

Non-cooperative Game Optimization of Urban Energy Systems Based on Inner-outer Iterative Method

Chenxu Yin, Yonghui Sun, Dongliang Xie, Fan Sheng, and Liang Zhao

Abstract—This paper addresses the increasingly tight energy coupling in urban energy systems, which are composed of the power distribution network (PDN), the transportation network (TN), and the gas distribution network (GDN). A non-cooperative game optimization model for the urban energy system is developed, to achieve coordinated optimization among these three different energy networks operated by independent stakeholders. To avoid excessive sharing of private information, an inner-outer iterative method is further proposed to obtain the Nash equilibrium and enhance solving efficiency. In the proposed method, each energy network is iteratively optimized by exchanging only the price and load information, and the network parameters are not disclosed. In the inner layer, the PDN-TN coupled subsystem and the PDN-GDN coupled subsystem are solved separately. In the outer layer, interactions between charging loads and gas prices are coordinated through the PDN, thereby enabling cross-network coordination of urban energy systems. Case studies demonstrate that the convergence speed of the PDN-TN coupled subsystem is significantly improved by dynamically adjusting the electricity price based on sensitivity coefficients. Balanced resource allocation of the urban energy system is achieved through the non-cooperative game, while autonomy of all stakeholders is preserved. Energy procurement costs are reduced by up to 25.56% compared with that of the PDN-TN coupled subsystem.

Index Terms—Urban energy system, power distribution network (PDN), non-cooperative game, coordinated optimization, inner-outer iterative method, Nash equilibrium, privacy preservation.

NOMENCLATURE

A. Functions

$\tau_{a,R}(t, x_{a,R}(t))$	Travel time and queuing time of transportation network (TN)
$\tau_{a,C}(t, x_{a,C}(t))$	

F_{PN}	Operation cost of power distribution network (PDN)
F_{pn}	Operation cost of PDN excluding gas purchase cost
F_{TN}	Operation cost of TN
F_{tn}	Travel-time cost of TN
F_{GN}	Operation cost of gas distribution network (GDN)
L_{TN}, L_{PN}	Lagrangian functions of TN and PDN

B. Sets and Indices

a, l, k	Indexes of road, path, and route for TN
i, j, h	Indexes of nodes for PDN
K_{OD}	Route of origin and destination (OD) pair
$m(j), n(j)$	Branch sets with node j as OD nodes
m, n, u, v	Indexes of nodes of GDN
OD	Index of OD pair
r	Index of iterations
$T_{A,C}, T_{A,R}$	Indexes of roads with and without charging stations

C. Parameters

χ_1, χ_2, χ_3	Coefficients of energy costs
$\alpha_1, \alpha_2, \alpha_3$	Coefficients of operation and maintenance costs
γ	Coefficient of self-dissipation
Δt	Time step of optimization
ω	Monetary value of travel time
η_C, η_{GT}	Coefficients of device efficiency
φ_{GT}	Coefficient of energy conversion
$\delta_{OD,a,k}(t)$	Incidence parameter (1 indicates an association, whereas 0 indicates no association)
$\sigma_{k,l}(t)$	Energy consumption on road
δ_{mn}	Incidence parameter of direction for flow
$\mu_1, \mu_2, \varsigma_1, \varsigma_2$	Operating coefficients of device

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$\tau_{a,R,0}(t), \tau_{a,C,0}(t)$	Congestion and waiting time for charging	$P_{i,j}(t), Q_{i,j}(t)$	Active power and reactive power of PDN
a_1, b_1, c_1	Coefficients of electricity price for charging station	$P_{C, re}(t), P_{C, \pi}(t)$	Auxiliary variables related to $P_C(t)$
a_2, b_2, c_2	Coefficients of gas price of gas turbine (GT)	$P_W(t)$	Power output of device
$c_{a,R}^{\max}(t), c_{a,C}^{\max}(t)$	The maximum numbers of non-congested vehicles and non-queued electric vehicles (EVs)	$Q_{j, COM}(t), Q_{j, WT}(t)$	Reactive power of compensation and WT
$E_{OD, ini}(t), E_{OD, end}(t)$	Total initial and terminal energies	$q_{gas}(t)$	Gas purchased from upstream GDN
$f_{mn, gas}^L(t), f_{mn, gas}^U(t)$	Lower and upper bounds of pipeline flow	$q_m(t), q_{m, gas}(t)$	Gas purchased from upstream and gas flow injected at node of GDN
M_{mn}	Parameter of pipeline	$q_{m, L, gas}(t), q_{m, L, GT}(t)$	Gas loads at node of GDN and GT
$p_{GT}(t), p_{r, GT}(t)$	Gas prices of GT	$S(t)$	Energy stored in storage device
$p_{TR}(t), p_{a, TR}(t), p_{a, r, TR}(t), p_{a, j, TR}(t)$	Electricity prices of charging station	$U_i(t), U_j(t)$	Squared voltages of PDN
$P_{W, min}(t), P_{W, max}(t)$	The minimum and maximum power of device	$x_a(t)$	Total traffic flow
$R_{i,j}$	Resistance of PDN	$x_{a, R}(t), x_{a, C}(t)$	Traffic flows on regular road and charging station road
$X_{i,j}$	Reactance of PDN		
$Z_{a, r, TR}(t), Z_{r, GT}(t)$	Sensitivity coefficients of price for charging station and GT		
<i>D. Variables</i>			
$\pi_m(t), \pi_n(t), \pi_u(t)$	Gas pressures of GDN		
$\pi_v(t)$			
$\varsigma_{mn, gas}(t)$	Auxiliary variable of gas flow		
$\Pi_m(t), \Pi_n(t)$	Squares of pressure term		
$f_{OD, k}(t)$	Traffic flow on route of OD pair		
$f_{k, l}(t)$	Traffic flow on path of route		
$f_{mn, gas}(t)$	Gas flow of pipeline		
$I_{i,j}(t)$	Current of PDN		
$L_{j, TR}(t), L_{a, TR}(t), L_{a, r, TR}(t), L_{a, j, TR}(t), L_{TR}(t)$	Electricity loads of charging station		
$L_{j, P}(t), L_{j, Q}(t)$	Active and reactive loads at node of PDN		
$L_k(t)$	Charging energy on path		
l_{TN}, l_{PN}	Lagrange terms associated with constraint of cost functions for TN and PN		
$P_{grid}(t)$	Electricity purchased from upstream network		
$P_{WT}(t), P_{GT}(t)$	Power output of wind turbine (WT) and GT		
$q_{GT}(t), q_{r, GT}(t)$	Gases consumed by GT		
$P_C(t)$	Power exchanged by storage device		
$P_{j, WT}(t), P_{j, GT}(t), P_{j, C}(t)$	Power generated at node of WT, GT and storage device		
$P_j(t), Q_j(t)$	Active power and reactive power injected at node of PDN		

I. INTRODUCTION

URBAN areas have been identified as one of the main centers of energy consumption [1]. The coupling among separate energy systems has been intensified by advances in energy technology. A multi-energy interconnected network has been established in urban areas [2]. Although this complex network provides abundant energy supply, it also creates new challenges for energy management and dispatch. The spread of electric vehicles (EVs) and charging infrastructure have strengthened the coupling between the power distribution network (PDN) and the transportation network (TN) [3]. Electricity and gas are both essential for urban users, so the PDN and the gas distribution network (GDN) have also become inseparable [4]. Coordinated optimization of the coupled PDN, TN, and GDN is essential for improving energy efficiency. It also reveals the coupling mechanisms of the system and provides a theoretical basis for coordinated urban energy management.

Current research on coordinated optimization of urban energy systems has mainly focused on two directions: modeling and optimization methods. Modeling of urban energy systems is divided into three categories. The first category incorporates only the PDN and GDN. The second category incorporates the PDN and TN. The third category integrates the PDN, GDN, and TN simultaneously. In [5] and [6], the model of gas turbine (GT) was built, and a coordinated optimization model was proposed for the coupled system of the PDN and the GDN. In [6], the costs of energy purchase, operation and maintenance, and carbon emission were converted into economic terms, and a corresponding optimization model was developed. To accommodate multiple objectives, multi-objective optimization models were built in [7] and [8], where the Pareto theory and the analytic hierarchy process were utilized, respectively. Traffic data were used to partition regions and a bi-level optimization model was proposed for the coupled system of the PDN and TN in [9] and [10].

In [11], the regulation capability of EV clusters was leveraged, and a two-stage optimization model that considered the coupling between the PDN and TN was proposed. In [12], an optimization model for scheduling coupled systems that included an EV aggregator was established. Although EV behavior was characterized by a clustered model, network constraints on transportation roads were not incorporated, leading to deviations from reality.

The method of origin and destination (OD) was often used to characterize network constraints on roads. In [13] and [14], traffic flow allocation was analyzed from an OD perspective, and a bi-level optimization model was built. The urban energy system in real society is associated with different stakeholders and objectives. In [6], [8], and [13], the goals of all stakeholders were transformed into a single overall objective, and in [7], a solution set was provided for the system. Even so, the final strategy could not ensure the interest balance among all stakeholders. In [9] and [14], non-cooperative game optimization models were proposed for the coupled system of the PDN and TN, achieving optimal power flow in the PDN and Nash equilibrium in the TN, while only two forms of energy were involved. Since the PDN, TN, and GDN were not covered simultaneously, the multi-energy coordinated optimization problem faced by modern cities remained unresolved. This gap has limited the realization of coordination potential, as existing optimizations fail to capture critical cross-network interactions. As a result, the full potential of each network is not realized, and overall system performance is compromised. More effective coordination and resource allocation, together with improved system efficiency, can be achieved by optimization of the three networks. In [15] and [16], a coordinated optimization model for the coupled system of the PDN, TN, and GDN was established. However, in aforementioned studies, a single objective function was used to represent the interests of all energy stakeholders. Consequently, the diverse interests of different stakeholders were not fully addressed.

Existing studies on solving urban energy systems are mainly classified into three methods. Centralized methods were adopted in [6]-[8], [13], [15], and [16]. Hierarchical iterative methods were adopted in [9]-[12]. Game-theoretic iterative methods were adopted in [5] and [14]. For centralized methods, computational efficiency can be hindered by the large number of decision variables. Additionally, sensitive information can be disclosed when network parameters are shared, raising privacy concerns. Small-scale optimization models can be solved with a single optimization run by hierarchical iterative methods and game-theoretic iterative methods. A more decentralized manner is allowed by hierarchical iterative methods, either by dividing the problem into manageable subproblems or by modeling the interactions of independent stakeholders. Required computational resources are significantly reduced by hierarchical iterative methods and game-theoretic iterative methods. However, when more stakeholders are included in the optimization model, convergence and convergence speed of the iterative process are not always guaranteed [17]. In particular, in game-theoretic iterative

methods, the computation of Nash equilibrium can be expensive and challenging in highly complex systems. Hierarchical iterative methods were applied in the aforementioned studies to solve coordinated optimization problems for coupled energy systems that include only two energy types. Therefore, these methods are not necessarily suitable for coordinated optimization of urban energy systems that consider the PDN, TN, and GDN. The main contributions of this paper are summarized as follows.

- 1) A non-cooperative game optimization model is formulated for the urban energy system comprising the PDN, TN, and GDN. The mutual interactions among electricity price, load, and gas price are accurately represented.
- 2) An inner-outer iterative method is proposed. In the inner layer, the optimal response is obtained by communicating only price and load information. In the outer layer, charging load and gas load are updated, and coordination among the three energy networks is achieved.
- 3) The proposed model is proved to guarantee the existence of a unique Nash equilibrium in the non-cooperative game. By applying the Karush-Kuhn-Tucker (KKT) conditions, the proposed method is proved to converge to the optimal solution of the original game optimization model for the urban energy system.

II. OPTIMIZATION MODELS FOR DIFFERENT ENERGY NETWORKS

The urban energy system proposed in this paper is composed of the PDN, GDN, and TN. The interaction structure of the urban energy system is illustrated in Fig. 1.

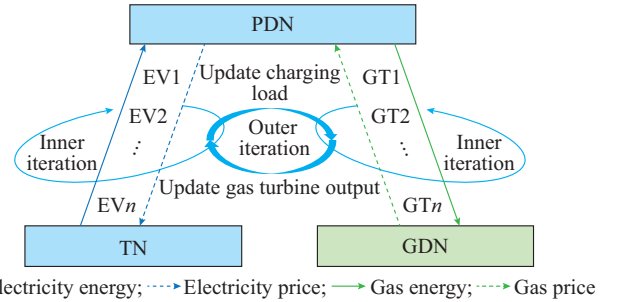


Fig. 1. Interaction structure of urban energy system.

A. Optimization Model of PDN

Without loss of generality, the cost function for the PDN is defined as:

$$\begin{aligned} \min F_{PN} = & \sum_{i=1}^{24} (\chi_1 P_{\text{grid}}(t) + \chi_2 I_{ij}(t) R_{ij}) + \\ & \sum_{i=1}^{24} (\alpha_1 P_{WT}(t) + \alpha_2 P_{GT}(t) + \alpha_3 |P_C(t)|) + \\ & \sum_{i=1}^{24} (p_{GT}(t) q_{GT}(t) - p_{TR}(t) L_{TR}(t)) \end{aligned} \quad (1)$$

To facilitate computation, power flow constraints based on second-order cone relaxation are employed [18]:

$$U_i(t) - U_j(t) = 2(R_{ij} P_{ij}(t) + X_{ij} Q_{ij}(t)) - I_{ij}(t)(R_{ij}^2 - X_{ij}^2) \quad (2)$$

$$I_{ij}(t)U_i(t) \geq P_{ij}^2(t) + Q_{ij}^2(t) \quad (3)$$

The model of the electrical energy storage device with self-dissipation is defined as:

$$S(t) = (1 - \gamma)S(t-1) + \frac{(|P_c(t)| + P_c(t))\eta_c \Delta t}{2} + \frac{(P_c(t) - |P_c(t)|)\Delta t}{2\eta_c} \quad (4)$$

Electrical energy at each node is required to satisfy the following balance constraints:

$$P_j(t) = P_{j,WT}(t) + P_{j,GT}(t) - L_{j,TR}(t) - L_{j,P}(t) - P_{j,C}(t) \quad (5)$$

$$Q_j(t) = Q_{j,WT}(t) + Q_{j,COM}(t) - L_{j,Q}(t) \quad (6)$$

The electricity price function of charging station is established according to the actual load, and is constrained in a quadratic form as [19]:

$$p_{TR}(t) = \chi_1 + a_1 \left(\frac{L_{TR}(t)}{\sum_{t=1}^{24} L_{TR}(t)} \right)^2 + b_1 \left(\frac{L_{TR}(t)}{\sum_{t=1}^{24} L_{TR}(t)} \right) + c_1 \quad (7)$$

The remaining constraints of operation and power flow in the PDN are detailed in Appendix A.

B. Optimization Model of TN

The cost function for the TN is defined as:

$$\min F_{TN} = F_{tn} + \sum_{t=1}^{24} \sum_{a \in T_{A,C}} p_{a,TR}(t) L_{a,TR}(t) \quad (8)$$

$$F_{tn} = \omega \sum_{t=1}^{24} \sum_{a \in T_{A,R}} \int_0^{x_{a,R}(t)} \tau_{a,R}(t, \theta) d\theta + \omega \sum_{t=1}^{24} \sum_{a \in T_{A,C}} \int_0^{x_{a,C}(t)} \tau_{a,C}(t, \theta) d\theta \quad (9)$$

The total traffic flow on each road is composed of traffic flows on the regular road and charging station road:

$$x_a(t) = x_{a,R}(t) + x_{a,C}(t) \quad (10)$$

The constraint linking the total traffic flow to its associated roads is given as:

$$x_a(t) = \sum_{OD} \sum_{k \in K_{OD}} f_{OD,k}(t) \delta_{OD,a,k}(t) \quad (11)$$

The charging state of each EV is required, and the following constraint must be satisfied to ensure normal vehicle operation:

$$E_{OD,ini}(t) + \sum_{k \in K_{OD}} L_k(t) - \sum_{k \in K_{OD}} \sum_{l \in k} \sigma_{k,l}(t) f_{k,l}(t) = E_{OD,end}(t) \quad (12)$$

The load of charging station is required to satisfy the following constraint:

$$L_{a,TR}(t) = \sum_{OD} \sum_{k \in K_{OD}} L_k(t) \delta_{OD,a,k}(t) \quad (13)$$

The specific mathematical expressions of $\tau_{a,R}(t, x_{a,R}(t))$ and $\tau_{a,C}(t, x_{a,C}(t))$, along with the remaining associated constraints, are provided in Appendix B.

C. Optimization Model of GDN

Without loss of generality, the cost function for the GDN is defined as:

$$\min F_{GN} = \sum_{t=1}^{24} (\chi_3 q_{gas}(t) - p_{GT}(t) q_{GT}(t)) \quad (14)$$

The constraint on gas pipelines and nodal pressures is specified as:

$$f_{mn,gas}(t) = M_{mn} \delta_{mn} \sqrt{\delta_{mn} ((\pi_m(t))^2 - (\pi_n(t))^2)} \quad (15)$$

The constraints of the gas pressure for pipelines equipped with compressors are defined as:

$$\mu_1 \pi_u(t) \leq \pi_v(t) \leq \mu_2 \pi_u(t) \quad (16)$$

The flow of gas at each node is required to satisfy the following balance constraint:

$$q_{m,gas}(t) + q_m(t) - q_{m,L,gas}(t) - q_{m,L,GT}(t) = 0 \quad (17)$$

The gas price is determined according to the actual gas consumption of the GT, analogous to how the electricity price of charging station is set in the PDN:

$$p_{GT}(t) = \chi_3 + a_2 \left(\frac{q_{GT}(t)}{\sum_{t=1}^{24} q_{GT}(t)} \right)^2 + b_2 \left(\frac{q_{GT}(t)}{\sum_{t=1}^{24} q_{GT}(t)} \right) + c_2 \quad (18)$$

III. NON-COOPERATIVE GAME OPTIMIZATION MODEL OF URBAN ENERGY SYSTEMS

The PDN, TN, and GDN in real society are governed as separate energy sectors, and distinct objectives are assigned to each sector. The charging load $L_{TR}(t)$, which is influenced by the charging price $p_{TR}(t)$, is transmitted from the TN to the PDN. This load is treated as an output variable of the TN and an input variable of the PDN. Similarly, the gas load $q_{m,L,gas}(t)$ of the GT, influenced by the gas price $p_{GT}(t)$, is transmitted from the PDN to the GDN. This load is treated as an output variable of the PDN and an input variable of the GDN. The cost functions presented in the previous section indicate that conflicts of the interest exist among the three energy networks. Therefore, from a practical perspective, the urban energy system is regarded as suitable for optimization through the non-cooperative game, so that the cost of each sector can be minimized.

A. Linearization and Convex Envelope Relaxation of Energy Network Optimization Models

According to the condition of the existence and uniqueness of a pure-strategy Nash equilibrium in non-cooperative game [20], the requirement for a finite number of players is satisfied by the optimization models presented in Section II. However, the optimization models fail to meet the conditions of having a differentiable cost function and a convex strategy space. Regarding the differentiability of the cost function, the cost functions of the TN and GDN are continuous convex functions, and therefore, differentiable. The cost function of the PDN includes absolute value terms, which do not satisfy the differentiability. Auxiliary variables are used to equivalently linearize the absolute value terms, as shown in (19)-(21). The equivalence of this transformation is proven in [21].

$$P_C(t) \leq P_{C, \text{re}}(t) \quad (19)$$

$$-P_{C, \text{re}}(t) \leq P_{C, \text{re}}(t) \quad (20)$$

$$P_{C, \text{rr}}(t) \leq P_C(t) - P_{C, \text{re}}(t) \quad (21)$$

According to (19)-(21), the cost function of the PDN and the constraint of the energy storage device can be transformed as:

$$\min F_{\text{PN}} = \sum_{t=1}^{24} (\chi_1 P_{\text{grid}}(t) + \chi_2 I_{i,j}(t) R_{i,j}) + \sum_{t=1}^{24} [\alpha_1 P_{\text{WT}}(t) + \alpha_2 P_{\text{GT}}(t) + \alpha_3 (P_{C, \text{re}}(t) - P_{C, \text{rr}}(t))] + \sum_{t=1}^{24} (p_{\text{GT}}(t) q_{\text{GT}}(t) - p_{\text{TR}}(t) L_{\text{TR}}(t)) \quad (22)$$

$$S(t) = (1 - \gamma)S(t-1) + \frac{P_{C, \text{re}}(t) \eta_C \Delta t}{2} + \frac{P_{C, \text{rr}}(t) \Delta t}{2 \eta_C} \quad (23)$$

The strategy space is rendered non-convex by the nonlinear term in (15). To enable the subsequent application of convex relaxation, it is reformulated as:

$$f_{mn, \text{gas}}^2(t) = M_{mn}^2 (\Pi_m(t) - \Pi_n(t)) \quad (24)$$

The quadratic term in (24) is convexly relaxed by the convex envelope. The relaxation accuracy is improved by updating the upper and lower bounds.

$$f_{mn, \text{gas}}^2(t) \leq \zeta_{mn, \text{gas}}(t) \leq (f_{mn, \text{gas}}^L(t) + f_{mn, \text{gas}}^U(t)) f_{mn, \text{gas}}(t) - f_{mn, \text{gas}}^L(t) f_{mn, \text{gas}}(t) \quad (25)$$

The constraint of the relaxed flow for pipeline is given as:

$$\zeta_{mn, \text{gas}}(t) = M_{mn}^2 (\Pi_m(t) - \Pi_n(t)) \quad (26)$$

After the above linearization and relaxation of convex envelope, the conditions requiring that cost functions be convex and differentiable are satisfied. Thus, the strategy space is a convex set. Consequently, a unique Nash equilibrium is ensured for the constructed non-cooperative game optimization model. Compared with the original non-convex optimization model, the relaxed optimization model can be solved optimally within polynomial time. Additionally, iterative updates ensure that the optimal solution is obtained within a reasonable time and under a specified convergence threshold.

B. Adjustment Strategy of Price Based on Sensitivity Coefficients

According to (7), the electricity price of the charging station is determined solely by the electricity load, so fluctuations in the load are directly reflected in the price. Unlike the PDN and GDN, the TN is characterized by a unique topology. The travel of EVs is regarded as an inelastic demand, and the required energy is typically obtained only from the PDN.

$$\min F_{\text{TN}} = F_{\text{tn}} + \sum_{t=1}^{24} \frac{Z_{a,r, \text{TR}}(t) L_{a,r, \text{TR}}^2(t)}{2} + \sum_{t=1}^{24} (p_{a,r, \text{TR}}(t) - Z_{a,r, \text{TR}}(t) L_{a,r-1, \text{TR}}(t)) L_{a,r, \text{TR}}(t) \quad (27)$$

From [22], the sensitivity coefficient is computed by fitting historical iteration data generated during the optimization process:

$$Z_{a,r, \text{TR}}(t) = \frac{\sum_{j=2}^{r-1} (L_{a,j, \text{TR}}(t) - L_{a,j-1, \text{TR}}(t)) (p_{a,j+1, \text{TR}}(t) - p_{a,j, \text{TR}}(t))}{\sum_{j=2}^{r-1} (L_{a,j, \text{TR}}(t) - L_{a,j-1, \text{TR}}(t))^2} \quad (28)$$

By comparing (8) with (27), it can be observed that introducing the sensitivity coefficient affects only the term of charging cost $p_{a,r, \text{TR}}(t) L_{a,r, \text{TR}}(t)$ in the cost function of the TN. However, all other cost components and constraints remain unchanged. Differences between the KKT conditions of the original optimization model for the TN and the modified optimization model incorporating the sensitivity coefficient are attributed solely to their respective Lagrange functions. The Lagrange function of the modified optimization model for the TN is expressed as:

$$L_{\text{TN}} = F_{\text{tn}} + l_{\text{TN}} + \sum_{t=1}^{24} \frac{Z_{a,r, \text{TR}}(t) L_{a,r, \text{TR}}^2(t)}{2} + \sum_{t=1}^{24} (p_{a,r, \text{TR}}(t) - Z_{a,r, \text{TR}}(t) L_{a,r-1, \text{TR}}(t)) L_{a,r, \text{TR}}(t) \quad (29)$$

The partial derivative of L_{TN} with respect to $L_{a,r, \text{TR}}(t)$ is expressed as:

$$\frac{\partial L_{\text{TN}}}{\partial L_{a,r, \text{TR}}(t)} = \frac{\partial F_{\text{tn}}}{\partial L_{a,r, \text{TR}}(t)} + \frac{\partial l_{\text{TN}}}{\partial L_{a,r, \text{TR}}(t)} + p_{a,r, \text{TR}}(t) + Z_{a,r, \text{TR}}(t) (L_{a,r, \text{TR}}(t) - L_{a,r-1, \text{TR}}(t)) \quad (30)$$

It can be observed that the l_{TN} terms in the Lagrange functions derived from the original optimization model of TN are identical to those in the modified optimization model. According to [22], the modified optimization model for the TN does not affect the optimal solution. This result is established by proving that the partial derivative of the Lagrange function with respect to $L_{a,r, \text{TR}}(t)$ in (30) is identical to that of the original optimization model for the TN. Equation (30) is satisfied at any time. Upon convergence of the coupled system of the PDN and TN, $L_{a,r, \text{TR}}(t)$ is identical to $L_{a,r-1, \text{TR}}(t)$. As a result, the term $Z_{a,r, \text{TR}}(t)$ is eliminated, and (30) becomes:

$$\frac{\partial L_{\text{TN}}}{\partial L_{a,r, \text{TR}}(t)} = \frac{\partial F_{\text{tn}}}{\partial L_{a,r, \text{TR}}(t)} + \frac{\partial l_{\text{TN}}}{\partial L_{a,r, \text{TR}}(t)} + p_{a,r, \text{TR}}(t) \quad (31)$$

In (31), the partial derivative of the Lagrange function in the original optimization model for the TN with respect to $L_{a,r, \text{TR}}(t)$ is obtained. The KKT conditions are therefore identical to those of the original optimization model. Accordingly, the optimal solution of the modified optimization model for the TN also serves as the optimal solution of the original optimization model. For the coupled system of the PDN and GDN, the gas price must likewise be adjusted. A similar proof applies, and the detailed steps are provided in Appendix C.

IV. INNER-OUTER ITERATIVE METHOD

In urban energy systems, the PDN, TN, and GDN are managed as separate sectors, and complete sharing of system information is usually not possible. Therefore, an inner-outer

iterative method is proposed to obtain Nash equilibrium. Local optimization can be conducted by each energy network without disclosing its network parameters or other private data. Only the load and corresponding price information need to be exchanged. Although no direct coupling exists between the TN and GDN, their strategies are reflected in the electricity price and energy procurement of the PDN, which in turn prompts adjustments to their respective strategies. The basic idea of the proposed method is summarized as follows.

In the inner layer, game processes for the PDN-TN and PDN-GDN coupled subsystems are conducted sequentially. This serial execution is required because the outcome of the PDN-TN coupled subsystem (charging load $L_{TR}(t)$) serves as a fixed input for the subsequent PDN-GDN coupled subsystem within the same inner iteration. In the outer layer, the overall process is coordinated by the PDN, which acts as a bridge. The gas price $p_{GT}(t)$ resulting from the PDN-GDN coupled subsystem is then used to influence the next inner iteration of the game for the PDN-TN coupled subsystem.

If absolute errors of the electricity price and gas price of the GT between two consecutive outer iterations fall below the specified thresholds, the strategy of the PDN remains unchanged. Consequently, the other networks remain unchanged, and the game is regarded as having reached a Nash equilibrium. Section III proves that, when convergence is reached for the PDN-TN coupled subsystem, $L_{a,r,TR}(t)$ is equal to $L_{a,r-1,TR}(t)$ in (30). The added sensitivity coefficient $Z_{a,r,TR}(t)$ is eliminated. At final convergence, the KKT conditions of the two subsystems remain consistent with those of the original optimization model. Similarly, if the convergence is reached for the PDN-GDN coupled subsystem, $q_{r,GT}(t)$ and $q_{r-1,GT}(t)$ in Appendix C (C5) become equal. By eliminating the additional sensitivity coefficient $Z_{r,GT}(t)$, the invariance of the KKT conditions is preserved. Consequently, the Nash equilibrium is equivalent to the optimal solution of the urban energy system. The flowchart of the proposed method is shown in Fig. 2.

V. CASE STUDIES

The test system is composed of an modified IEEE 33-bus PDN, a Nguyen-Dupuis TN, and a GDN. The simulation step and period are set to be 1 hour and 24 hours, respectively. The wind turbine (WT), energy storage device, and static var compensator (SVG) are installed at the relevant nodes of the PDN. In the TN, the unit time cost is set to be 0.42 CNY/h. The base traffic flow is 100 EVs. Each EV has a rated battery capacity of 57.56 kWh, and the average charging energy is 18.42 kWh. In the PDN, the operation and maintenance cost for the WT is 0.04 CNY/kWh. The base electrical load is 1 MWh. In the GDN, the base gas load is 1000 m³. The absolute error threshold for the outer layer is set to be 10^{-8} . The placement of charging stations, GTs, WTs, and SVGs is shown in Appendix D Table DI. The remaining parameter information is shown in Appendix D Tables DII and DIII.

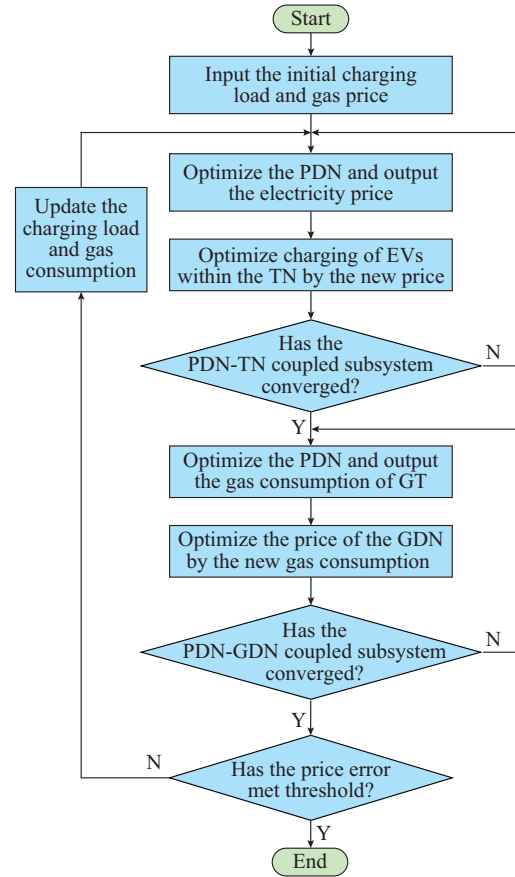


Fig. 2. Flowchart of proposed method.

A. Analysis of Method Convergence

In Fig. 3, curves of electricity price error of the PDN-TN coupled subsystem in the inner layer are plotted with and without considering the sensitivity coefficient. Without considering the sensitivity coefficient, the electricity price error decreases slightly after the first iteration but then stagnates, causing the system to oscillate and fail to converge. However, when considering the sensitivity coefficient, the electricity price error diminishes rapidly to nearly zero within a few iterations. The results demonstrate that dynamically adjusting the electricity price through the sensitivity coefficient influences the charging load, thereby enabling the PDN-TN coupled subsystem to reach a Nash equilibrium rapidly.

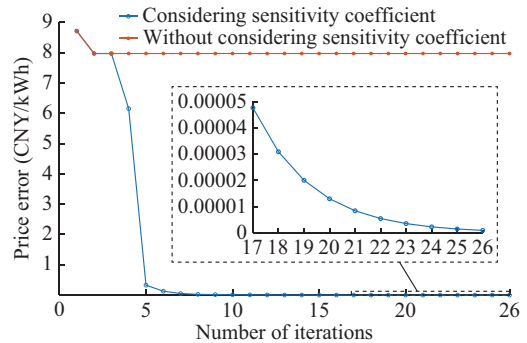


Fig. 3. Curves of electricity price error of PDN-TN coupled subsystem in inner layer with and without considering sensitivity coefficient.

As shown in Table I, the error decreases almost exponentially with the progression of outer iterations. After the first outer iteration, the error is 1.2459. In the second outer iteration, it drops to 1.6974×10^{-5} . In the third and fourth outer iterations, it further drops to the levels of 10^{-7} and 10^{-9} , respectively. The game problem is divided into an outer layer and an inner layer. The outer layer handles coordination among the subsystems of the urban energy system, while the inner layer optimizes the games of price and load between coupled subsystems. After the PDN-TN and PDN-GDN coupled subsystems reach their Nash equilibrium in each inner iteration, the next outer iteration starts from more accurate initial conditions. The error therefore converges quickly to a negligible range, indicating that there is no incentive for the PDN, TN, and GDN have no incentive to deviate from the current strategy profile. This outcome indicates that the Nash equilibrium of the urban energy system has been obtained successfully.

TABLE I
INNER AND OUTER ITERATION RESULTS OF GAME OPTIMIZATION IN URBAN ENERGY SYSTEM

Number of outer iterations	Number of inner iterations	Error
1	143	1.2459×10^0
2	87	1.6974×10^{-5}
3	109	7.5770×10^{-7}
4	88	8.6013×10^{-9}

TABLE II
ITERATION OPTIMIZATION RESULTS UNDER DIFFERENT CATEGORIES OF INITIAL PARAMETERS AND CONVERGENCE THRESHOLDS FOR OUTER LAYER

Category of parameters	Convergence threshold for outer layer	Method	Number of outer iterations	Number of inner iterations	Total solving time (s)	Cost of PDN (CNY)	Cost of GDN (CNY)	Cost of TN (CNY)
Category 1	10^{-8}	Proposed	4	427	157.3732	475948.5501	484991.5534	13579.8967
Category 1	10^{-8}	Convex optimization			>1000.0000			
Category 2	10^{-8}	Proposed	4	374	113.2886	475948.9868	484991.1540	13579.8967
Category 3	10^{-8}	Proposed	4	491	176.8309	475948.3593	484991.9161	13579.8967
Category 1	10^{-6}	Proposed	3	339	124.6084	475950.3577	484991.2995	13579.8967
Category 1	10^{-10}	Proposed	5	611	406.0756	475949.0449	484991.9145	13579.8967

When identical initial parameters are used, the convergence thresholds differ, and the number of outer iterations increases. This leads to a corresponding increase in the number of inner iterations, thereby extending the total solving time. This trend reflects the inherent trade-off: seeking higher precision prolongs the iterative process. Despite varying the convergence threshold from 10^{-6} to 10^{-10} , only minor cost differences are observed among the three energy networks. This suggests that if strict accuracy is not required, a convergence threshold of 10^{-6} or 10^{-8} is sufficient. These looser convergence thresholds yield nearly the same Nash equilibrium while keeping the total solving time relatively short. In summary, once convergence is achieved, the final

To further validate the proposed method, we configure three categories of initial parameters and convergence thresholds for the outer layer. Additionally, to demonstrate its advantage over the centralized optimization method, a convex optimization method is reconstructed based on the energy network optimization model in [21]. It employs the Nikaido-Isoda function with category 1, which includes parameters such as the initial electricity price, gas price, and charging station load, and a threshold of 10^{-8} .

The iteration optimization results in Table II indicate that the number of outer iterations remains at 4 under the same convergence threshold, even when the initial parameters differ. The final costs of the PDN, GDN, and TN are almost identical. These findings indicate that initial parameters have a negligible impact on both the number of outer iterations and the final Nash equilibrium. A comparison of the number of inner iterations and total solving time is also performed. The proposed method exhibits strong stability, with the number of inner iterations and the total solving time being only slightly influenced by the initial parameters, and the final Nash equilibrium remains unaffected. Furthermore, as summarized in Table II, the convex optimization method fails to achieve convergence within 1000 s due to its large-scale structure. In contrast, the proposed method converges stably in just 157.3732 s. This contrast highlights the superior computational tractability and practical feasibility of the proposed method, making it particularly suitable for coordinated optimization of large-scale, multi-stakeholder urban energy systems where centralized optimization is intractable.

costs of the energy networks remain very close, regardless of adjustments to the initial parameters or thresholds. These results underscore the strong stability of the proposed method in non-cooperative game optimization of the urban energy system.

Furthermore, the operational security of the Nash equilibrium is validated by the nodal voltage shown in Fig. 4. The results demonstrate that voltages across the PDN lie strictly within a safe range of 0.9829 p.u. to 1.0009 p.u., corresponding to a maximum deviation of only 1.71%. This comfortably complies the standard security constraints of the PDN [23].

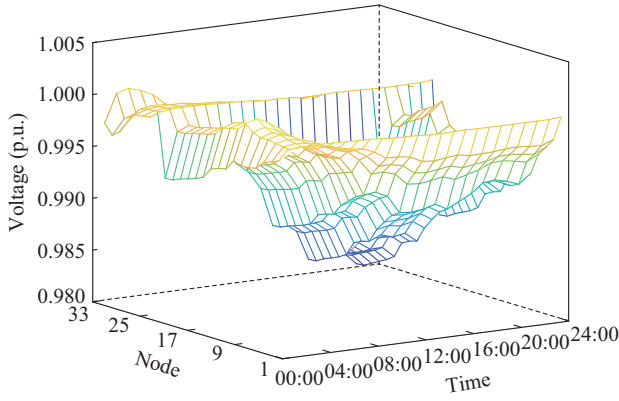


Fig. 4. Nodal voltage.

B. Analysis of Equilibrium for Urban Energy Systems

To analyze the equilibrium advantages of the proposed model of urban energy system, three coupled system scenarios are defined for comparison.

Scenario 1: an urban energy system that couples the PDN and GDN.

Scenario 2: an urban energy system that couples the PDN and TN.

Scenario 3: an urban energy system that couples the PDN, TN, and GDN.

As shown in Table III, the cost of the PDN varies significantly across scenarios. Scenario 3 achieves the lowest cost at 475948.5501 CNY, followed by Scenario 1 at 482355.5393 CNY, and Scenario 2 at the highest cost of 508767.8836 CNY. This result highlights that Scenario 3 achieves superior cost control. In Fig. 5, for gas purchase, Scenarios 1 and 3 exhibit nearly identical costs, both of which are 25.56% lower than that of Scenario 2. This suggests that the non-cooperative game optimization in Scenario 3 effectively minimizes gas expenses, significantly outperforming Scenario 2 where only the PDN and TN are coupled. For revenue of selling electricity, scenario 1 has significantly lower revenue compared with the other scenarios. This is attributed to the absence of traffic flow regulation feedback from the TN and the electricity price of charging station in Scenario 1 is fixed. In summary, Scenario 3 optimizes energy allocation through the non-cooperative game optimization among the three energy networks, thereby avoiding the resource waste inherent in independent network operation. The integration of the TN proves critical: it assists the PDN in peak-load regulation, alleviating stress during periods of high electricity demand. Consequently, the reliance on the GDN is reduced, maintaining a lower overall operation cost.

C. Analysis of Energy Supply and Electricity Prices

Under (7), the electricity price for the charging station is determined by the peak-valley profile of the load, with adjustments made primarily according to the actual charging energy. Specifically, the electricity price rises during periods of high demand and falls when demand decreases. This dynamic guides the allocation of charging peaks and valleys, as illustrated in Fig. 6.

TABLE III
OPERATION COSTS OF ENERGY NETWORKS UNDER NON-COOPERATIVE GAME OPTIMIZATION IN DIFFERENT SCENARIOS

Scenario	Cost of PDN (CNY)	Cost of GDN (CNY)	Cost of TN (CNY)
1	482355.5393	484991.0631	
2	508767.8836		13579.8967
3	475948.5501	484991.5534	13579.8967

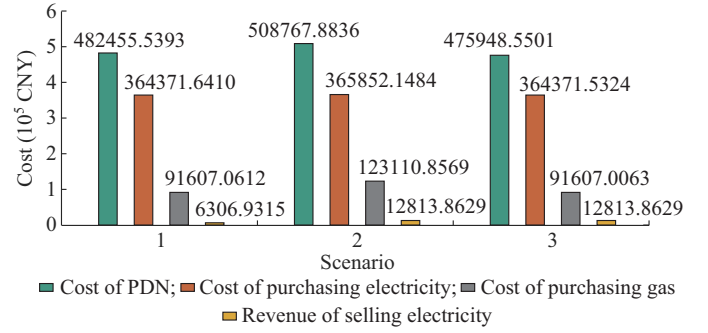


Fig. 5. Comparison of costs and revenues for energy networks in different models of urban energy system.

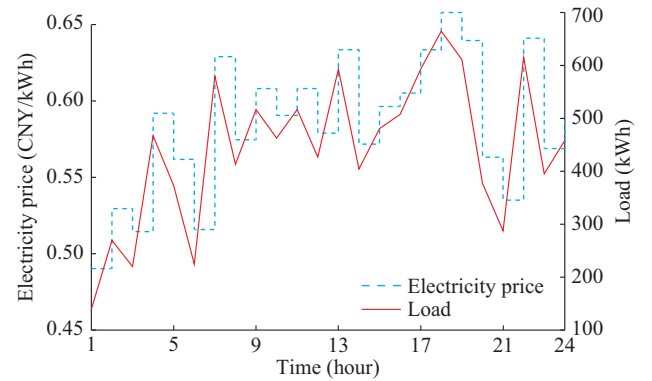


Fig. 6. Load and electricity price of charging station 1.

Since all load in the figure is supplied by the charging station, load peaks correspond directly to travel demand. Travel demand—and consequently the resulting charging load—is generally regarded as inelastic. Although time-of-use pricing provides an incentive, EV users are constrained by their travel schedules and range requirements. Therefore, electricity price alone cannot be expected to fully suppress peak demand. Nevertheless, Fig. 6 shows that some EVs shift their charging to off-peak periods in response to the flexible electricity price signal. Through this electricity price leverage, the PDN effectively seeks to reduce peak load. In the TN, however, the charging peak cannot be fully shifted due to rigid EV demand, although users still pursue cost minimization. Consequently, the curves of load and electricity price converge to equilibrium. The analysis of charging station 2 follows the same pattern and is therefore omitted.

As shown in Fig. 7, the power outputs of GT1 and GT2 (located at node 2 and node 12 of the PDN, respectively) increase over time and peak at 21:00, coinciding with the peak period of gas prices. This reflects the rigid electricity de-

mand in the PDN, which requires the GT to ensure supply reliability, even when gas prices are high. As gas demand increases, supply costs rise, leading to a significant rise in gas price. The simultaneous increase in power outputs of GTs confirms the interaction between the PDN and GDN: higher electricity demand boosts power outputs of GTs, which in turn drives up gas consumption and prices. After the peak, both gas prices and power outputs of GTs decline, indicating reduced electricity and travel demand. The rise in gas prices during peak periods reflects the supply-demand balance of the GDN and serves as a price signal to the PDN. If cheaper alternative power sources are available, part of the power output of GTs can be replaced without compromising system security, thereby reducing overall energy purchase cost.

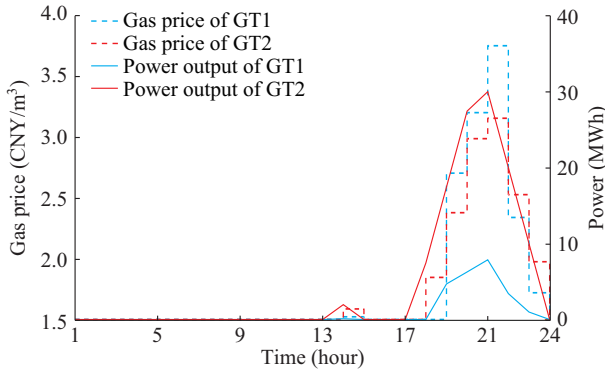


Fig. 7. Power outputs of GTs and corresponding gas prices.

D. Analysis of EV Penetration Rate

To further validate scalability of the proposed method un-

der increasing transportation electrification, the following scenarios with varying EV penetration rates are defined, and all scenarios are simulated on the same urban energy system comprising the PDN, TN, and GDN.

Scenario 1: base EV penetration rate.

Scenario 2: 30% increase of EV penetration rate relative to the base case.

Scenario 3: 60% increase of EV penetration rate relative to the base case.

Scenario 4: 90% increase of EV penetration rate relative to the base case.

As shown in Table IV, the maximum voltage deviation of the system increases slightly with the rising EV penetration rate, from 1.71% in Scenario 1 to 1.75% in Scenario 4. This minor degradation is inherent to the power flow characteristics of the PDN: higher EV penetration rate directly increases the aggregate load at charging stations. According to (2) and (3), a higher load level leads to higher line currents, consequently causing greater voltage drops across the line resistances. Based on the power flow model, these voltage drops are partially compensated by adjusting upstream power injections. However, complete elimination is not achievable. Consequently, the minimum system voltage is slightly reduced under increased loading, leading to an increase in the voltage deviation. Importantly, even with this increase, the voltage deviation remains strictly within the safe operating range in all scenarios and is well below the typical threshold of the PDN. These results confirm that the proposed method consistently ensures safe and stable operation of the PDN as EV penetration rate grows.

TABLE IV
SYSTEM PERFORMANCE UNDER DIFFERENT EV PENETRATION RATES

Scenario	Cost of PDN (CNY)	Cost of GDN (CNY)	Cost of TN (CNY)	Total solving time (s)	Voltage deviation (%)
1	475948.5501	484991.5534	13579.89670	157.3732	1.71
2	474631.1377	484812.6043	17666.27754	167.9755	1.72
3	473345.9677	484604.4387	21759.11900	188.3590	1.73
4	471994.7241	484349.3357	26148.17190	183.8918	1.75

The total solving time exhibits an overall upward trend with increasing EV penetration rate, except for a slight reduction from Scenario 3 to Scenario 4. This phenomenon reveals a key property of the proposed method: the increase in total solving time is primarily driven by the strengthened subsystem couplings. Higher charging loads amplify the impact of the TN on the PDN, which in turn alters electricity prices and supply capability, feeding back into the path selection of the TN. Initially, this stronger feedback requires more inner and outer iterations to reach a new equilibrium. However, at very high penetration rates, the system approaches a saturated state where subsystem interactions become less sensitive. This saturation stabilizes the number of iterations, offsetting the marginal increase in per-iteration computational cost and resulting in a stabilized total solving time in Scenario 4.

Furthermore, the TN is formulated as an energy flow model subject to road network constraints, where the per-iteration solution complexity depends on the network topology and the number of OD pairs, rather than on the number of EVs. Consequently, the growth in total solving time with increasing EV penetration rate is driven primarily by a rise in the number of iterations, not by an increase in the per-iteration computational cost. This inherent characteristic ensures that the proposed method maintains good computational efficiency and stable convergence even under high EV penetration rate, with no risk of the solving time scaling intractably with problem size.

The cost of the PDN decreases continuously with increasing EV penetration rate, primarily driven by the rise in revenue of selling electricity. The additional charging load generates increased revenue for the PDN. Although energy pur-

chase cost and network losses may increase, the revenue gain sufficiently offsets these increments, resulting in a net cost reduction. The reduction in the cost of the GDN, by contrast, stems from a shift in the dispatch strategy of the GTs within the PDN. As the total load of the PDN increases, the demand for the power output of GTs is adjusted. The operation of GTs is shifted toward periods with lower gas prices, thereby reducing the need for high-cost gas procurement from upstream sources. The marked increase in the cost of the TN is straightforward: a larger number of EVs directly increases total travel time cost and total energy consumption.

VI. CONCLUSION

A non-cooperative game optimization model is developed for an urban energy system comprising the PDN, TN, and GDN, whose energy interactions are increasingly intertwined in real society. In practice, each energy network is operated by an independent stakeholder with distinct objectives, necessitating a solution method that preserves information privacy. To address this issue, an inner-outer iterative method is proposed to obtain Nash equilibrium of the urban energy system, requiring only the exchange of price and load information among the energy networks. This method is suitable for urban energy systems with non-cooperative characteristics. It hierarchically incorporates the electricity price, gas price, traffic flow, and load allocation into the iterative procedure. All parties update their decisions based on the strategies of others, and the process repeats until convergence is achieved in the outer layer.

The results show that a stricter convergence threshold increases both the number of outer iterations and the total solving time, while yielding only marginal cost reductions for energy networks. In practice, the convergence accuracy can therefore be relaxed to improve solution efficiency. A comparison of game outcomes of the urban energy systems reveals mutual influences among the three energy networks. This interaction provides the PDN with a broader strategy space, allowing it to retain autonomy while engaging in balanced games with the TN and GDN. The non-cooperative game pressures each energy network to make concessions without sacrificing individual autonomy, ultimately leading to a mutual Nash equilibrium. In terms of computational performance, the proposed method achieves a total solving time of 157.3732 s, which is significantly faster than the centralized optimization method that fails to converge within 1000 s. Moreover, the maximum voltage deviation in the PDN is only 1.75%, which is well within safe operating limits. However, this study considers only the load characteristics of EVs, without accounting for their potential role as mobile energy storage, which is a direction for future work.

APPENDIX A

$$P_{GT}(t) = \eta_{GT} \phi_{GT} q_{GT}(t) \quad (A1)$$

$$P_{W,min}(t) \leq P_W(t) \leq P_{W,max}(t) \quad (A2)$$

$$\varsigma_1 P_{W,min}(t) \leq P_W(t) - P_W(t-1) \leq \varsigma_2 P_{W,max}(t) \quad (A3)$$

$$S(1) = S(25) \quad (A4)$$

$$\begin{cases} P_j(t) = \sum_{h \in m(j)} P_{i,h}(t) - \sum_{i \in n(j)} (P_{i,j}(t) - R_{i,j} I_{i,j}(t)) \\ Q_j(t) = \sum_{h \in m(j)} Q_{i,h}(t) - \sum_{i \in n(j)} (Q_{i,j}(t) - X_{i,j} I_{i,j}(t)) \end{cases} \quad (A5)$$

APPENDIX B

$$\tau_{a,R}(t, x_{a,R}(t)) = \tau_{a,R,0} \left(t \right) \left[1 + 0.15 \left(\frac{x_{a,R}(t)}{c_{a,R}^{\max}} \right)^4 \right] \quad (B1)$$

$$\tau_{a,C}(t, x_{a,C}(t)) = \tau_{a,C,0} \left(t \right) \left(\frac{x_{a,C}(t)}{c_{a,C}^{\max}} \right)^3 \quad (B2)$$

$$f_{OD,k}(t) \geq 0 \quad (B3)$$

APPENDIX C

The original cost function of the PDN is as follows:

$$\min F_{PN} = F_{pn} + \sum_{t=1}^{24} p_{GT}(t) q_{GT}(t) \quad (C1)$$

The expression of the gas price considering the sensitivity coefficient adjustment is as follows:

$$p_{r,GT}(t) = p_{r-1,GT}(t) + Z_{r,GT}(t)(q_{r,GT}(t) - q_{r-1,GT}(t)) \quad (C2)$$

The cost function of the PDN after substituting (C2) into (C1) is as follows:

$$\begin{aligned} \min F_{PN} = F_{pn} + \sum_{t=1}^{24} \frac{Z_{r,GT}(t) q_{r,GT}^2(t)}{2} + \\ \sum_{t=1}^{24} (p_{r,GT}(t) - Z_{r,GT}(t) q_{r-1,GT}(t)) q_{r,GT}(t) \end{aligned} \quad (C3)$$

The Lagrange function of the optimization model for the PDN after introducing the price adjustment mechanism is as follows:

$$\begin{aligned} L_{PN} = F_{pn} + l_{PN} + \sum_{t=1}^{24} \frac{Z_{r,GT}(t) q_{r,GT}^2(t)}{2} + \\ \sum_{t=1}^{24} (p_{r,GT}(t) - Z_{r,GT}(t) q_{r-1,GT}(t)) q_{r,GT}(t) \end{aligned} \quad (C4)$$

The partial derivative of L_{PN} with respect to $q_{r,GT}(t)$ is expressed as:

$$\frac{\partial L_{PN}}{\partial q_{r,GT}(t)} = \frac{\partial F_{pn}}{\partial q_{r,GT}(t)} + \frac{\partial l_{PN}}{\partial q_{r,GT}(t)} + p_{r,GT}(t) + Z_{r,GT}(t)(q_{r,GT}(t) - q_{r-1,GT}(t)) \quad (C5)$$

When the coupled system of the PDN and GDN reaches iterative convergence, $q_{r,GT}(t)$ and $q_{r-1,GT}(t)$ become equal. Substituting this relationship into (C5) yields:

$$\frac{\partial L_{PN}}{\partial q_{r,GT}(t)} = \frac{\partial F_{tn}}{\partial q_{r,GT}(t)} + \frac{\partial l_{PN}}{\partial q_{r,GT}(t)} + p_{r,GT}(t) \quad (C6)$$

According to (C5), when the system converges, the introduced coefficient $Z_{r,GT}(t)$ is eliminated. Consequently, the resulting optimal solution is equivalent to that obtained from the original model.

APPENDIX D

TABLE DI
PLACEMENT OF DEVICES

Device	Node of PDN	Node of TN	Node of GDN
Charging station	5, 10	4, 11	
GT	2, 12		2, 5
WT	8, 26		
SVG	5, 15, 31		

TABLE DII
SYSTEM PARAMETERS

Parameter	Value
χ_1, χ_2, χ_3 (CNY/kWh)	0.35, 0.30, 1.40
a_1, a_2, a_3 (CNY/kWh)	0.05, 0.04, 0.05
a_1, b_1, c_1 (CNY/kWh)	5, 3, 0.1
a_2, b_2, c_2 (CNY/m ³)	5, 5, 0.1
$\mu_1, \mu_2, \varsigma_1, \varsigma_2$	1.1, 2, 0.2, 0.2
η_C, η_{GT}	0.95, 0.45
Δt (hour)	1
γ	0.005
$P_{GT, \min}, P_{GT, \max}$ (MWh)	0, 5

TABLE DIII
PIPELINE PARAMETERS

Pipeline number	Initial node	Terminal node	Pipe diameter (mm)	Pipe length (km)
1	1	2	100	4
2	2	3	50	4
3	2	4	50	3
4	4	5	50	2
5	5	6	50	3
6	5	7	50	3

REFERENCES

- [1] L. Ge, B. Zhang, W. Huang *et al.*, "A review of hydrogen generation, storage, and applications in power system," *Journal of Energy Storage*, vol. 75, p. 109307, Jan. 2024.
- [2] W. Xu, Y. Guo, T. Meng *et al.*, "Coordinated dispatch based on distributed robust optimization for interconnected urban integrated energy and transmission systems," *Journal of Modern Power Systems and Clean Energy*, vol. 12, no. 3, pp. 840-851, May 2024.
- [3] Y. Tang, W. Liu, K. T. Chau *et al.*, "Stochastic behavior modeling and optimal bidirectional charging station deployment in EV energy network," *IEEE Transactions on Intelligent Transportation Systems*, vol. 26, no. 5, pp. 6231-6247, May 2025.
- [4] H. Wang, C. Bai, Z. Wang *et al.*, "Resilient preparation and restoration strategy for integrated electric-gas distribution systems considering mobile energy storage," *IEEE Transactions on Smart Grid*, vol. 16, no. 2, pp. 1127-1141, Mar. 2025.
- [5] J. Wang, Y. Sun, R. J. Mahfoud *et al.*, "Co-optimization of multi-stakeholders in multi-heterogeneous energy system considering gas flow dynamics," *Sustainable Energy, Grids and Networks*, vol. 36, p. 101175, Dec. 2023.
- [6] J. Xiong, Y. Sun, J. Wang *et al.*, "Multi-stage equipment optimal configuration of park-level integrated energy system considering flexible loads," *International Journal of Electrical Power and Energy Systems*, vol. 140, p. 108050, Sept. 2022.
- [7] Y. Li, Y. Cai, T. Zhao *et al.*, "Multi-objective optimal operation of centralized battery swap charging system with photovoltaic," *Journal of Modern Power Systems and Clean Energy*, vol. 10, no. 1, pp. 149-162, Jan. 2022.
- [8] Y. Wang, F. Lu, C. Gu *et al.*, "Multi-objective optimization for customer flexibility management to mitigate consecutive days of power supply shortages," *IEEE Transactions on Power Systems*, vol. 40, no. 4, pp. 3306-3319, Jul. 2025.
- [9] J. Wang, Y. Sheng, H. Ge *et al.*, "Charging pricing for autonomous mobility-on-demand fleets based on game theory," *Journal of Modern Power Systems and Clean Energy*, vol. 12, no. 6, pp. 2006-2018, Nov. 2024.
- [10] X. Ge, S. Cao, Y. Fu *et al.*, "Optimal scheduling of space-time dual-scale electric vehicles based on regional decoupling," *Proceedings of the CSEE*, vol. 43, no. 19, pp. 7383-7396, Oct. 2023.
- [11] C. Lou, C. Li, L. Zhang *et al.*, "Two-stage bidding strategy with dispatch potential of electric vehicle aggregators for mitigating three-phase imbalance," *Journal of Modern Power Systems and Clean Energy*, vol. 13, no. 5, pp. 1823-1835, Sept. 2025.
- [12] X. Xu, Z. Qiu, T. Zhang *et al.*, "Distributed source-load-storage cooperative low-carbon scheduling strategy considering vehicle-to-grid aggregators," *Journal of Modern Power Systems and Clean Energy*, vol. 12, no. 2, pp. 440-453, Mar. 2024.
- [13] K. Li, C. Shao, Z. Hu *et al.*, "A MILP method for optimal planning of electric vehicle charging stations in coordinated urban power and transportation networks," *IEEE Transactions on Power Systems*, vol. 38, no. 6, pp. 5406-5419, Nov. 2023.
- [14] Y. Cui, Z. Hu, X. Duan *et al.*, "A multi-period charging service pricing game for public charging network operators considering the dynamics of coupled traffic-power systems," *IEEE Transactions on Intelligent Transportation Systems*, vol. 24, no. 9, pp. 9355-9367, Sept. 2023.
- [15] K. Li, C. Shao, Y. Wang *et al.*, "Optimal planning of urban integrated energy systems considering electricity-gas-transportation interactions," *Proceedings of the CSEE*, vol. 43, no. 6, pp. 2263-2273, Mar. 2023.
- [16] S. Xie, J. Zheng, Z. Hu *et al.*, "Urban multi-energy network optimization: an enhanced model using a two-stage bound-tightening approach," *Applied Energy*, vol. 277, p. 115577, Nov. 2020.
- [17] Z. Zhang and K. S. Fedorovich, "Optimal operation of multi-integrated energy system based on multi-level Nash multi-stage robust," *Applied Energy*, vol. 358, p. 122557, Mar. 2024.
- [18] Y. Sun, B. Zhang, L. Ge *et al.*, "Day-ahead optimization schedule for gas-electric integrated energy system based on second-order cone programming," *CSEE Journal of Power and Energy Systems*, vol. 6, no. 1, pp. 142-151, Mar. 2020.
- [19] Z. Tong, Z. Ren, J. Wu *et al.*, "Multi-agent game operation of electricity-hydrogen-transportation system considering HFCV hydrogen refueling uncertainty," *Automation of Electric Power Systems*, vol. 49, no. 2, pp. 154-164, Jan. 2025.
- [20] Y. Si, L. Chen, X. Zhang *et al.*, "Capacity allocation of hybrid power system with hot dry rock geothermal energy, thermal storage, and PV based on game approaches," *Journal of Modern Power Systems and Clean Energy*, vol. 11, no. 1, pp. 290-301, Jan. 2023.
- [21] C. Yin, Y. Sun, D. Xie *et al.*, "Multi-objective cooperative optimization of electricity-gas interconnected integrated energy system from game perspective," *Automation of Electric Power Systems*, vol. 48, no. 11, pp. 173-183, Jun. 2024.
- [22] Z. Li, Q. Guo, H. Sun *et al.*, "A new LMP-sensitivity-based heterogeneous decomposition for transmission and distribution coordinated economic dispatch," *IEEE Transactions on Smart Grid*, vol. 9, no. 2, pp. 931-941, Mar. 2018.
- [23] *Power Quality – Deviation of Supply Voltage*, GB/T 12325-2008, Jun. 2008.

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