltage and frequency
$P_{dc, Ld}^{(0)}$	Active power of load at DC node d under nominal voltage
$P_{vsc, i}^{sp}, Q_{vsc, i}^{sp}$	Active and reactive power injected into AC node i connected to AC side of VSC
pol	Number of poles in DC network
$U_{ac, i0}$	Initial voltage of AC node i
$U_{ac, i}^{(0)}$	Nominal voltage magnitude of AC node i
$U_{dc, d}^{(0)}$	Nominal voltage magnitude of DC node d

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$U_{dc,max}, U_{dc,min}$	Upper and lower limits of voltage on DC side
$U_{ac,i}^{sp}$	Reference voltage magnitude of AC node i
$\bar{U}_{ac,i}^{sp}$	Reference voltage phasor of AC node i
$U_{dc,d}^{sp}$	Known voltage of DC node d
$Y_{ac,ij}$	Admittance of branch between AC nodes i and j
$Y_{ac,i}^{sh}$	Parallel admittance connected to AC node i
$Y_{ac,ij}^{tr}$	Series admittance between AC nodes i and j

C. Variables

Δf	Deviation between actual frequency and rated frequency
$P_{ac,c}, Q_{ac,c}$	Active and reactive power of AC side of VSC c
$P_{ac,Gi}, Q_{ac,Gi}$	Active and reactive power of generation at AC node i
$P_{ac,Li}, Q_{ac,Li}$	Active and reactive power of load at AC node i
$P_{ac,i}, Q_{ac,i}$	Active and reactive power of AC node i
$P_{dc,Gd}, P_{dc,Ld}$	Active power of generation and load at DC node d
$P_{dc,d}$	Active power of DC node d
$Q_{vsc,i}$	Reactive power injected into AC node i by VSC
s, α	Embedded variables of AC and DC networks
ΔS	Imbalance complex power in power flow calculation
$\tilde{S}_{ac,Gi}$	Complex power generation of AC node i
$\tilde{S}_{ac,Li}$	Complex power load of AC node i
$U_{ac,c}$	Voltage magnitude of AC side of VSC c
$U_{ac,i}$	Voltage magnitude of AC node i
$\bar{U}_{ac,i}$	Voltage phasor of AC node i
$U_{dc,d}$	Voltage of DC node d
$W_{ac,i}, W_{dc,d}$	Reciprocal for voltages of AC node i and DC node d

I. INTRODUCTION

WITH the fast growth in load-side re-electrification, the increasing penetration of renewable energy, and the global energy transition, the power grid is evolving towards zero emission. Toward this trend, the renewable distributed generators (DGs), e.g., photovoltaic (PV) panels and wind turbines (WTs), have been widely deployed in the distribution networks. However, integrating these resources into traditional AC distribution networks requires multiple power conversion stages. Therefore, the DC distribution network, as an alternative to existing AC distribution network, could largely reduce the power losses in electric power conversion and improve the energy efficiency. However, given that the AC distribution network has dominated the power system over a century and is expected to remain prevalent in the coming decades, the hybrid AC/DC active distribution networks (ADNs) have emerged as an attractive option for up-

grading existing AC distribution networks. AC/DC ADNs consist of both AC and DC distribution networks, along with converters that interconnect these networks and regulate the power flow between them [1], [2]. This hybrid architecture introduces new challenges in power flow analysis compared with traditional AC distribution networks.

The Newton-Raphson (NR) method has been widely used for solving the power flow of different systems including hybrid AC/DC transmission networks [3], [4]. Research on NR-based power flow calculation for hybrid AC/DC systems can be divided into unified iterative methods [5] and sequential iterative methods [6]. Compared with transmission networks, AC/DC ADNs usually have higher R/X ratios and radial network topologies such that the Jacobian matrices generated by the NR method are prone to be singular [7], causing the early termination of power flow calculation [8]. To enhance the performance of NR method in power flow calculation, various improved NR methods have been developed [9]–[12]. In [9], an optimal multiplier method was proposed to calculate the power flow of ill-conditioned systems, which develops an adaptive step-size multiplier to prevent overshooting of the optimal point, but the computational processing is complex. In [10], a damped Newton method was adopted by using a damping factor to limit the iteration step size, avoiding the need for solving the optimal multiplier in each iteration, but it cannot guarantee a strictly optimal update at every step. In [11], the Levenberg-Marquardt method was applied with curvature and gradient steps to improve the robustness in ill-conditioned systems, while its applicability is limited in large-scale systems. In [12], the Newton-trust region method was proposed to confine the correction updates within a trust radius, which yields strong global convergence. Although these methods enhance the convergence and computational efficiency compared with the conventional NR method, several limitations remain: ① they are highly dependent on initial values, and inappropriate values may lead to the divergence; ② when the divergence occurs, it is often unclear whether the method fails or there is no feasible power flow solution; and ③ even when the convergence is achieved, the obtained solution is not always the desired high-voltage solution.

Another power flow calculation method, the holomorphic embedding load-flow method (HELM), was developed based on the complex analysis theory [13]. The advantages of HELM are as follows: ① it does not need initial values to start computation, thus avoiding the issue of initial value selection; ② it guarantees to find the solution if there exists one [14], while giving a clear signal if a solution does not exist [15]; and ③ the potential issues of a singular Jacobian matrix are avoided since the Jacobian matrix is not needed in this method. Hence, the HELM has been extensively discussed and applied in hybrid AC/DC power systems. In [16], the holomorphic embedding method (HEM) was used to solve the power flow of a hybrid AC/DC transmission system with a line-commutated converter (LCC), and the holomorphic embedding (HE) models of LCCs with different control modes were constructed. In [17], a unified HE formu-

$$\begin{cases} U_{ac,c} = \frac{M_c}{\sqrt{2}} U_{dc,d} \\ P_{ac,c} = \frac{P_{dc,d}}{\eta_c} \\ Q_{ac,c} = P_{ac,c} \tan \varphi_c \end{cases} \quad (1)$$

If the AC/DC ADN operates in the islanded mode, the VSC can automatically control the power flow between the AC and DC networks. In this scenario, the AC frequency f and $U_{dc,d}$ serve as the loading indicators for the AC and DC networks, respectively. The normalized indicators $\hat{f}$ and $\hat{U}_{dc,d}$ are [29] expressed as:

$$\begin{cases} \hat{f} = \frac{f - 0.5(f_{\max} - f_{\min})}{0.5(f_{\max} + f_{\min})} \\ \hat{U}_{dc,d} = \frac{U_{dc,d} - 0.5(U_{dc,\max} - U_{dc,\min})}{0.5(U_{dc,\max} + U_{dc,\min})} \end{cases} \quad (2)$$

B. AC Network

The power flow model of AC network in the AC/DC ADN can be expressed as:

$$\sum_{j=1}^N Y_{ac,ij} \dot{U}_{ac,j} = \left(\frac{\tilde{S}_{ac,Gi} - \tilde{S}_{ac,Li}}{\dot{U}_{ac,i}} \right)^* \quad (3)$$

For a traditional AC network, there are two types of nodes: slack and PQ nodes. Their models are expressed as:

$$\begin{cases} \dot{U}_{ac,i} = U_{ac,i}^{sp} \angle 0^\circ & i \in SL \\ \sum_{j=1}^N Y_{ac,ij} \dot{U}_{ac,j} = \left(\frac{-\tilde{S}_{ac,Li}}{\dot{U}_{ac,i}} \right)^* = \left(\frac{-P_{ac,Li} - jQ_{ac,Li}}{\dot{U}_{ac,i}} \right)^* & i \in PQ_{AC} \end{cases} \quad (4)$$

Regarding the influence of voltage and frequency on the load, the following load model is used to represent the static load characteristics of AC node i [31].

$$\begin{cases} P_{ac,Li} = P_{ac,Li}^{(0)} \left[a_{pi} \left(\frac{U_{ac,i}}{U_{ac,i}^{(0)}} \right)^2 + b_{pi} \frac{U_{ac,i}}{U_{ac,i}^{(0)}} + c_{pi} \right] (1 + k_{pi} \Delta f) \\ Q_{ac,Li} = Q_{ac,Li}^{(0)} \left[a_{qi} \left(\frac{U_{ac,i}}{U_{ac,i}^{(0)}} \right)^2 + b_{qi} \frac{U_{ac,i}}{U_{ac,i}^{(0)}} + c_{qi} \right] (1 + k_{qi} \Delta f) \end{cases} \quad (5)$$

It should be noted that, if the AC/DC ADN operates in the grid-connected mode, $\Delta f = 0$ and the impact of the frequency on the load will not be included; otherwise, $\Delta f \neq 0$ and the influence of frequency should be considered.

C. DC Network

The power flow model of DC network in the AC/DC ADN can be expressed as:

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k} = \frac{P_{dc,Gd} - P_{dc,Ld}}{pol \cdot U_{dc,d}} \quad (6)$$

There are two types of nodes in traditional DC networks: constant voltage nodes and constant active power injection nodes. Their models can be expressed as:

$$U_{dc,d} = U_{dc,d}^{sp} \quad d \in U_{DC} \quad (7a)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k} = \frac{-P_{dc,Ld}}{pol \cdot U_{dc,d}} \quad d \in P_{DC} \quad (7b)$$

Regarding the impact of voltage on load, the load model is [32]:

$$P_{dc,Ld} = P_{dc,Ld}^{(0)} \left[a_{pd} \left(\frac{U_{dc,d}}{U_{dc,d}^{(0)}} \right)^2 + b_{pd} \frac{U_{dc,d}}{U_{dc,d}^{(0)}} + c_{pd} \right] \quad d \in P_{DC} \quad (8)$$

D. DG

In the AC/DC ADN, the DGs can operate in both AC and DC networks, their models can be developed as follows.

1) DG Models in AC Network

In the AC network, the DG can operate in ① constant active and reactive power mode, ② constant active power and voltage mode, or ③ AC droop control mode.

1) The DG model in the constant active power and reactive power mode is:

$$\sum_{j=1}^N Y_{ac,ij} \dot{U}_{ac,j} = \left(\frac{\tilde{S}_{ac,Gi}}{\dot{U}_{ac,i}} \right)^* = \left(\frac{P_{ac,Gi} + jQ_{ac,Gi}}{\dot{U}_{ac,i}} \right)^* \quad i \in PQ_{AC} \quad (9)$$

2) The DG model in the constant active power and voltage mode is:

$$\begin{cases} P_{ac,Gi} = \text{Re} \left(\dot{U}_{ac,i} \sum_{j=1}^N Y_{ac,ij}^* \dot{U}_{ac,j} \right) \\ U_{ac,i} = U_{ac,i}^{sp} \end{cases} \quad i \in PV_{AC} \quad (10)$$

3) The DG model in the AC droop control mode, which could share the demand when the AC network operates in the islanded mode, is:

$$\begin{cases} P_{ac,Gi} = \frac{1}{m_i} (f_0 - f) \\ Q_{ac,Gi} = \frac{1}{n_i} (U_{ac,i}^{(0)} - U_{ac,i}) \end{cases} \quad i \in DROOP_{AC} \quad (11)$$

2) DG Models in DC Network

In the DC network, the DG can operate in ① constant voltage mode, ② constant active power mode, or ③ constant DC droop control mode [33].

1) The DG model in the constant voltage mode is the same as (7a).

2) The DG model in the constant active power mode is:

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k} = \frac{P_{dc,Gd} - P_{dc,Ld}}{pol \cdot U_{dc,d}} \quad d \in P_{DC} \quad (12)$$

3) The DG model in the constant DC droop control mode is:

$$\begin{cases} \sum_{k=1}^M G_{dc,dk} U_{dc,k} = \frac{P_{dc,Gd} - P_{dc,Ld}}{pol \cdot U_{dc,d}} \\ P_{dc,Gd} = \frac{1}{k_d} (U_{dc,d}^{(0)} - U_{dc,d}) \end{cases} \quad d \in DROOP_{DC} \quad (13)$$

III. HE MODELS AND POWER FLOW CALCULATION OF AC/DC ADNs

This section develops the HE models of AC/DC ADNs

and the FHES method to calculate the power flow of AC/DC ADNs in the grid-connected and islanded modes.

A. HE Models of VSC

This subsection develops the HE models of VSC in various control modes.

1) Active Power Control Mode

Regarding the active power control mode of VSC, the HE models for the DC power control, DC voltage control, and DC voltage droop control are given as follows.

1) DC power control

The active power injected into constant active power injection node d connected to the DC side of VSC $P_{vsc,d}^{sp}$ is given. The HE model is:

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}(\alpha) = \frac{\alpha P_{vsc,d}^{sp}}{pol \cdot U_{dc,d}(\alpha)} \quad d \in P_{DC} \quad (14)$$

2) DC voltage control

The voltage of constant voltage node d connected to DC side of VSC $U_{dc,d}^{sp}$ is given, and the type of DC node d belongs to U_{DC} . The HE model is:

$$U_{dc,d}(\alpha) = 1 + \alpha(U_{dc,d}^{sp} - 1) \quad d \in U_{DC} \quad (15)$$

3) DC voltage droop control

The voltage and power of DC droop control node d connected to the DC side of VSC satisfy the given droop relationship. The HE model is:

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}(\alpha) = \frac{1}{pol \cdot k_d} \left(\frac{U_{dc,d}^{(0)}}{U_{dc,d}(\alpha)} - 1 \right) \quad d \in DROOP_{DC} \quad (16)$$

2) Reactive Power Control Mode

Similarly, the HE models of VSC in the reactive power control mode are expressed as follows.

1) Reactive power control

The reactive power injected into AC node i connected to the AC side of VSC $Q_{vsc,i}^{sp}$ is given, and AC node i is a PQ node. The HE model is:

$$\sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}(s) = \frac{s(P_{vsc,i}^{sp} - jQ_{vsc,i}^{sp})}{\dot{U}_{ac,i}^*(s^*)} - sY_{ac,i}^{sh} \dot{U}_{ac,i}(s) \quad i \in PQ_{AC} \quad (17)$$

2) AC voltage control

The voltage magnitude of AC node i connected to the AC side of VSC $U_{ac,i}^{sp}$ is given, and AC node i is a PV node. The HE model is:

$$\left\{ \begin{aligned} \sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}(s) &= \frac{sP_{vsc,i}^{sp} - jQ_{vsc,i}^{sp}}{\dot{U}_{ac,i}^*(s^*)} - sY_{ac,i}^{sh} \dot{U}_{ac,i}(s) \\ \dot{U}_{ac,i}(s) \dot{U}_{ac,i}^*(s^*) &= U_{ac,i0} + s((U_{ac,i}^{sp})^2 - U_{ac,i0}) \end{aligned} \right. \quad i \in PV_{AC} \quad (18)$$

B. HE Models of AC Network

Taking into account the voltage and frequency characteristics of the load and the node types, the HE models of loads, slack node, PQ node, PV node, and AC droop control node in the AC network are developed as follows.

1) Loads. The HE model of loads considering the static

voltage and frequency characteristics is:

$$\begin{cases} P_{ac,Li} = P_{ac,Li}^{(0)} A_{Pi}(s) [1 + k_{pi}(f(s) - f_0)] \\ Q_{ac,Li} = Q_{ac,Li}^{(0)} A_{Qi}(s) [1 + k_{qi}(f(s) - f_0)] \end{cases} \quad (19)$$

where the expressions of $A_{Pi}(s)$ and $A_{Qi}(s)$ can be found in Supplementary Material A (SA1).

2) Slack node. The HE model of a slack node is:

$$\dot{U}_{ac,i}(s) = 1 + s(\dot{U}_{ac,i}^{sp} - 1) \quad i \in SL \quad (20)$$

3) PQ node. The HE model of a PQ node is:

$$\begin{aligned} \sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}(s) &= \{-A_{Pi}(s)[sP_{ac,Li}^{(0)} + P_{ac,Li}^{(0)} k_{pi}(f(s) - f_0)] + \\ &\quad jA_{Qi}(s)[sQ_{ac,Li}^{(0)} + Q_{ac,Li}^{(0)} k_{qi}(f(s) - f_0)]\} W_{ac,i}^*(s^*) - \\ &\quad sY_{ac,i}^{sh} \dot{U}_{ac,i}(s) + s(\tilde{S}_{ac,i}^* + P_{vsc,i}^{sp} - jQ_{vsc,i}^{sp}) W_{ac,i}^*(s^*) \\ &\quad i \in PQ_{AC} \end{aligned} \quad (21)$$

4) PV node. The HE model of a PV node is:

$$\left\{ \begin{aligned} \sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}(s) &= (sP_{ac,Gi} + sP_{vsc,i}^{sp} - jQ_{ac,Gi}(s)) W_{ac,i}^*(s^*) - \\ &\quad sY_{ac,i}^{sh} \dot{U}_{ac,i}(s) + \{-A_{Pi}(s)[sP_{ac,Li}^{(0)} + P_{ac,Li}^{(0)} k_{pi}(f(s) - f_0)] + \\ &\quad jA_{Qi}(s)[sQ_{ac,Li}^{(0)} + Q_{ac,Li}^{(0)} k_{qi}(f(s) - f_0)]\} W_{ac,i}^*(s^*) \\ \dot{U}_{ac,i}(s) \dot{U}_{ac,i}^*(s^*) &= U_{ac,i0} + s((U_{ac,i}^{sp})^2 - U_{ac,i0}) \\ &\quad i \in PV_{AC} \end{aligned} \right. \quad (22)$$

5) AC droop control node. The HE model of an AC droop control node is:

$$\left\{ \begin{aligned} \sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}(s) &= \frac{1}{m_i} (f_0 - f(s)) W_{ac,i}^*(s^*) - sY_{ac,i}^{sh} \dot{U}_{ac,i}(s) + \\ &\quad j \frac{1}{n_i} (U_{ac,i}(s) - U_{ac,i}^{(0)}) W_{ac,i}^*(s^*) + \{-A_{Pi}(s)[sP_{ac,Li}^{(0)} + \\ &\quad P_{ac,Li}^{(0)} k_{pi}(f(s) - f_0)] + jA_{Qi}(s)[sQ_{ac,Li}^{(0)} + \\ &\quad Q_{ac,Li}^{(0)} k_{qi}(f(s) - f_0)]\} W_{ac,i}^*(s^*) \\ U_{ac,i}(s) U_{ac,i}(s) &= \dot{U}_{ac,i}(s) \dot{U}_{ac,i}^*(s^*) \\ &\quad i \in DROOP_{AC} \end{aligned} \right. \quad (23)$$

According to the Taylor series, $f(s)$, $\dot{U}_{ac,i}(s)$, $U_{ac,i}(s)$, and $W_{ac,i}(s)$ in (19)-(23) can be further expressed as:

$$f(s) = \sum_{n=0}^{\infty} f[n] s^n = f[0] + f[1]s + \dots + f[n]s^n + \dots \quad (24)$$

$$\left\{ \begin{aligned} \dot{U}_{ac,i}(s) &= \sum_{n=0}^{\infty} \dot{U}_{ac,i}[n] s^n = \dot{U}_{ac,i}[0] + \dot{U}_{ac,i}[1]s + \dots + \dot{U}_{ac,i}[n]s^n + \dots \\ U_{ac,i}(s) &= \sum_{n=0}^{\infty} U_{ac,i}[n] s^n = U_{ac,i}[0] + U_{ac,i}[1]s + \dots + U_{ac,i}[n]s^n + \dots \\ U_{ac,i}(s) U_{ac,i}(s) &= \dot{U}_{ac,i}(s) \dot{U}_{ac,i}^*(s^*) \end{aligned} \right. \quad (25)$$

$$W_{ac,i}(s) = \frac{1}{\dot{U}_{ac,i}(s)} = \sum_{n=0}^{\infty} W_{ac,i}[n] s^n = W_{ac,i}[0] + W_{ac,i}[1]s + \dots + W_{ac,i}[n]s^n + \dots \quad (26)$$

Comparing the coefficients of the same power series in

(25) and (26), the recurrence relationship of $U_{ac,i}[n]$ and $W_{ac,i}[n]$ is:

$$U_{ac,i}[n] = \text{Re}(\dot{U}_{ac,i}[n]) + \frac{1}{2U_{ac,i}} \sum_{r=1}^{n-1} \dot{U}_{ac,i}[r] \dot{U}_{ac,i}^*[n-r] - \frac{1}{2U_{ac,i}} \sum_{r=1}^{n-1} U_{ac,i}[r] U_{ac,i}[n-r] \quad (27)$$

$$\begin{cases} W_{ac,i}[0] \dot{U}_{ac,i}[0] = 1 & n=0 \\ W_{ac,i}[1] \dot{U}_{ac,i}[0] + W_{ac,i}[0] \dot{U}_{ac,i}[1] = 0 & n=1 \\ W_{ac,i}[n] \dot{U}_{ac,i}[0] + W_{ac,i}[0] \dot{U}_{ac,i}[n] = - \sum_{r=1}^{n-1} W_{ac,i}[r] \dot{U}_{ac,i}[n-r] & n>1 \end{cases} \quad (28)$$

If $s=0$, (19) - (23) are hereby linear equations. Hence, $\dot{U}_{ac,i}(s)$, $U_{ac,i}(s)$, $W_{ac,i}(s)$, $Q_{ac,Gi}(s)$, and $f(s)$ in (19)-(23) are all solved and their values are named the germ solution. Different from the upper grid, which balances the power and the constant frequency of the AC/DC ADN in the grid-connected mode, the DGs will operate in the droop control mode to balance the power, and the frequency will vary for the AC/DC ADN in the islanded mode. Therefore, the germ solution for the AC/DC ADN in the grid-connected and islanded modes is different.

In the grid-connected mode, the germ solution is:

$$\begin{cases} \dot{U}_{ac,i}[0] = 1 & i \in SL \cup PQ_{AC} \cup PV_{AC} \\ W_{ac,i}[0] = 1 & i \in SL \cup PQ_{AC} \cup PV_{AC} \\ Q_{ac,Gi}[0] = 0 & i \in PV_{AC} \\ f[0] = f_0 \end{cases} \quad (29)$$

In the islanded mode, the germ solution is:

$$\begin{cases} \dot{U}_{ac,i}[0] = U_{ac,i}^{(0)} & i \in PQ_{AC} \cup DROOP_{AC} \\ W_{ac,i}[0] = \frac{1}{U_{ac,i}^{(0)}} & i \in PQ_{AC} \cup DROOP_{AC} \\ U_{ac,i}[0] = U_{ac,i}^{(0)} & i \in PQ_{AC} \cup DROOP_{AC} \\ Q_{ac,Gi}[0] = 0 & i \in DROOP_{AC} \\ f[0] = f_0 \end{cases} \quad (30)$$

If the germ solution of the AC network is achieved from (29) or (30), the recursive relationship between the coefficients of the power series for the nodes of the AC/DC ADN in the grid-connected and islanded modes can be obtained by comparing the homogeneous items on both sides of (20)-(23).

In the grid-connected mode, there are slack nodes, PQ nodes, and PV nodes in the AC network, and their recursive relationship between the coefficients of power series is given as (31)-(33), respectively.

$$\dot{U}_{ac,i}[n] = \delta(n)(U_{ac,i}^{sp} - \dot{U}_{ac,i}[0]) \quad i \in SL \quad (31a)$$

$$\delta(n) = \begin{cases} 1 & n=1 \\ 0 & n>1 \end{cases} \quad (31b)$$

$$\sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}[n] = -Y_{ac,i}^{sh} \dot{U}_{ac,i}[n-1] + (\tilde{S}_{ac,Gi}^{*} + P_{ac,i}^{sp} - jQ_{ac,i}^{sp}) W_{ac,i}^*[n-1] + \gamma_i \quad i \in PQ_{AC} \quad (32)$$

$$\begin{cases} \sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}[n] + jW_{ac,i}^*[0] Q_{ac,Gi}[n] = \gamma_i - Y_{ac,i}^{sh} \dot{U}_{ac,i}[n-1] + (P_{ac,Gi}^{sp} + P_{ac,i}^{sp}) W_{ac,i}^*[n-1] - j \sum_{r=1}^{n-1} Q_{ac,Gi}[r] W_{ac,i}^*[n-r] \\ \text{Re}(\dot{U}_{ac,i}[0]) \text{Re}(\dot{U}_{ac,i}[n]) + \text{Im}(\dot{U}_{ac,i}[0]) \text{Im}(\dot{U}_{ac,i}[n]) = \beta(n) \left(-\frac{1}{2} \sum_{r=1}^{n-1} \dot{U}_{ac,i}[r] \dot{U}_{ac,i}^*[n-r] \right) + \frac{1}{2} \delta(n) ((U_{ac,i}^{sp})^2 - U_{ac,i0}) \\ i \in PV_{AC} \end{cases} \quad (33a)$$

$$\delta(n) = \begin{cases} 1 & n=1 \\ 0 & n>1 \end{cases} \quad (33b)$$

where the expression of γ_i can be found in Supplementary Material A (SA2).

Unlike the slack node of the AC network in the grid-connected mode, the reference node, whose voltage angle is set to be 0° , is required in the islanded mode. Moreover, DGs will switch to the AC droop control mode to collectively meet the demand and the demand is significantly impacted by the voltage and frequency in the islanded mode [34]. Hence, in the islanded mode, the recursive relationship between the coefficients of power series for the reference node, PQ node, and AC droop control node is derived as (34)-(36), respectively.

$$\text{Im}(\dot{U}_{ac,i}[n]) = 0 \quad n \geq 1, i \in REF \quad (34)$$

$$\sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}[n] + F_i f[n] = -Y_{ac,i}^{sh} \dot{U}_{ac,i}[n-1] + (\tilde{S}_{ac,Gi}^{*} + P_{ac,i}^{sp} - jQ_{ac,i}^{sp}) W_{ac,i}^*[n-1] + \chi_i \quad i \in PQ_{AC} \quad (35)$$

$$\begin{cases} \sum_{j=1}^N Y_{ac,ij}^{tr} \dot{U}_{ac,j}[n] + \frac{1}{m_i} f[n] W_{ac,i}^*[0] - j \frac{1}{n_i} U_{ac,i}[n] W_{ac,i}^*[0] + F_i f[n] = -\frac{1}{m_i} \sum_{r=1}^{n-1} f[r] W_{ac,i}^*[n-r] + j \frac{1}{n_i} \sum_{r=1}^{n-1} U_{ac,i}[r] W_{ac,i}^*[n-r] + \chi_i - Y_{ac,i}^{sh} \dot{U}_{ac,i}[n-1] \\ U_{ac,i}[n] - \text{Re}(\dot{U}_{ac,i}[n]) = \frac{1}{2U_{ac,i}^{(0)}} \sum_{r=1}^{n-1} (\dot{U}_{ac,i}[r] \dot{U}_{ac,i}^*[n-r] - U_{ac,i}[r] U_{ac,i}[n-r]) \\ i \in DROOP_{AC} \end{cases} \quad (36)$$

where the expressions of F_i and χ_i can be found in Supplementary Material A (SA3) and (SA4), respectively.

According to the germ solutions in (29) and (30) and the recursive relationships in (27) and (31)-(36), the procedure for solving the coefficients of power series $\dot{U}_{ac,i}[n]$, $U_{ac,i}[n]$, $Q_{ac,Gi}[n]$, $W_{ac,i}[n]$, and $f[n]$ is presented in Supplementary Material A Fig. SA1. Through the recursive relationships in (31)-(36), the coefficients of any desired r^{th} order in the power series can be obtained for the grid-connected and the islanded modes.

It is obvious that, if the embedded complex variable s equals 1, the HE equations (19)-(23) are completely transferred to the original equations, $\dot{U}_{ac,i}(1)$ and $f(1)$ are hence the solutions of the voltage and frequency, respectively. Howev-

er, using the power series expansion in (24)-(26) to solve $\dot{U}_{ac,i}(s)$, $U_{ac,i}(s)$, $Q_{ac,i}(s)$, $W_{ac,i}(s)$, and $f(s)$ may encounter slow convergence or even divergence issues. To accelerate the convergence and enlarge the convergence region, the Padé approximation is used to solve these holomorphic functions [13]. The Padé approximation employed in this study is based on the Epsilon algorithm, with the detailed procedure provided in Supplementary Material B.

C. HE Models of DC Network

Considering the voltage characteristic of the loads and the node types, the HE models of loads, constant voltage nodes, constant active power injection nodes, and DC droop control nodes are represented as (37)-(40), respectively.

$$P_{dc,Ld}(\alpha) = P_{dc,Ld}^{(0)} \left[a_{pd} \left(\frac{U_{dc,d}(\alpha)}{U_{dc,d}^{(0)}} \right)^2 + b_{pd} \frac{U_{dc,d}(\alpha)}{U_{dc,d}^{(0)}} + c_{pd} \right] \quad (37)$$

$$U_{dc,d}(\alpha) = U_{dc,d}^{(0)} + \alpha(U_{dc,d}^{sp} - U_{dc,d}^{(0)}) \quad d \in U_{DC} \quad (38)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}(\alpha) = \frac{\alpha(P_{vsc,d}^{sp} + P_{dc,Gd})}{pol \cdot U_{dc,d}(\alpha)} - \frac{\alpha D_{dc,Ld}(\alpha)}{pol \cdot U_{dc,d}(\alpha)} \quad d \in P_{DC} \quad (39)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}(\alpha) = \frac{1}{pol \cdot k_d} \left(\frac{U_{dc,d}^{(0)}}{U_{dc,d}(\alpha)} - 1 \right) - \frac{\alpha D_{dc,Ld}(\alpha)}{pol \cdot U_{dc,d}(\alpha)} \quad d \in DROOP_{DC} \quad (40)$$

Similar to the HE models of AC network, the HE models of DC network in (38)-(40) can be reformulated as (41) and (42) via Taylor series.

$$U_{dc,d}(\alpha) = \sum_{n=0}^{\infty} U_{dc,d}[n] \alpha^n = U_{dc,d}[0] + U_{dc,d}[1] \alpha + \dots + U_{dc,d}[n] \alpha^n + \dots \quad (41)$$

$$W_{dc,d}(\alpha) = \frac{1}{U_{dc,d}(\alpha)} = \sum_{n=0}^{\infty} W_{dc,d}[n] \alpha^n = W_{dc,d}[0] + W_{dc,d}[1] \alpha + \dots + W_{dc,d}[n] \alpha^n + \dots \quad (42)$$

Likewise, the recursive relationship between $U_{dc,d}[n]$ and $W_{dc,d}[n]$ can be further derived from (41) and (42), respectively. Similar to the AC network, set $\alpha=0$ in (37)-(40) to derive the germ solution for the DC network.

1) Grid-connected Mode

In the grid-connected mode, there are constant voltage nodes, constant active power injection nodes, and DC droop control nodes in the DC network. The germ solution in the grid-connected mode is:

$$\begin{cases} U_{dc,d}[0] = U_{dc,d}^{(0)} \\ W_{dc,d}[0] = \frac{1}{U_{dc,d}^{(0)}} \end{cases} \quad d \in U_{DC} \cup P_{DC} \cup DROOP_{DC} \quad (43)$$

For $n=1$, by comparing the homogeneous items on both sides of (38)-(40), the recursive relationship between the coefficients of power series for constant voltage nodes, constant active power injection nodes, and DC droop control nodes are expressed as (44)-(46), respectively.

$$U_{dc,d}[1] = U_{dc,d}^{sp} - U_{dc,d}^{(0)} \quad d \in U_{DC} \quad (44)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}[1] = \frac{P_{vsc,d}^{sp} + P_{dc,Gd}}{pol} W_{dc,d}[0] - \zeta_d \quad d \in P_{DC} \quad (45)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}[1] - \frac{U_{dc,d}^{(0)}}{pol \cdot k_d} W_{dc,d}[1] = -\zeta_d \quad d \in DROOP_{DC} \quad (46)$$

$$\zeta_d = \frac{P_{dc,Ld}^{(0)}}{pol} \left(a_{pd} \frac{U_{dc,d}[0]}{U_{dc,d}^{(0)}} + \frac{b_{pd}}{U_{dc,d}^{(0)}} + c_{pd} W_{dc,d}[0] \right) \quad (47)$$

Further, for $n>1$, the recursive relationships between the coefficients of power series for constant voltage nodes, constant active power injection nodes, and droop control nodes are given in (48)-(50), respectively.

$$U_{dc,d}[n] = 0 \quad d \in U_{DC} \quad (48)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}[n] = \frac{P_{vsc,d}^{sp} + P_{dc,Gd}}{pol} W_{dc,d}[n-1] - \zeta_d \quad d \in P_{DC} \quad (49)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}[n] - \frac{U_{dc,d}^{(0)}}{pol \cdot k_d} W_{dc,d}[n] = -\zeta_d \quad d \in DROOP_{DC} \quad (50)$$

$$\zeta_d = \frac{P_{dc,Ld}^{(0)}}{pol} \left(a_{pd} \frac{U_{dc,d}[n-1]}{U_{dc,d}^{(0)}} + c_{pd} W_{dc,d}[n-1] \right) \quad (51)$$

2) Islanded Mode

In the islanded mode, there are only constant active power injection nodes and DC droop control nodes in the DC network. The germ solution is:

$$\begin{cases} U_{dc,d}[0] = U_{dc,d}^{(0)} \\ W_{dc,d}[0] = \frac{1}{U_{dc,d}^{(0)}} \end{cases} \quad d \in P_{DC} \cup DROOP_{DC} \quad (52)$$

For $n=1$, by comparing the homogeneous items on both sides of (39) and (40), the recursive relationships between the coefficients of power series for constant active power injection nodes and DC droop control nodes are represented as (53) and (54), respectively.

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}[1] = -\zeta_d \quad d \in P_{DC} \quad (53)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}[1] - \frac{U_{dc,d}^{(0)}}{pol \cdot k_d} W_{dc,d}[1] = -\zeta_d \quad d \in DROOP_{DC} \quad (54)$$

Further, for $n>1$, the recursive relationships between the coefficients of power series for constant active power injection nodes and DC droop control nodes are given as:

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}[n] = -\zeta_d \quad d \in P_{DC} \quad (55)$$

$$\sum_{k=1}^M G_{dc,dk} U_{dc,k}[n] - \frac{U_{dc,d}^{(0)}}{pol \cdot k_d} W_{dc,d}[n] = -\zeta_d \quad d \in DROOP_{DC} \quad (56)$$

For the coefficients of power series $U_{dc,d}[n]$ and $W_{dc,d}[n]$, the solution is similar to that in AC network proposed in Section III-B.

D. Power Flow Calculation Process of AC/DC ADNs

The flowchart of the proposed FHES method for power flow calculation of AC/DC ADNs in the grid-connected and islanded modes can be found in Supplementary Material A Fig. SA2.

IV. NUMERICAL EXAMPLES

In this section, the performance of the proposed FHES method for AC/DC ADNs is evaluated by two modified hybrid AC/DC test systems, i.e., the modified IEEE 33-node and IEEE 123-node AC/DC ADNs. The NR method is employed as a baseline to evaluate the performance of the proposed FHES method. Both the proposed FHES method and NR method are based on the sequential iterative method. In the power flow calculations for both AC and DC networks, a permissible convergence error for power is set to be $e = 1 \times 10^{-8}$. The convergence criteria in the grid-connected and islanded modes are set to be $e_1 = 1 \times 10^{-8}$ and $e_2 = 0.05$, respectively.

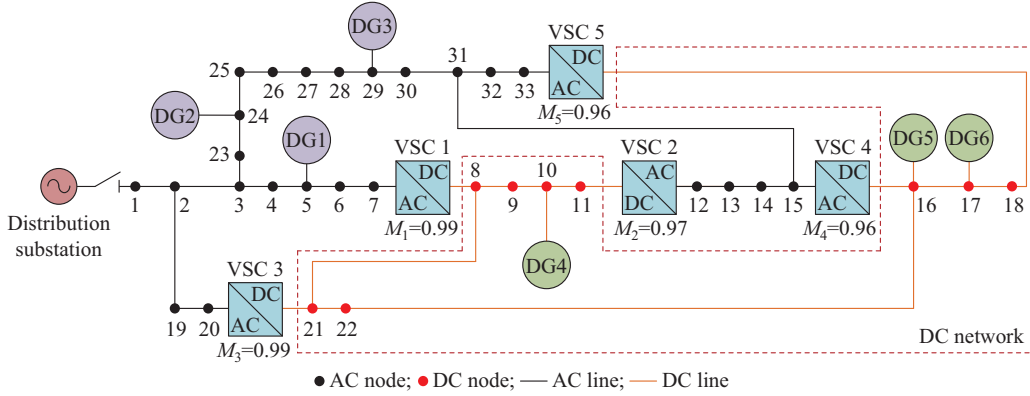


Fig. 2. Modified IEEE 33-node AC/DC ADN.

1) Grid-connected Mode

Using the sequential iterative method to solve the power flow of AC/DC ADNs requires solving the power flow of the AC network. The HE models of the AC network in Section III-B are used to solve the power flow. According to (29), the germ solution is obtained and the 1st-order coefficient of voltage power series $\dot{U}_{ac}[1]$ of each AC node can be obtained via the germ solution and the recursive relationships in (31)-(33). The 0th-order and 1st-order coefficients of the voltage power series of each node are analytically continued using Padé approximation at $s=1$, then the approximated voltage of each node in the AC network can be obtained. Since the maximum power imbalance of AC network $|\Delta S_{\max}| = 0.0050$ p.u. is greater than the threshold 1×10^{-8} p.u., a higher order of voltage power series is required. Supplementary Material D Fig. SD1 shows the relationship between the order of power series and the power imbalance.

As illustrated in Supplementary Material D Fig. SD1, the power imbalance decreases as the order of power series increases. In this case, the power series cannot meet the convergence threshold until the 8th order is reached. The power flow calculation of the AC network is hence completed.

After calculating the power flow of the AC network, the power flow information on the PCC needs to be transmitted to the DC network. The HE models of the DC network in Section III-C are used to solve its power flow, which is similar to the process of AC network. The 1st-order coefficient of the voltage power series $\dot{U}_{dc}[1]$ of each DC node is obtained via the germ solution (43) and the recursive relationship

ively. Detailed discussions on the selection of e_1 and e_2 can be found in Supplementary Material C.

A. Modified IEEE 33-node AC/DC ADN

The modified IEEE 33-node AC/DC ADN is shown in Fig. 2 and the corresponding parameters are available in [29]. The area enclosed by the dotted line in Fig. 2 represents the DC network, which is connected to the AC network through VSC 1-VSC 5. The parameters of DGs and the control parameters of VSCs are listed in Supplementary Material D Table SDI and Table SDII, respectively. The initial parameters correspond to the output of generators in the grid-connected mode.

(44)-(46). Then, the voltage of each DC node is obtained via Padé approximation at $\alpha=1$. Since the maximum power imbalance of DC network $|\Delta P_{\max}| = 0.0001$ p.u. is greater than the threshold 1×10^{-8} p.u., the higher order of voltage power series is required. The error is satisfied when the order of power series increases to 3.

The PCC power of the AC network is updated according to the power flow calculation results of DC network. Since the maximum power imbalance of the PCC $|\Delta S_{\max}| = 0.0388$ p.u. is greater than the threshold 1×10^{-8} p.u., the next sequential process needs to be calculated. The sequential process alternates only 4 times and makes the power imbalance meet the convergence threshold. The sequential process is illustrated in Supplementary Material D Table SDIII. Hence, the power flow calculation of the proposed FHES method for the modified IEEE 33-node AC/DC ADN is completed.

1) Verification of accuracy

In Supplementary Material D Table SDIV, the results of the proposed FHES method are compared with those of the NR method. The power flow results of NR method and the proposed FHES method are almost identical, and the maximum relative error of the proposed FHES method is less than 1×10^{-7} . This result demonstrates the accuracy of the proposed FHES method in calculating the power flow of AC/DC ADNs.

Figure 3 compares the active power and relative errors of the top 8 branches with the largest errors calculated by the proposed FHES method and NR method, and Fig. 4 shows the corresponding reactive power results. It is clear that the

results obtained by the proposed FHES method still closely match those of NR method, which confirms the accuracy of the proposed FHES method in solving the power flow of the AC/DC ADNs in the grid-connected mode.

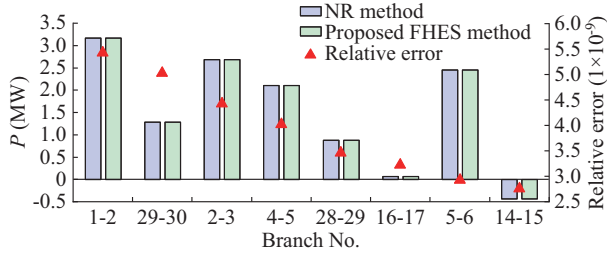


Fig. 3. Active power and relative errors of top 8 branches.

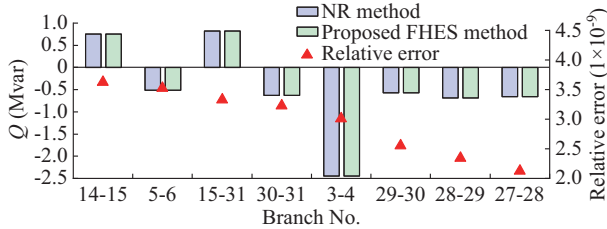


Fig. 4. Reactive power and relative errors of top 8 branches.

2) Verification of efficiency and robustness

The proposed FHES method does not require initial values because initial values are uniquely determined by the formulated HE equations. In contrast, the NR method requires predefined initial values. Figure 5 compares the computational efficiency of the NR method under different initial voltages. When the initial voltages are set below 0.74 p.u., the NR method diverges. This demonstrates the deficiency of the NR method in the initial value selection.

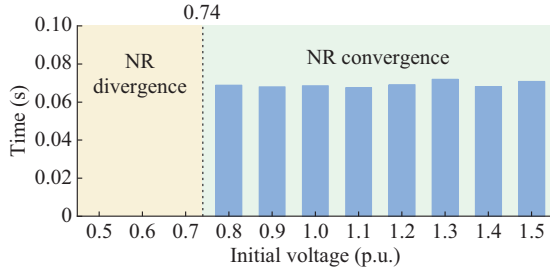


Fig. 5. Computational efficiency of NR method under different initial voltages.

The efficiency and robustness of the proposed FHES method are also evaluated by increasing the load levels in the modified IEEE 33-node AC/DC ADN, and the results are listed in Supplementary Material D Table SDV. Supplementary Material D Table SDV shows that the calculated power flow results of the proposed FHES method are identical to those of the NR method until the load increases to 2.71 times the baseline. However, as the load continues to increase, the NR method fails to converge due to its dependency on proper initial values, whereas the proposed FHES method successfully solves the power flow. In addition, compared with the NR method, the proposed FHES method

achieves higher computational efficiency while maintaining calculation accuracy. This comparison demonstrates the accuracy, efficiency, and robustness of the proposed FHES method in the power flow calculation.

Figure 6 compares the calculation results of the NR method and the proposed FHES method with the load at node 6 increasing to 3 times its original value. It is obvious from Fig. 6 that the voltage magnitudes of the modified IEEE 33-node AC/DC ADN calculated by the NR method are mostly below 0.95 p.u., which may mislead operators to make incorrect control decision. In contrast, the voltage solutions obtained by the proposed FHES method remain within the secure operating range. The comparative results demonstrate the robustness of the proposed FHES method while the NR method fails in this scenario.

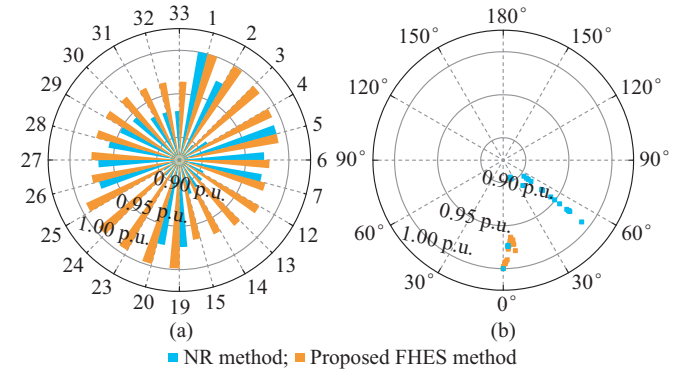


Fig. 6. Calculation results of NR method and proposed FHES method with load at node 6 increasing to 3 times its original value. (a) Voltage magnitude. (b) Voltage phase angle.

3) VSC control mode switching

During the normal operation, the VSC should adjust its control mode to maintain the nodal voltage within the secure operating range. The upper and lower limits for the nodal voltage magnitude of VSC are 1.05 p.u. and 0.95 p.u., respectively. When the reactive loads at AC nodes 15 and 33 increase to 5 times their nominal values, the reactive support capacity at these nodes becomes insufficient. The nodal voltages of VSC 4 and VSC 5 are 0.9364 p.u. and 0.9226 p.u., respectively, both below the lower voltage limit. To improve the voltages of VSC 4 and VSC 5, their control modes of AC sides are switched from reactive power control to AC voltage control, with the voltage setpoint specified as 1.0 p.u..

Table I lists the nodal voltage magnitudes and relative errors of top 8 branches with the largest errors calculated by the proposed FHES method and NR method. The maximum relative errors of the AC and DC nodes are 9.4037×10^{-10} and 1.9377×10^{-10} , respectively, demonstrating that the proposed FHES method can accurately calculate the power flow of AC/DC ADNs under various control modes.

2) Islanded Mode

Assume that a fault occurring in the upper grid of the modified IEEE 33-node AC/DC ADN causes the whole AC/DC ADN operating from the grid-connected mode to islanded mode.

TABLE I
NODAL VOLTAGE MAGNITUDES AND RELATIVE ERRORS OF TOP 8
BRANCHES WITH THE LARGEST ERRORS CALCULATED BY
PROPOSED FHES METHOD AND NR METHOD

Node type	Node No.	Voltage (p.u.)		Error
		NR	FHES	
AC node	15	1.0000	1.0000	9.4037×10^{-10}
	14	0.9938	0.9938	9.1576×10^{-10}
	13	0.9883	0.9883	8.9252×10^{-10}
	33	1.0000	1.0000	8.7120×10^{-10}
	12	0.9800	0.9800	8.4536×10^{-10}
	32	0.9970	0.9970	8.3670×10^{-10}
	31	0.9954	0.9954	8.0871×10^{-10}
	30	0.9910	0.9910	6.6267×10^{-10}
DC node	17	0.9976	0.9976	1.9377×10^{-10}
	18	0.9973	0.9973	1.8048×10^{-10}
	16	0.9978	0.9978	1.6103×10^{-10}
	22	0.9983	0.9983	1.1672×10^{-10}
	21	0.9983	0.9983	7.9355×10^{-11}
	10	0.9981	0.9981	2.2126×10^{-11}
	11	0.9979	0.9979	1.6775×10^{-11}
	9	0.9989	0.9989	7.5053×10^{-12}

Supplementary Material D Table SDVI compares the results of the NR method and the proposed FHES method in the islanded mode. As shown in Table SDVI, in the islanded mode, the results calculated by the proposed FHES method are essentially consistent with those by the NR method. The frequencies calculated by the NR method and the proposed FHES method are both 0.9902 p.u.. These results confirm the accuracy of the proposed FHES method for the power flow calculation in the AC/DC ADN in the islanded mode. Supplementary Material D Table SDVII compares the outputs of DGs calculated by the NR method and the proposed FHES method in the islanded mode. The outputs of DGs calculated by the proposed FHES method are the same as those of NR method. These comparisons confirm the accuracy of the proposed FHES method and the power flow calculation of AC/DC ADNs.

B. Modified IEEE 123-node AC/DC ADN

The applicability of the proposed FHES method is evaluated by the modified IEEE 123-node AC/DC ADN in Fig. 7. The parameters of DGs and VSCs are listed in Supplementary Material D Table SDVIII and Table SDIX, respectively. Other parameters can be found in [35], [36].

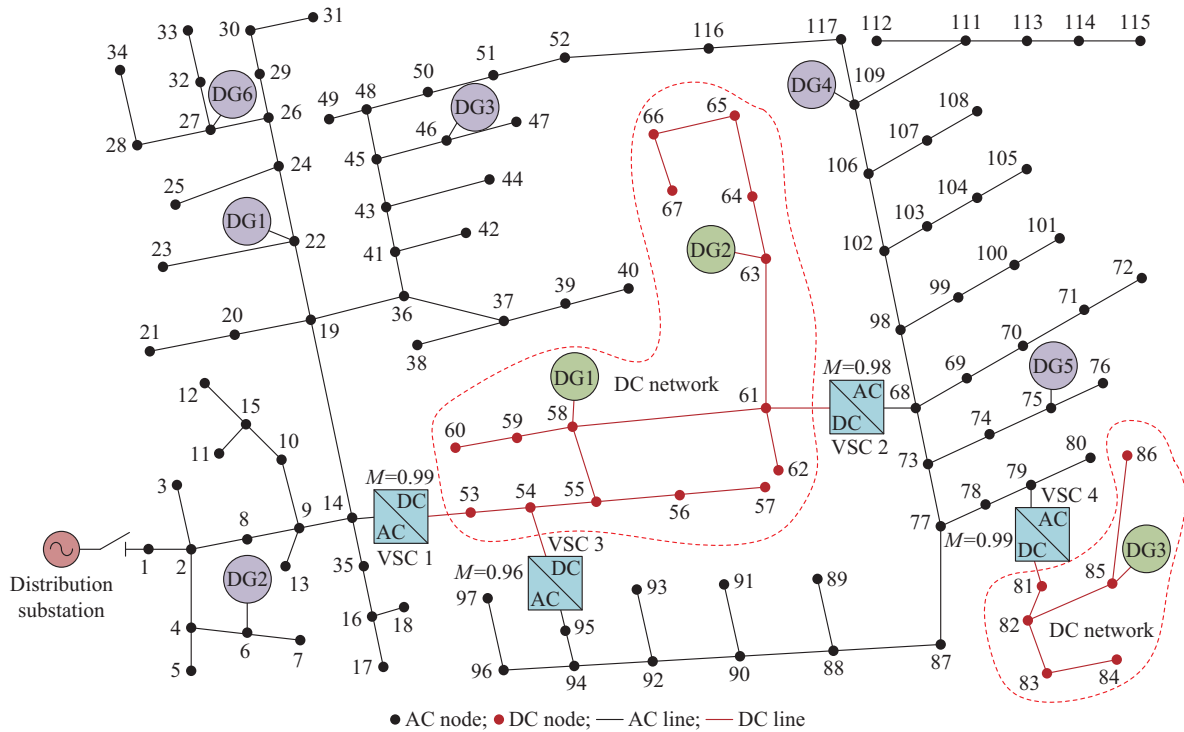


Fig. 7. Modified IEEE 123-node AC/DC ADN.

1) Grid-connected Mode

The power flow of the modified IEEE 123-node AC/DC ADN in the grid-connected mode is calculated by the proposed FHES method and NR method. Using the result of NR method as the baseline, Supplementary Material D Table SDX lists the voltage magnitudes and active power of the top 8 branches with the largest relative errors in the DC network, and the voltage magnitudes and angles of top 8 nodes

with the largest relative errors in the AC network are illustrated in Supplementary Material D Fig. SD3 and Fig. SD4, respectively.

The comparisons of results in Table SDX, Fig. SD3, and Fig. SD4 show that the power flow results of the proposed FHES method closely match those of the NR method, with the relative errors below 1×10^{-7} . These comparisons prove the accuracy of the proposed FHES method in the power

flow calculation of the larger-scale AC/DC ADN in the grid-connected mode.

2) Islanded Mode

The power flow of the modified IEEE 123-node AC/DC ADN in the islanded mode is calculated by the proposed FHES method and the NR method. The frequencies of this AC/DC ADN in the islanded mode calculated by the proposed FHES method and the NR method are both 0.9953 p. u.. Supplementary Material D Fig. SD5 presents the voltage magnitudes, phase angles, active power, and reactive power of top 8 nodes with the largest errors by employing the NR method as the baseline. As shown in Fig. SD5, the maximum relative errors are all less than 1×10^{-6} . This further confirms the accuracy of the proposed FHES method in calculating the power flow of larger AC/DC ADNs in the islanded mode.

Supplementary Material D Table SDXI compares the power injection of DGs calculated by the proposed FHES method and the NR method in the islanded mode. The results calculated by the proposed FHES method match those of NR method very well, which confirms the capability of the proposed FHES method in the power flow calculation of the large-scale AC/DC ADNs.

C. Computational Efficiency

Figures 8 and 9 compare the mismatch of the proposed FHES method and the NR method at each iteration in the modified IEEE 33-node and IEEE 123-node AC/DC ADNs, respectively. The results indicate that the NR method satisfies the convergence conditions within fewer iterations due to its quadratic convergence property. Table II further compares the computational time of the NR method and the proposed FHES method. The results indicate that, although the the proposed FHES method involves higher orders of power series, it achieves significant advantages in the computational efficiency compared with the NR method in AC/DC ADNs with different scales and operation modes.

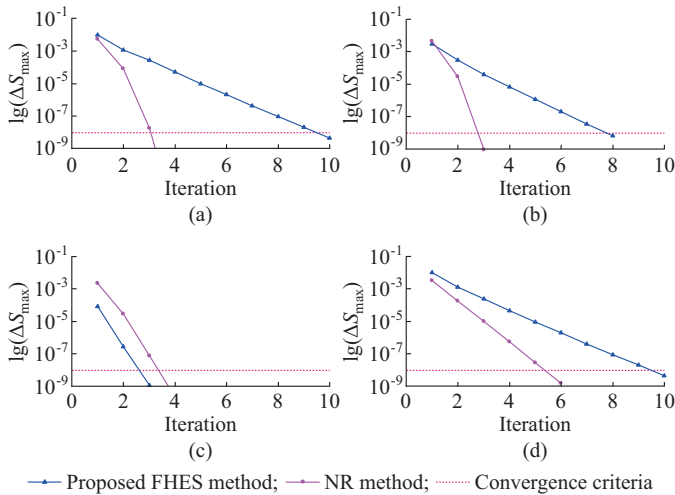


Fig. 8. Mismatch of proposed FHES method and NR method at each iteration in modified IEEE 33-node AC/DC ADN. (a) AC node in grid-connected mode. (b) AC node in islanded mode. (c) DC node in grid-connected mode. (d) DC node in islanded mode.

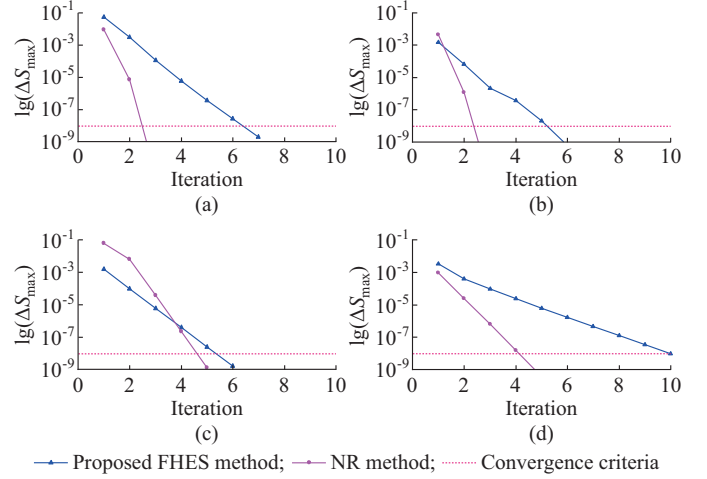


Fig. 9. Mismatch of proposed FHES method and NR method at each iteration in modified IEEE 123-node AC/DC ADN. (a) AC node in grid-connected mode. (b) AC node in islanded mode. (c) DC node in grid-connected mode. (d) DC node in islanded mode.

TABLE II
COMPUTATIONAL TIME COMPARISON BETWEEN PROPOSED FHES METHOD AND NR METHOD

AC/DC ADN	Operation mode	Time (s)		Improvement (%)
		NR	FHES	
IEEE 33-node	Grid-connected	0.0686	0.0367	46.56
	Islanded	0.0841	0.0485	42.36
IEEE 123-node	Grid-connected	0.0734	0.0444	39.57
	Islanded	0.0737	0.0503	31.68

The computational time of the proposed FHES method is mainly determined by the following processes: ① solving high-order coefficients of power series; and ② analytical continuation process calculations. In the first process, the matrix used to compute the coefficients of power series remains constant through the solution process. During the sequential process, only a single LU decomposition of this matrix is required. The decomposition results are stored and repeatedly utilized, thereby significantly reducing the per-step computational time. In contrast, the NR method requires recalculating the Jacobian matrix and performing LU decomposition at each iteration, thereby increasing the computational burden. In the second process, the Padé approximation is employed to obtain voltage solutions. Epsilon algorithm, which only requires the embedded variables $s=1$ and $\alpha=1$, is adopted. The matrix vectorization is developed to calculate the voltage approximations of all nodes simultaneously, further improving the computational efficiency. The detailed procedure of this process is provided in Supplementary Material A.

V. CONCLUSION

This paper proposes an FHES method to calculate the power flow of AC/DC ADNs. The performance of the proposed FHES method is evaluated by the modified IEEE 33-node and IEEE 123-node AC/DC ADNs, the conclusions are drawn as follows.

1) The proposed FHES method can effectively solve the power flow of AC/DC ADNs in both the grid-connected and islanded modes, which exhibits great advantages of flexibility over the existing HELMs.

2) The accuracy of HE models for the AC/DC ADN, which take account of the static voltage and frequency characteristics of the load and the different control modes of VSCs and DGs, and the performance of the proposed FHES method has been demonstrated.

3) The proposed FHES method has advantages in the robustness and computational efficiency over NR method in calculating the power flow of AC/DC ADNs in different scenarios.

The proposed FHES method preserves the classical HELM characteristic of requiring no initial values. However, when performing power flow calculations for heavily loaded systems, the existing HELM often requires high-order coefficients of power series, resulting in slower convergence and increased computational complexity. Future work will incorporate multi-stage HELM into power flow calculations to alleviate this issue. Furthermore, the three-phase balanced HE models for various DGs, VSCs, and ZIP loads developed in this work could serve as references for analyzing unbalanced three-phase power flow of AC/DC ADNs. This will be an important focus of future research.

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