

Robust Stability Analysis and Design of Parallel Grid-forming Converters in Islanded Microgrid

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Abstract—The stability of microgrid (MG) is affected by the interaction between parallel inverter-based resources (IBRs), especially when grid-forming (GFM) control is implemented in islanded MG to provide voltage support. In this paper, the robust stability analysis using structured singular value (SSV) approach and design of parallel GFM converters in islanded MG by structured μ synthesis are presented, catering to its multi-input multi-output (MIMO) nature. Specifically, a modular state-space model of parallel GFM converters in islanded MG is derived with component connection method (CCM). Model uncertainty is captured against the nominal model, delivering a certain degree of robustness for stability analysis, in the face of potential parametric uncertainty and resonance effects induced by parallel GFM converters. Moreover, the effects of system parameters on robust stability are comprehensively analyzed, and a robust design is proposed for virtual impedance (VI) tuning. The accuracy of modelling and robust stability analysis, as well as the effectiveness of the robust design is validated in MATLAB/Simulink/PLECS and real-time digital simulator (RTDS).

Index Terms—Islanded microgrid, grid-forming (GFM) converter, robust stability, uncertainty, structured singular value (SSV), structured μ synthesis, component connection method (CCM), virtual impedance (VI).

NOMENCLATURE

A. Sets and Symbols

$\tilde{(\cdot)}$	Perturbed signal around equilibrium point
$(\cdot)_0$	Equilibrium point
$(\cdot)_L$	Variable of open-form aggregation for component connection method (CCM)
$(\cdot)_m$	Variable of the m^{th} subsystem for CCM
$(\cdot)_n$	Variable of the n^{th} virtual synchronous machine (VSM)

$ \cdot $	Absolute value (2-norm)
$\ \cdot\ _\infty$	Infinity (∞)-norm
δ_i	The i^{th} parametric uncertainty
Δ_j	The j^{th} unstructured uncertainty
Δ	Structured uncertainty
μ	Structured singular value
$\bar{\sigma}(\cdot)$	The largest singular value
$\mathbb{C}$	Set of complex numbers
$\mathbb{R}$	Set of real numbers
$\det(\cdot)$	Determinant of matrix
$\text{diag}(\cdot)$	Diagonal matrix
$\mathbf{e}_i^T(\cdot)$	The i^{th} row of matrix
$\mathcal{F}_l, \mathcal{F}_u$	Lower and upper linear fractional transformations
F	Number of unstructured uncertainties
$\mathbf{I}$	Identity matrix
$\mathbf{L}$	Algebraic connection matrix for CCM
m_j	Order of Δ_j
M	Number of subsystems for CCM
N	Number of parallel VSMs
$\mathbf{O}$	Zero matrix
$P(s), \bar{P}(s)$	Actual and nominal transfer functions in single-input single-output (SISO) form
$\mathbf{P}, \bar{\mathbf{P}}$	Actual and nominal transfer function matrices in multi-input multi-output (MIMO) form
r_i	Number of occurrences of δ_i
S	Number of parametric uncertainties

B. Power System Specifications

δ_n	Lead angle of the n^{th} dq -frame relative to DQ -frame
θ	Virtual rotor angle
ω	Virtual rotor speed
ω_s	Nominal frequency of power system
ω_{com}	Virtual rotor speed of common reference frame
C	Capacitance of shunt filter
$\mathbf{i}_c$	Converter-side current vector
i_{cd}, i_{cq}	d - and q -axis converter-side currents
i_{load2d}, i_{load2q}	d - and q -axis currents of load $R_{load2} + j\omega_s L_{load2}$
$\mathbf{i}_t$	Terminal current vector at point of common

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	coupling (PCC)
i_{td}, i_{tq}	d - and q -axis terminal currents at PCC
i_{tD}, i_{tQ}	D - and Q -axis terminal currents at PCC
i_{tDsum}, i_{tQsum}	Sums of D - and Q -axis currents from parallel VSMs
L_{load2}	Public load inductance
$M_f i_f$	Virtual mutual flux
P	Measured value of active power at PCC
Q	Measured value of reactive power at PCC
R, L	Nominal resistance and inductance of uncertain single-phase load
R_f, L_f	Filter resistance and inductance
R_l, L_l	Line resistance and inductance
R_{load1}, R_{load2}	Public load resistances
S_n	Rated power of the n^{th} VSM
T_m, T_e	Mechanical torque and electrical torque
$\mathbf{v}_b$	Voltage vector at public bus
$\mathbf{v}_{bd}, \mathbf{v}_{bq}$	d - and q -axis voltages at public bus
$\mathbf{v}_{bD}, \mathbf{v}_{bQ}$	D - and Q -axis voltages at public bus
$\mathbf{v}_c$	Voltage vector at alternating current (AC) side of VSM
$\mathbf{v}_{cd}, \mathbf{v}_{cq}$	d - and q -axis voltages at AC side of VSM
$\mathbf{v}_{dc}$	Voltage at direct current (DC) side
$\mathbf{v}_{td}, \mathbf{v}_{tq}$	d - and q -axis voltages at PCC
V_c	Virtual back electromotive force
V_n	Rated voltage of the n^{th} VSM
V_t	Measured value of terminal voltage at PCC
$\mathbf{v}_t$	Terminal voltage vector at PCC
Z_f	Filter impedance
Z_l	Line impedance

C. Control Parameters

ω_c	Cut-off frequency of low-pass filter (LPF)
ω_{ref}	Reference frequency
$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$	State, input, output, and feedforward matrices
$\mathbf{C}_m$	Equivalent MIMO controller of islanded microgrid (MG)
D_p	Droop coefficient of active power loop
D_q	Droop coefficient of reactive power loop
$\mathbf{G}_s$	General uncertainty model for stability analysis
$\mathbf{G}_{s11}$	Submodule of general uncertainty model that determines stability
$\mathbf{G}_\mu$	General uncertainty model for control synthesis
J, J_n	Virtual inertia and its nominal value
K, K_n	Integral coefficient and its nominal value
$\mathbf{K}_\mu$	Static gain matrix of virtual impedance (VI) parameters
$\mathbf{P}_\mu$	Interconnection matrix for control synthesis
$\mathbf{P}_m$	Equivalent MIMO plant of islanded MG
$\mathbf{P}_s$	Interconnection matrix for stability analysis
P_{ref}, Q_{ref}	References of active power and reactive power

R_V, L_V	Virtual resistance and inductance
$\mathbf{u}, \mathbf{y}$	Input and output variable vectors
$\mathbf{u}_\Delta, \mathbf{y}_\Delta$	Output and input variables of uncertainty block
$\mathbf{u}_{cm}, \mathbf{y}_{cm}$	Output and input variables of equivalent MIMO controller
V_{tref}	Reference of terminal voltage at PCC
$\mathbf{w}, \mathbf{z}$	Input and output variables of uncertain system
$\mathbf{w}_\mu, \mathbf{z}_\mu$	Input and output of performance specifications for VI tuning by robust design
$\mathbf{w}_m, \mathbf{z}_m$	Input and output of general uncertainty model
$W(s), \mathbf{W}$	Uncertainty weighting function and matrix
W_μ	Weighting function of control synthesis
$\mathbf{W}_\mu$	Weighting function matrix of control synthesis
$\mathbf{x}$	State variable vector

I. INTRODUCTION

POWER systems are undergoing tremendous transition from the dominance of traditional synchronous machines (SMs) to the integration of massive inverter-based resources (IBRs), e.g., photovoltaics (PVs), wind turbines (WTs), and energy storage systems (ESSs) [1]. A typical islanded microgrid (MG) could be a potential commercial or residential MG with single point-of-common-coupling (PCC) bus [2]-[4]. A regional microgrid cluster (MGC) is generally organized by numerous elementary MG modules. Therefore, it is essential to warrant the stability of islanded MG composed of multiple parallel IBRs, which is still an open problem in the research community. With the increasing proportion of IBRs controlled as current sources, the inertia decreases since the grid strength cannot be enhanced by grid-following (GFL) converters [5].

The virtual inertia and damping feature of grid-forming (GFM) converters such as virtual synchronous machines (VSMs) can offer low-inertial voltage support in islanded mode, which is regarded as a favorable solution to tackle the above issues [1], [6], [7]. However, the emerging low-frequency oscillation (LFO) and even instability risk will deteriorate the system performance when system parameters are not coordinated properly [8]-[10].

Plenty of current studies investigate the cause of LFO issues of active power and relevant frequency stability problems, which is attributed to the mismatch between control loop parameters, the capacity of IBRs, and the line impedance [11]-[16]. However, the conclusion is based on a quasi-stationary model and the premise of purely inductive filter and line impedance, which restricts the application of these proposed control strategies when resistance cannot be ignored. More importantly, in these studies, reactive power control (RPC) and alternating current (AC) voltage control (AVC) are neglected for simplicity, which have been proven to affect the damping feature of the system, also potentially inducing LFO issues [8], [9], [17].

Indeed, the full-order islanded MG behaves as a typical multi-input multi-output (MIMO) system when active power control (APC), RPC, AVC, and power dynamics are consid-

ered [18]-[20]. The state-space modeling technique is widely used to reveal the high-field dynamic characteristics of the system [18], [21], [22]. Based on this, eigenvalue analysis on state matrix is adopted to figure out the oscillation frequency and damping ratio, whilst participation factor can offer the ranking of each mode, in view of its sensitivity to all states. However, they only represent the natural or unforced response of the system, rather than input-output characteristics. Thus, to study the response of input-output pairs of the MIMO system, it is necessary to consider the closed-form essence of the system. Moreover, the computational burdens will increase exponentially with the additional consideration of uncertainty, which could originate from model refinement and unmodeled system dynamics. Guardian map method and bifurcation theory based on state-space modeling are used to explore the robust stability of MG with multiple GFM converters, but the variability of control parameters is only assessed with respect to a specific plant linearized around a particular operating point [21], [22]. The results also lack robustness in practical contexts.

Impedance-based stability analysis has been extensively studied for parallel GFM converters with single-input single-output (SISO) [23] and MIMO manners [20]. However, there exists the limitation of analyzing an MIMO system using SISO manners [24], [25]. Specifically, overly optimistic or erroneous conclusions might be obtained when multiple uncertainties are simultaneously activated. Even the MIMO essence of MG could be reflected by impedance or admittance matrix [20], the iterative assessment on a large number of potential plants is still needed when the uncertainty cannot be ignored.

Singular value of an MIMO system represents the largest (worst) transfer gain, capturing all potential interactive effects among different input-output pairs [24], [25]. Furthermore, structured singular value (SSV, also called μ) clearly quantifies the largest singular value of a system subject to a structured uncertainty, which integrates multiple uncertainties into a robust stability analysis framework, catering for practical operation. Moreover, there is no stiff voltage support from the grid in islanded MG, and the tuning of control loop will affect the system stabilization [26], [27], not mentioning the effect of load as well. Accordingly, a modular state-space model of parallel GFM converters in islanded MG is derived with component connection method (CCM), then the SSV approach could be used for robust stability analysis of islanded MG, offering the tuning guidance of control loop parameters with respect to a set of possible plants.

Robust stability analysis and design of parallel GFM converters in islanded MG have not yet been addressed, which calls for a confident level of control robustness and stabilizes the system even though the model inaccuracy takes effect. Based on this, robust stability analysis using SSV approach and design of parallel GFM converters in islanded MG by structured μ synthesis are comprehensively investigated in this paper.

The key contributions of this paper are outlined as follows.

1) A modular state-space model of parallel GFM converters in islanded MG is established with CCM. With the appropriate

selection of state variables and reference frame, a non-singular system model is derived, with which robust stability analysis can be performed.

2) Robust stability and margin of islanded MG are analyzed considering a set of frequency-dependent multiple uncertainties. The impact of the variations of control parameters and supplied load on system robustness have been investigated with the proposed robust stability analysis using SSV approach, offering the authentic MIMO behaviors of islanded MG.

3) Robust design by structured μ synthesis is proposed for virtual impedance (VI) tuning based on D - K iteration while maintaining enhanced robust stability of islanded MG under the same uncertainty assumption. Consequently, the resonance peaks can be suppressed by the auxiliary damping, and oscillation issues in islanded MG can be prevented effectively.

The remainder of this paper is organized as follows. Section II elaborates the system description and VSM. Section III establishes a modular state-space model of parallel GFM converters in islanded MG. Section IV conducts robust stability analysis using SSV approach for islanded MG. Robust design of parallel GFM converters in islanded MG by structured μ synthesis is presented in Section V. Validations are conducted in Section VI. Finally, Section VII concludes this paper.

II. SYSTEM DESCRIPTION AND VSM

An equivalent structure of islanded MG is explained in this section, and voltage support service supplied by GFM converters is emphasized, by introducing the operating principles of VSM.

A. Islanded MG

For the studied islanded MG with multiple VSMs, let subscript n denote the index of VSM ($n=1, 2, \dots, N$). The inductive filter is denoted as Z_{fn} , which connects the AC side of VSM n and the n^{th} PCC (denoted as PCC n), whereas line impedance is denoted as Z_{ln} and the capacitive shunt filter is denoted as C_n . Public load is represented as the combination of a resistive load R_{load1} paralleled with a resistive-inductive load $R_{load2} + j\omega_s L_{load2}$. If there is no resistive load R_{load1} in practice, a virtual large value could be assumed, which helps eliminate the common bus voltage as the input variable [18], [21], [22]. Figure 1 shows the single-line schematic diagram of the studied islanded MG with multiple VSMs, which are connected to the common bus through respective cables.

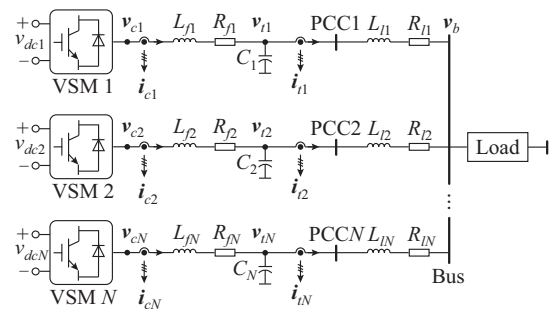


Fig. 1. Single-line schematic diagram of studied islanded MG with multiple VSMs.

In order to simplify the modeling and analysis of MG system, several assumptions are made as follows.

1) AC-direct current (DC) coupling effects are ignored since the constant DC-link voltage of VSM can be guaranteed with the connection of batteries or DC-DC converters [8]-[10], [13], [18], [21], [22], [26].

2) Current control (CC) is dedicated to limiting the fault current. However, it remains inactive and transparent in normal situations, and does not influence the small-signal stability [8], [9], [13], [22], [26], which is the main concern of this paper.

3) Modulation effects are neglected, which is reasonable for studies on slow-interaction converter-driven stability (SICDS) that focus on low-frequency stability issues such as synchronous resonance (SR) and sub-synchronous resonance (SSR) [8]-[14], [21], [22], [26], [28].

B. VSM

Figure 2 shows the structure and control diagram of a single VSM, including APC, RPC, AVC, and VI (if used). Open-loop vector voltage control (VVC) is employed, which is beneficial for meeting the functional specifications of GFM converters. The examples employing this structure can be found in [8], [9], [13], [26].

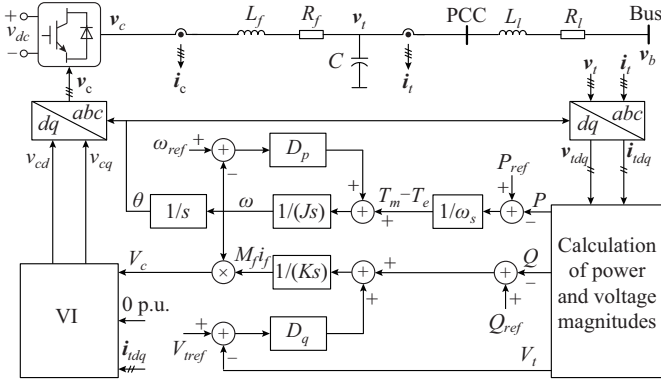


Fig. 2. Structure and control diagram of single VSM.

1) APC

The APC emulates the active power-frequency droop control of an SM, as also known as the swing equation:

$$J \frac{d\omega}{dt} = T_m - T_e + D_p (\omega_{ref} - \omega) \quad (1)$$

The active power-frequency (or virtual rotor speed ω) droop control is realized by D_p . Virtual rotor angle θ is the output of APC. $T_m = P_{ref}/\omega_s$, $T_e = P/\omega_s$, and $\omega_{ref} = \omega_s$.

2) RPC

RPC emulates the reactive power-voltage droop control as:

$$K \frac{d(M_f i_f)}{dt} = Q_{ref} - Q + D_q (V_{ref} - V_t) \quad (2)$$

The reactive power-voltage droop control is regulated by D_q . The amplitude of the virtual back electromotive force (EMF) V_c is the product of $M_f i_f$ and ω .

3) VI

VI serves as a multi-functional unit in MG, including power decoupling, current limitation, harmonics suppression, and stability enhancement [12], [13], [26]. There are different

ways to implement the VI, e.g., virtually increasing the filter impedance by proportionally decreasing converter-side voltages v_{cd} and v_{cq} [26], or using current feedback control to virtually modify the line impedance [12], [13]. The most commonly used is the latter, which is adopted in this paper as:

$$\begin{cases} v_{cd} = V_c - R_f i_{td} + \omega_0 L_f i_{tq} \\ v_{cq} = -\omega_0 L_f i_{td} - R_f i_{tq} \end{cases} \quad (3)$$

where v_{cd} and v_{cq} in the control stage correspond to the variables in the power network if the modulation process and time delay are ignored.

III. MODULAR STATE-SPACE MODEL OF PARALLEL GFM CONVERTERS IN ISLANDED MG

A. Transformation of Reference Frame

To model the MG system consisting of multiple VSMs represented in their own dq -frame, a common dq -frame (denoted as DQ -frame) is required to transform the input and output quantities of VSMs. Then, all VSMs are integrated by aligning them with respect to the angle difference between each dq -frame and the DQ -frame [18], [21], [22]. Without loss of generality, the rotating dq -frame of the first VSM in Fig. 1 is used to represent the DQ -frame in Supplementary Material A Fig. SA1 [18], [21], [22].

Specifically, the output current of the n^{th} non-referenced VSM should be transformed into the DQ -frame as:

$$\begin{bmatrix} i_{tdn} \\ i_{tqn} \end{bmatrix} = \begin{bmatrix} \cos(\delta_n) & -\sin(\delta_n) \\ \sin(\delta_n) & \cos(\delta_n) \end{bmatrix} \begin{bmatrix} i_{tdn} \\ i_{tqn} \end{bmatrix} \quad (4)$$

where subscript n represents the index of VSM. The linearized form of (4) is shown in Supplementary Material A SA1.

The input bus voltages v_{bD} and v_{bQ} in the DQ -frame should be transformed into the dq -frame of the n^{th} non-referenced VSM as:

$$\begin{bmatrix} v_{bdn} \\ v_{bqn} \end{bmatrix} = \begin{bmatrix} \cos(\delta_n) & \sin(\delta_n) \\ -\sin(\delta_n) & \cos(\delta_n) \end{bmatrix} \begin{bmatrix} v_{bD} \\ v_{bQ} \end{bmatrix} \quad (5)$$

The linearized form of (5) is also shown in Supplementary Material A SA1.

B. Modeling of Single GFM Converter

1) VSM

The state-space model of the non-referenced VSM control is derived by linearizing (1) and (2) and adding the new state δ , where $d\delta/dt = \omega - \omega_{com}$.

$$\begin{cases} \frac{d\tilde{\mathbf{x}}_c}{dt} = \mathbf{A}_c \tilde{\mathbf{x}}_c + \mathbf{B}_c \tilde{\mathbf{u}}_c \\ \tilde{\mathbf{y}}_c = \mathbf{C}_c \tilde{\mathbf{x}}_c + \mathbf{D}_c \tilde{\mathbf{u}}_c \end{cases} \quad (6)$$

where $\mathbf{x}_c = [M_f i_f, \omega, \delta]^T$; $\mathbf{u}_c = [P_{ref}, P, Q_{ref}, Q, V_t, \omega_{com}]^T$; $\mathbf{y}_c = [V_c, \omega, \delta]^T$; and $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$, and $\mathbf{D}_c$ are detailed in Supplementary Material A. For the first (referenced) VSM, (6) should be revised. Specifically, δ in the state vector and the output vector is eliminated, along with ω_{com} in the input vector. Otherwise, one of the eigenvalues remains zero [21].

2) LC Filter and Coupling Line Impedance

Different from grid-connected mode in [26], the dq -frame

is oriented to the voltage vector at AC side of VSM in this paper, eliminating the redundant state variable θ and a resulting eigenvalue of zero. The state-space model of LC filter and coupling line impedance is derived without and with the VI implementation, respectively.

$$\begin{cases} \frac{d\tilde{\mathbf{x}}_f}{dt} = \mathbf{A}_f \tilde{\mathbf{x}}_f + \mathbf{B}_f \tilde{\mathbf{u}}_f \\ \tilde{\mathbf{y}}_f = \mathbf{C}_f \tilde{\mathbf{x}}_f + \mathbf{D}_f \tilde{\mathbf{u}}_f \end{cases} \quad (7)$$

where $\mathbf{x}_f = [i_{cd}, i_{cq}, v_{td}, v_{tq}, i_{td}, i_{tq}]^T$; $\mathbf{u}_f = [V_c, \omega, v_{bd}, v_{bq}]^T$; $\mathbf{y}_f = [v_{td}, v_{tq}, i_{td}, i_{tq}]^T$; and $\mathbf{A}_f$, $\mathbf{B}_f$, $\mathbf{C}_f$, and $\mathbf{D}_f$ are detailed in Supplementary Material A.

3) Calculation of Power and Voltage Magnitude

A low-pass filter (LPF) with a high cut-off frequency $\omega_c = 190$ Hz is inserted into the original algebraic equations for the calculation of power and voltage magnitude, thus CCM can be employed for the modeling of MG system. The state-space expression of a single VSM is obtained in per-unit (p.u.) system as:

$$\begin{cases} \frac{d\tilde{\mathbf{x}}_{cal}}{dt} = \mathbf{A}_{cal} \tilde{\mathbf{x}}_{cal} + \mathbf{B}_{cal} \tilde{\mathbf{u}}_{cal} \\ \tilde{\mathbf{y}}_{cal} = \mathbf{C}_{cal} \tilde{\mathbf{x}}_{cal} + \mathbf{D}_{cal} \tilde{\mathbf{u}}_{cal} \end{cases} \quad (8)$$

where $\mathbf{x}_{cal} = [P, Q, V_t]^T$; $\mathbf{u}_{cal} = [v_{td}, v_{tq}, i_{td}, i_{tq}]^T$; $\mathbf{y}_{cal} = [P, Q, V_t]^T$; and $\mathbf{A}_{cal}$, $\mathbf{B}_{cal}$, $\mathbf{C}_{cal}$, and $\mathbf{D}_{cal}$ are detailed in Supplementary Material A. For islanded MG with parallel VSMs, Supplementary Material A (SA5) should be augmented diagonally, which is omitted for simplicity.

C. Load Modeling

Apart from a resistive-inductive load, i.e., $R_{load2} + j\omega_s L_{load2}$, a resistive load R_{load1} is introduced in Fig. 1 to eliminate bus voltage v_{bd} and v_{bq} as the input variables [18], [21], [22]. The linearized state-space model is derived as:

$$\begin{cases} \frac{d\tilde{\mathbf{x}}_{load}}{dt} = \mathbf{A}_{load} \tilde{\mathbf{x}}_{load} + \mathbf{B}_{load} \tilde{\mathbf{u}}_{load} \\ \tilde{\mathbf{y}}_{load} = \mathbf{C}_{load} \tilde{\mathbf{x}}_{load} + \mathbf{D}_{load} \tilde{\mathbf{u}}_{load} \end{cases} \quad (9)$$

where $\mathbf{x}_{load} = [i_{load2d}, i_{load2q}]^T$; $\mathbf{u}_{load} = [i_{tdsum}, i_{tqsum}, \omega]^T$; $\mathbf{y}_{load} = [v_{bd}, v_{bq}]^T$; and $\mathbf{A}_{load}$, $\mathbf{B}_{load}$, $\mathbf{C}_{load}$, and $\mathbf{D}_{load}$ are detailed in Supplementary Material A.

D. Modular State-space Modeling of Complete System

For each subsystem described above, the general state-space model can be obtained for the m^{th} subsystem ($m = 1, 2, \dots, M$) as:

$$\begin{cases} \frac{d\tilde{\mathbf{x}}_m}{dt} = \mathbf{A}_m \tilde{\mathbf{x}}_m + \mathbf{B}_m \tilde{\mathbf{u}}_m \\ \tilde{\mathbf{y}}_m = \mathbf{C}_m \tilde{\mathbf{x}}_m + \mathbf{D}_m \tilde{\mathbf{u}}_m \end{cases} \quad (10)$$

The open-form aggregation is first conducted to group all the subsystems as:

$$\begin{cases} \frac{d\tilde{\mathbf{x}}_L}{dt} = \mathbf{A}_L \tilde{\mathbf{x}}_L + \mathbf{B}_L \tilde{\mathbf{u}}_L \\ \tilde{\mathbf{y}}_L = \mathbf{C}_L \tilde{\mathbf{x}}_L + \mathbf{D}_L \tilde{\mathbf{u}}_L \end{cases} \quad (11)$$

where $\mathbf{A}_L = \text{diag}(\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_M)$; $\mathbf{B}_L = \text{diag}(\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_M)$; $\mathbf{C}_L = \text{diag}(\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_M)$; $\mathbf{D}_L = \text{diag}(\mathbf{D}_1, \mathbf{D}_2, \dots, \mathbf{D}_M)$; and $\mathbf{x}_L$, $\mathbf{u}_L$, and $\mathbf{y}_L$ are the vertical aggregations of the related vectors,

as shown in Fig. 3.

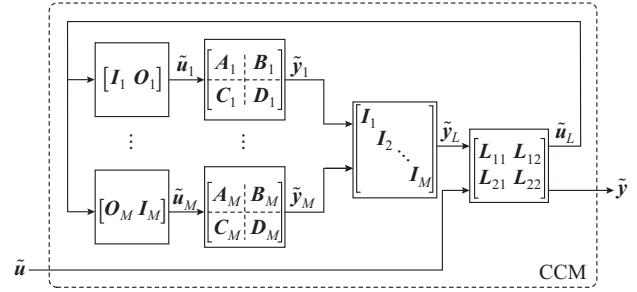


Fig. 3. Schematic diagram of CCM.

Then, an algebraic connection matrix $\mathbf{L} = [\mathbf{L}_{11}, \mathbf{L}_{12}; \mathbf{L}_{21}, \mathbf{L}_{22}]$ connecting the input $\mathbf{u}_L$ and output $\mathbf{y}_L$ of the open-form aggregated model, as well as the input $\mathbf{u}$ and output $\mathbf{y}$ of the whole closed-form system is described as:

$$\begin{cases} \tilde{\mathbf{u}}_L = \mathbf{L}_{11} \tilde{\mathbf{y}}_L + \mathbf{L}_{12} \tilde{\mathbf{u}} \\ \tilde{\mathbf{y}} = \mathbf{L}_{21} \tilde{\mathbf{y}}_L + \mathbf{L}_{22} \tilde{\mathbf{u}} \end{cases} \quad (12)$$

A modular state-space model of the complete system is derived in a closed-form as:

$$\begin{cases} \frac{d\tilde{\mathbf{x}}}{dt} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}\tilde{\mathbf{u}} \\ \tilde{\mathbf{y}} = \mathbf{C}\tilde{\mathbf{x}} + \mathbf{D}\tilde{\mathbf{u}} \end{cases} \quad (13)$$

where $\mathbf{x} = \mathbf{x}_L$; and $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are determined by CCM [26], [28] as:

$$\begin{cases} \mathbf{A} = \mathbf{A}_L + \mathbf{B}_L \mathbf{L}_{11} (\mathbf{I} - \mathbf{D}_L \mathbf{L}_{11})^{-1} \mathbf{C}_L \\ \mathbf{B} = \mathbf{B}_L \mathbf{L}_{11} (\mathbf{I} - \mathbf{D}_L \mathbf{L}_{11})^{-1} \mathbf{D}_L \mathbf{L}_{12} + \mathbf{B}_L \mathbf{L}_{12} \\ \mathbf{C} = \mathbf{L}_{21} (\mathbf{I} - \mathbf{D}_L \mathbf{L}_{11})^{-1} \mathbf{C}_L \\ \mathbf{D} = \mathbf{L}_{21} (\mathbf{I} - \mathbf{D}_L \mathbf{L}_{11})^{-1} \mathbf{D}_L \mathbf{L}_{12} + \mathbf{L}_{22} \end{cases} \quad (14)$$

For example, when islanded MG with two parallel VSMs ($N=2$) is concerned, $\mathbf{A}_L = \text{diag}(\mathbf{A}_{c1}, \mathbf{A}_{f1}, \mathbf{A}_{c2}, \mathbf{A}_{f2}, \mathbf{A}_{load}, \mathbf{A}_{cal})$, $\mathbf{B}_L = \text{diag}(\mathbf{B}_{c1}, \mathbf{B}_{f1}, \mathbf{B}_{c2}, \mathbf{B}_{f2}, \mathbf{B}_{load}, \mathbf{B}_{cal})$, $\mathbf{C}_L = \text{diag}(\mathbf{C}_{c1}, \mathbf{C}_{f1}, \mathbf{C}_{c2}, \mathbf{C}_{f2}, \mathbf{C}_{load}, \mathbf{C}_{cal})$, and $\mathbf{D}_L = \text{diag}(\mathbf{D}_{c1}, \mathbf{D}_{f1}, \mathbf{D}_{c2}, \mathbf{D}_{f2}, \mathbf{D}_{load}, \mathbf{D}_{cal})$. The subscripts “1” and “2” denote the corresponding matrices of the first and the second VSMs, respectively. The detailed expressions of the used matrices are given in Supplementary Material A. It is worth noting that $\mathbf{A}_{cal}$, $\mathbf{B}_{cal}$, $\mathbf{C}_{cal}$, and $\mathbf{D}_{cal}$ are obtained by diagonally augmenting in Supplementary Material A (SA5), which represents the case of a single VSM. Moreover, $\mathbf{L}_{11} \in \mathbb{R}^{31 \times 21}$, $\mathbf{L}_{12} \in \mathbb{R}^{31 \times 5}$, $\mathbf{L}_{21} \in \mathbb{R}^{6 \times 21}$, and $\mathbf{L}_{22} \in \mathbb{R}^{6 \times 5}$ are the sparse connection matrices obtained from simple algebraic operations, with which all subsystems could be integrated as the complete system and they are omitted here for simplicity.

IV. ROBUST STABILITY ANALYSIS USING SSV APPROACH

A. SSV Approach

Consider a general complex MIMO system. The general uncertainty model $\mathbf{G}$ is given in (15), with output dimensions p_1 and p_2 for its submatrices $\mathbf{G}_{11}$ and $\mathbf{G}_{21}$, respectively, and input dimensions q_1 and q_2 for its submatrices $\mathbf{G}_{12}$ and $\mathbf{G}_{22}$, respectively.

$$\mathbf{G} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \in \mathbb{C}^{(p_1+p_2) \times (q_1+q_2)} \quad (15)$$

Figure 4(a) shows the general uncertainty model $\mathbf{G}$ in transfer function form, perturbed by uncertainty block $\Delta \in \mathbb{C}^{q_1 \times p_1}$. Obviously, only general uncertainty model $\mathbf{G}_{11} = [\mathbf{A}, \mathbf{B}_1; \mathbf{C}_1, \mathbf{D}_{11}]$ in Fig. 4(b) links uncertainty block Δ . To this extent, robust stability of uncertain system only depends on the corresponding $\mathbf{G}_{11}$ [25], which will be derived in next subsection for the studied system.

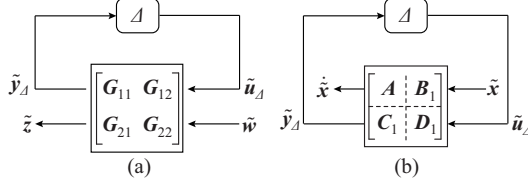


Fig. 4. Uncertain system with general uncertainty model and uncertainty block. (a) General uncertainty model $\mathbf{G}$ in transfer function form. (b) General uncertainty model $\mathbf{G}_{11}$ in state-space form.

Another important step is uncertainty modeling. There are two different methods to address this issue in robust control theory: parametric uncertainty δ_i and unstructured uncertainty Δ_j . One advantage of SSV approach is that it is compatible with above two elements, behaving less conservative compared with the small gain theory, which can only deal with one unstructured uncertainty block. Specifically, the SSV approach measures robust stability and margin with respect to structured uncertainty $\Delta \in \mathbb{C}^{n \times n}$, which is arranged as:

$$\Delta = \text{diag}(\delta_1 \mathbf{I}_{r_1}, \dots, \delta_i \mathbf{I}_{r_i}, \dots, \delta_s \mathbf{I}_{r_s}, \Delta_1, \dots, \Delta_j, \dots, \Delta_F) \quad (16)$$

$$\delta_i \in \mathbb{C}, \Delta_j \in \mathbb{C}^{m_j \times m_j}, \sum_{i=1}^S r_i + \sum_{j=1}^F m_j = n$$

Uncertainties are first modeled in a decentralized manner at their respective locations, and they should be extracted from the system and arranged in a diagonal structure as shown in (16). $\mathbf{G}_{11}$ can then be derived from the resulting system excluding the uncertainties. For an MG, the uncertainty modeling and the formulation of $\mathbf{G}_{11}$ will be explained in Section IV-B and IV-C, respectively.

The definition of μ for $\mathbf{M} \in \mathbb{C}^{n \times n}$ is formally given as [25]:

$$\mu_{\Delta}(\mathbf{M}) = \frac{1}{\min \{ \bar{\sigma}(\Delta) : \det(\mathbf{I} - \mathbf{M}\Delta) = 0, \Delta \in \Delta \}} \quad (17)$$

where $\mu_{\Delta}(\mathbf{M}) = 0$ if $\mathbf{I} - \mathbf{M}\Delta$ is nonsingular for all $\Delta \in \Delta$.

The computational complexity involved in determining the exact value of μ persists as an open challenge [25], [29]. However, in this paper, robust stability and margin are derived from the peak value of μ , where the upper and lower bounds exhibit close proximity, suggesting a tight numerical estimation [30], [31]. Thereafter, the lower bound is employed and μ refers to the related peak value.

Therefore, the criterion for system robust stability with respect to μ and the size of structured uncertainty Δ (in sense of largest singular value) is defined as [25]:

$$\bar{\sigma}(\Delta(j\omega)) \leq \frac{1}{\sup_{\omega \in \mathbb{R}} \mu_{\Delta}(\mathbf{M}(j\omega))} \quad (18)$$

In particular, structured uncertainty Δ is initially normalized by pulling out any weights and integrating them into $\mathbf{M}$. To this extent, $\bar{\sigma}(\Delta(j\omega)) \leq 1$ holds, and the normalized variant of robust stability condition is accordingly promoted as:

$$\sup_{\omega \in \mathbb{R}} \mu_{\Delta}(\mathbf{M}(j\omega)) \leq 1 \quad (19)$$

Robust stability analysis is conducted. The system is robustly stable if $\mu \leq 1$, even though the defined uncertain ranges are expanded to $1/\mu$ times. Otherwise, $\mu > 1$ means that only by reducing the defined uncertainty range to $1/\mu$ times can the stability of the system be guaranteed. Therefore, the normalized robust stability margin can be defined as $1/\mu$, and the predefined uncertain ranges can be adjusted to $1/\mu$ times accordingly in order to warrant the system stability.

B. Model Uncertainty Representation in Islanded MG

Robust stability analysis using SSV approach should be conducted regarding some real uncertainties, which can be usually presented in the form of parametric uncertainty, e.g., impedance of filters and operating points. However, unstructured uncertainty is used in this paper regarding its simpler form, which is useful for stability analysis and control synthesis despite of its conservativeness [25], [26], [32], [33]. The grey frequency-dependent multiplicative uncertainty in SISO form is given as (20) [25], [26], [32], [33], which is quantified by several factors.

$$P(s) = \bar{P}(s)(1 + W(s)\Delta(s)) \quad (20)$$

1) Transfer function $\bar{P}(s)$ of the nominal plant. It changes with different operating points, which are dependent on parameters of power stage, as well as parameters of control stage in islanded MG, where droop mechanisms of active power-frequency (P - ω) and reactive power-PCC voltage magnitude (Q - V_f) have been activated.

2) Uncertainty weighting function $W(s)$. It describes the frequency-dependent uncertain ranges between the uncertainty model and the nominal model, in sense of the largest singular value. An initial selection can be obtained using MATLAB command “makeweight” by defining low-frequency gain and high-frequency gain according to well-consolidated techniques in [26].

3) Unstructured uncertainty block $\Delta(s)$. It refers to the uncertain linear time-invariant dynamics where the only known property is bound of related frequency response. In general, $\|\Delta(s)\|_{\infty} \leq 1$ holds since any weight of model uncertainty can be absorbed by uncertainty weighting function $W(s)$. $\Delta(s)$ is defined by MATLAB command “ultidyn”.

A hypothetical case of the uncertain single-phase load under the nominal expression is given to explain the model uncertainty representation with an unstructured form as:

$$\bar{P}(s) = R + sL \quad (21)$$

Supplementary Material A Fig. SA2(a) shows the frequency response of the nominal model and the uncertainty models, where the resistance/reactance ratio, load impedance amplitude, and high-frequency effects specified by two resonant filters are considered. Obviously, all potential uncertainty models lie in the allowable ranges, defined by the employed

uncertainty weighting function $W(s)$. The absolute value of relative difference between the perturbed models and the nominal one is given in Supplementary Material A Fig. SA2(b), intuitively determined by:

$$W(s) = \left| \frac{P(s) - \bar{P}(s)}{\bar{P}(s)} \right| \quad (22)$$

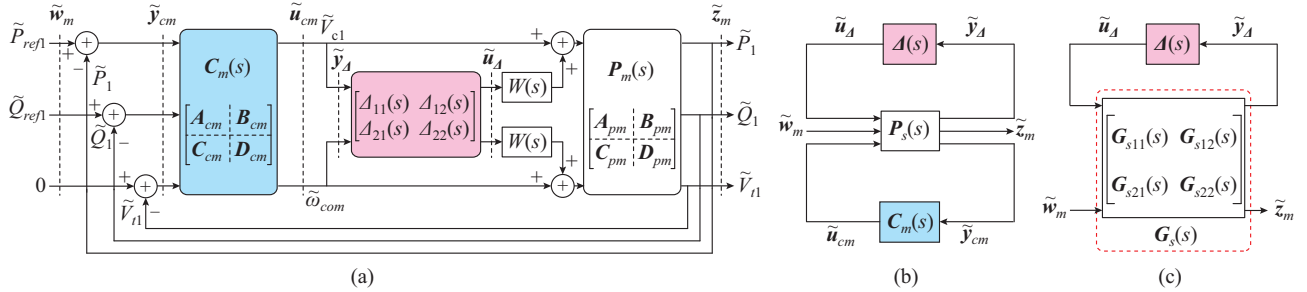


Fig. 5. Formulation of general uncertainty model. (a) Equivalent MIMO controller and plant of islanded MG. (b) General framework of robust control. (c) Robust stability analysis framework.

Obviously, the plant $P_m(s)$ in Fig. 5(a), which consists of 3×2 transfer functions, is an MIMO system with MIMO nature. Therefore, a full structured uncertainty block $\Delta(s)$ consisting of four unstructured uncertainty blocks is configured at the input of MIMO plant, as shown in Fig. 5(a). To this extent, the coupling effects between different input channels of the plant could be reflected. Then, the description of the uncertain ranges boils down to the parametrization of uncertainty weighting function $W(s)$, which is expected to depict the worst cases in uncertain contexts considering all potential parametric uncertainties, i.e., the envelope of the Bode plot.

For islanded MG, uncertainty weighting function for each input channel of MIMO plant is selected under the same principle. Specifically, the maximum deviation ratio of 0.5 (≈ -6 dB) is dedicated to representing parametric uncertainty in low-frequency range. Besides, high-frequency uncertainty with the maximum deviation ratio of 5 (≈ 14 dB) is used to account for the neglected dynamics since the system behavior is often better understood in low-frequency range [26]. The resulting uncertainty weighting function for each input channel is given as:

$$W(s) = \frac{5s + 3861}{s + 7722} \quad (23)$$

The rationality of $W(s)$ can be verified by checking whether the envelope of the amplitude-frequency response configured with $W(s)$ can cover those corresponding to all real parametric uncertainties. The MIMO form of (20) is given as:

$$P(s) = \bar{P}(s)(I + W(s)\Delta(s)) \quad (24)$$

where $W(s) = \text{diag}(W(s), W(s))$.

The upper bound can be explicitly derived as:

$$\max_{\|\Delta\|_\infty \leq 1} |P_{ij}(j\omega)| = |\bar{P}_{ij}(j\omega)| + |e_i^T(\bar{P}W(j\omega))| \quad (25)$$

where $e_i^T(\bar{P}W(j\omega)) = \bar{P}W(i, :)$.

$W(s)$ in (23) is just an initial assumption in this paper,

To conduct robust stability analysis using SSV approach for islanded MG, the equivalent MIMO controller $C_m(s)$ and plant $P_m(s)$ in Fig. 5(a) are first developed with CCM in Fig. 3. Note that in this continuous-time modeling framework, all variables are linearized. The blue block represents the equivalent MIMO controller of islanded MG, and the red block represents structured uncertainty.

which should be adjusted under different uncertainty conditions, until the obtained envelope from (25) is capable of describing all real parametric uncertainties. For example, system response with different parametric uncertainties in Supplementary Material A Fig. SA3 can be covered by the upper bound of the uncertainty model parametrized by $W(s)$. Therefore, robust stability analysis using SSV approach can be conducted and the results ensure a certain level of robustness toward some real uncertainties.

C. Formulation of G_{s11}

The controller of VSM1 is treated as the equivalent controller of the rearranged system, and all the rest loops are aggregated as the equivalent MIMO plant of islanded MG. $W(s)$ in Fig. 5(a) for both channels has been pulled out and integrated into $P_s(s)$ in Fig. 5(b), which is derived as:

$$\begin{bmatrix} \tilde{y}_A \\ \tilde{z}_m \\ \tilde{y}_{cm} \end{bmatrix} = P_s(s) \begin{bmatrix} \tilde{u}_A \\ \tilde{w}_m \\ \tilde{u}_{cm} \end{bmatrix} = \begin{bmatrix} O_{2 \times 2} & O_{2 \times 3} & W(s) \\ P_m(s) & O_{3 \times 3} & P_m(s) \\ -P_m(s) & I_{3 \times 3} & -P_m(s) \end{bmatrix} \begin{bmatrix} \tilde{u}_A \\ \tilde{w}_m \\ \tilde{u}_{cm} \end{bmatrix} \quad (26)$$

Then, $P_s(s)$ can be partitioned as:

$$P_s(s) = \begin{bmatrix} O_{2 \times 2} & O_{2 \times 3} & W(s) \\ P_m(s) & O_{3 \times 3} & P_m(s) \\ -P_m(s) & I_{3 \times 3} & -P_m(s) \end{bmatrix} = \begin{bmatrix} P_{s11}(s) & P_{s12}(s) \\ P_{s21}(s) & P_{s22}(s) \end{bmatrix} \quad (27)$$

The robust stability analysis deals with the analysis framework in Fig. 5(c), where $C_m(s)$ is absorbed by $P_s(s)$. The resulting connection transfer function matrix is denoted as $G_s(s)$, which can be derived with $C_m(s)$ and $P_s(s)$ using lower linear fractional transformation (LFT) as (28) [25], provided $I - P_{s22}(s)C_m(s)$ is invertible.

$$G_s(s) = \mathcal{F}_l(P_s(s), C_m(s)) = P_{s11}(s) + P_{s12}(s)C_m(s)(I - P_{s22}(s)C_m(s))^{-1}P_{s21}(s) \quad (28)$$

$G_s(s)$ in (28) can be partitioned as the form in Fig. 5(c), which is the same as that in Fig. 4(a). As mentioned earlier, the property of robust stability is only dependent on the sub-

system $G_{s11}(s)$, and it can be eventually derived based on the above performed analysis.

D. Robust Stability Analysis

Islanded MG with two parallel VSMs ($N=2$) is investigated with the proposed robust stability analysis using SSV approach. Default system parameters are given in Supplementary Material A Table SAI. First, robust stability analysis under the default condition is performed and μ plots of islanded MG with one and two VSMs under different uncertainty weighting functions are shown in Fig. 6, where a single VSM with the same configuration is also investigated for comparison. Two cases are both stable regarding the assumed uncertainty since μ remains smaller than 1 across the whole frequency range. However, the significant difference is that a resonance peak of 0.63 at a frequency of 17.5 Hz is observed in the case with two VSMs, and robust stability margin is 1.59 (1/0.63), which is reduced compared with 2 (1/0.499) in the case with a single VSM. It means that the oscillation issues with a frequency of 17.5 Hz would occur, e.g., responding to step changes on the references. This phenomenon can be interpreted as the decrease of the electrical distance between the two voltage sources, since the output impedance of the converter is generally much smaller than the stator impedance of real SM. A practical way to improve the robust stability of parallel VSMs is the use of VI, which would be conducted through robust design by structured μ synthesis in Section V.

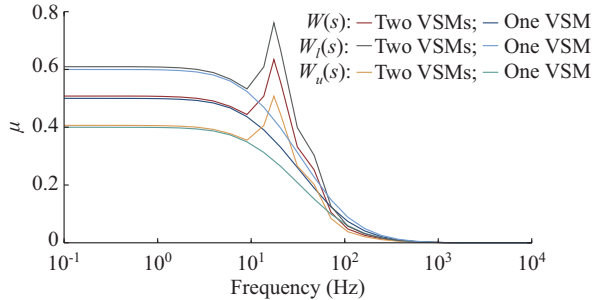


Fig. 6. μ plots of islanded MG with one and two VSMs under different uncertainty weighting functions.

Moreover, the impact of uncertainty weighting function is also investigated, where $\pm 20\%$ change of $W(s)$ is configured using MATLAB command “makeweight” [34]. Specifically, $W(s)=(5s+3861)/(s+7722)$ in (23) is sequentially replaced by $W_l(s)=(4s+3089)/(s+7722)$ and $W_u(s)=(6s+4633)/(s+7722)$, corresponding to -20% and $+20\%$ changes, respectively. The μ plots in Fig. 6 indicate that the increase of the uncertainty will evidently deteriorate the robust stability and margin regardless of whether there is a parallel operation of the converter.

The robust stability of two parallel VSMs is investigated with a sweep of virtual inertia J in the range of $[0.01J_n, 4J_n]$. The results are shown in Fig. 7(a). A resonance peak of μ shifts from 0.4 at a frequency of 47.1 Hz to 1.12 at a frequency of 9.5 Hz with the increase of J , where the latter is not robustly stable with respect to the predefined uncertainty

range since $\mu=1.12>1$. The robust stability margin is 0.89 (1/1.12), which means that the predefined uncertainty ranges should be reduced to 0.89 times. This can be done by shaping the initial uncertainty weighting function $W(s)$. Otherwise, the additional enhancement solution such as VI can be used to guarantee the stability with the unchanged $W(s)$.

In a similar manner, a sweep of integral coefficient K in the range of $[0.3K_n, 5K_n]$ is studied by the proposed robust stability analysis using SSV approach. The results are shown in Fig. 7(b), where the line colors have the same meaning with respect to the changes of K . Two resonance peaks occur when $K=0.3K_n$, which means that the system will experience oscillations of two different frequencies (around 20 Hz and 50 Hz, respectively). The robustness of islanded MG within the whole investigated frequency range can be enhanced by increasing K . However, performance limits occur when K is larger than specific value, e.g., $K>0.8K_n$ in this case. Thus, the resonance peak will increase even though μ decreases in low-frequency and high-frequency ranges.

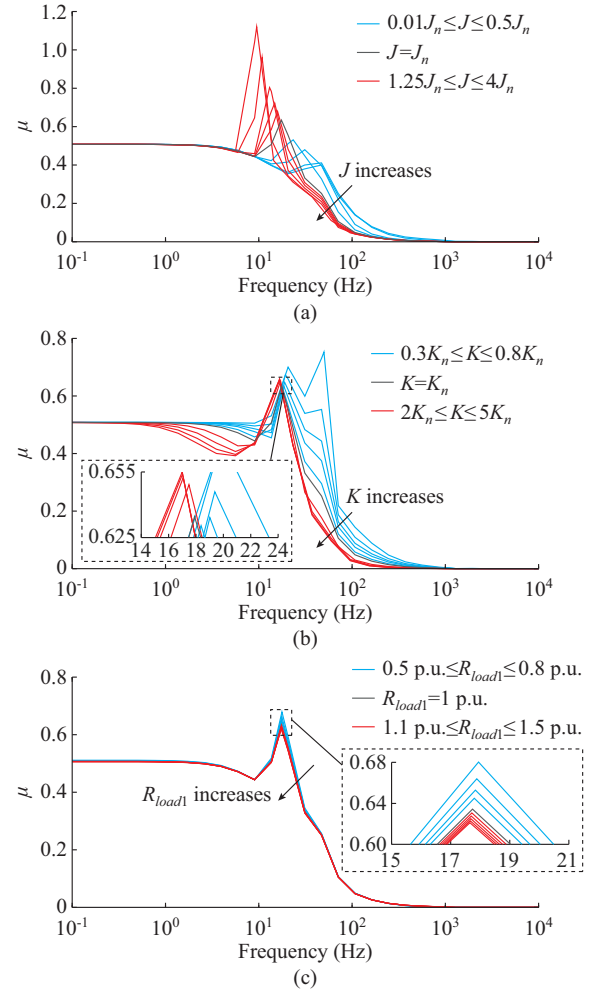


Fig. 7. μ plots of islanded MG with two VSMs under parameter variations. (a) J changes. (b) K changes. (c) R_{load1} changes.

Moreover, the influence of load variations on robust stability of islanded MG is analyzed with the proposed robust stability analysis using SSV approach. The results are shown in

Fig. 7(c) regarding a sweep of parallel resistive load R_{load1} in the range of [0.5, 1.5]p.u.. Obviously, μ is not relevantly reduced with the increase of resistance R_{load1} . Moreover, the changes of load will not affect the peak resonance frequency, with different mechanisms compared with J and K .

Sensitivity analysis of the above parameters is also provided, offering the guidance of parameter tuning. The sensitivity measures to what extent a given change in the normalized uncertain range of the related element affects the robust stability margin ($1/\mu$). The normalized form is given as [34]:

$$S_i = \frac{\left| \frac{d(1/\mu)}{d\delta_i} \right|}{\sum_{k=1}^S \left| \frac{d(1/\mu)}{d\delta_k} \right|} \times 100\% \quad (29)$$

The above equation is also applicable to unstructured uncertainty factor Δ_j in structured uncertainty Δ in (16).

For a fair comparison, 50% parametric uncertain ranges are assumed for J , K , and R_{load1} with the nominal values in Supplementary Material A Table SAI, and unstructured uncertainty with three different weighting functions in Fig. 6 are also considered simultaneously. Supplementary Material A Fig. SA4 shows the results of the sensitivity analysis, which indicate that J has the most significant impact on μ , apart from the unstructured uncertainty. Therefore, more attention should be paid to J during parameter tuning.

V. ROBUST DESIGN OF PARALLEL GFM CONVERTERS IN ISLANDED MG BY STRUCTURED μ SYNTHESIS

One important function of VI is to stabilize the islanded MG, providing more degrees of freedom to the parameter tuning, and avoiding physical limitations on passive impedance implementation. As mentioned in Section II, different methods have been developed to realize this technique. In this paper, current feedback control elaborated in (3) is employed to virtually modify the values of line impedance.

To this end, robust design by structured μ synthesis for VI tuning based on D - K iteration is proposed to optimize the performance specifications, and robust stability is enhanced under the same model uncertainty. In general, μ synthesis returns a robust full-order controller if its structure is not pre-defined, then model reduction such as balanced stochastic truncation model reduction is used to achieve a low-order realization [34]. However, μ synthesis also allows for direct design of the fixed-structure controller, where the structured H_∞ controller synthesis algorithm in [35] is used in the step K of μ synthesis. Therefore, VI parameters are regarded as tunable elements for convenience, with which the structured μ synthesis can be used to find the optimal solution.

A. Formulation of G_μ

VI parameters to be designed are diagonally formed, as shown in Fig. 8(a), where the same values are configured for VSM1 and VSM2 to fulfill the prerequisite for power sharing [13], [36]. The linear input $\tilde{w}_\mu$ and output $\tilde{z}_\mu$ of performance specifications are defined as:

$$\begin{cases} \tilde{w}_\mu = [\tilde{P}_{ref1} \quad \tilde{Q}_{ref1}]^T \\ \tilde{z}_\mu = \begin{bmatrix} (\tilde{P}_{ref1} + D_{p1}\tilde{\omega}_s) - (\tilde{P}_1 + D_{p1}\tilde{\omega}_{com}) \\ (\tilde{Q}_{ref1} + D_{q1}\tilde{V}_{iref1}) - (\tilde{Q}_1 + D_{q1}\tilde{V}_{t1}) \end{bmatrix} \end{cases} \quad (30)$$

where $\tilde{\omega}_s = 0$ and $\tilde{V}_{iref1} = 0$. In other words, the proposed robust design by structured μ synthesis would not affect the above droop characteristics.

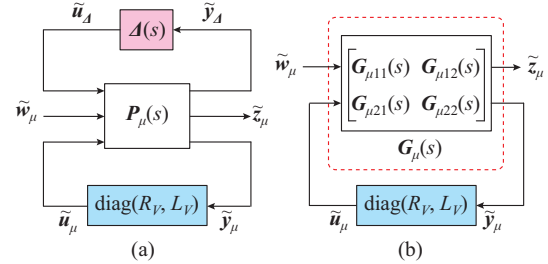


Fig. 8. Robust design by structured μ synthesis for VI tuning. (a) VI parameter formulation. (b) Synthesis framework.

Based on the above definitions, the synthesis framework is arranged in Fig. 8(b), where the uncertainty block $\Delta(s)$ is collapsed into general uncertainty model G_μ using upper LFT as (31), provided $I - P_{\mu11}(s)\Delta(s)$ is invertible. Note that $P_{\mu11}$, $P_{\mu12}$, $P_{\mu21}$, and $P_{\mu22}$ are submatrices of P_μ .

$$G_\mu(s) = \mathcal{F}_u(P_\mu(s), \Delta(s)) = \begin{bmatrix} P_{\mu22}(s) + P_{\mu21}(s)\Delta(s)(I - P_{\mu11}(s)\Delta(s))^{-1}P_{\mu12}(s) \end{bmatrix} \quad (31)$$

B. Selection of Weighting Functions

Weighting functions are required for H_∞ optimization, which serve as performance specifications and are generally constructed as the inversed shape of the desired closed-loop response [25], [37]. Based on the definitions of $\tilde{w}_\mu$ and $\tilde{z}_\mu$ in (30), the satisfactory closed-loop response to power references of $\tilde{w}_\mu$ should be the smallest possible (in a sense of ∞ -norm), tracking errors of $\tilde{z}_\mu$, especially in the low-frequency range. Therefore, $W_\mu(s)$ is selected as (32), and $\tilde{W}_\mu(s) = \text{diag}(W_\mu(s), W_\mu(s))$ is cascaded with $\tilde{z}_\mu$.

$$W_\mu(s) = \frac{s + 0.1}{s + 0.03} \quad (32)$$

C. VI Tuning

The robust design by structured μ synthesis for VI tuning is formulated as:

$$\min_{K_\mu} \mu_{\Delta_F}(\tilde{W}_\mu(s)T_\mu(s)) \quad (33)$$

where $T_\mu(s) = \mathcal{F}_l(G_\mu(s), K_\mu)$ is the closed-loop transfer functions from $\tilde{w}_\mu$ to $\tilde{z}_\mu$; $K_\mu = \text{diag}(R_V, L_V)$ is the static VI to be designed; and $\Delta_F(s) = \text{diag}(\Delta(s), \Delta_F(s))$, and $\Delta_F(s)$ is a fictitious uncertainty block with the compatible dimensions [25].

The initial values of VI are set to be $K_\mu = \text{diag}(0.02, 0.02)$ for optimization, and the final results can be obtained by solving the optimization problem in (33) using MATLAB command “musyn” [34], yielding (34), which means that VI parameters $R_V = 0$ p.u. and $L_V = 0.257$ p.u. should be configured for VSM1 and VSM2 to virtually increase the line impedance.

$$\mathbf{K}_\mu = \text{diag}(0, 0.257) \quad (34)$$

Moreover, a sweep of integral coefficient K as the same as in Fig. 7(b) is reevaluated with VI implementation. The results are shown in Supplementary Material A Fig. SA5(b), where all resonance peaks are damped, and the robust stability of islanded MG is enhanced. The damping effects on diverse J and public load are omitted for simplicity and will be demonstrated in the next section.

VI. VALIDATIONS

To validate the above analysis and design results, the comparisons among the theoretical results, time-domain simulation results from MATLAB/Simulink/PLECS, and real-time simulation results from real-time digital simulator (RTDS) are presented. Supplementary Material A Fig. SA6 shows the RTDS platform used in the validations, and all parameters are identical with theoretical and time-domain simulation tests.

By using the default parameters ($J_n = 0.2$ p.u., $K_n = 0.2$ p.u., $R_{load1} = 1$ p.u.) in Supplementary Material A Table SAI, the reference active power P_{ref1} of VSM1 is first increased from 1 p.u. to 1.2 p.u. at 0.5 s, and decreases back to 1 p.u. at 1 s. Then, the reference reactive power Q_{ref2} of VSM2 is increased from 0.8 p.u. to 1 p.u. at 1.5 s, and decreases back

to 0.8 p.u. at 2 s. The oscillation period of 0.057 s in Supplementary Material A Fig. SA7 corresponds to the frequency of 17.5 Hz related to the resonance peak of 0.63 in Fig. 6. Theoretical dynamics of the derived linear model are consistent with the other two results.

To validate the robust stability analysis, J , K , and R_{load1} have been changed to the worst cases regarding their respective ranges, i.e., $J = 4J_n$, $K = 0.3K_n$, and $R_{load1} = 0.5$ p.u.. In Fig. 9, the increase of J_n leads to more severe LFO issues compared with the default setting, and the lower oscillation frequency around 9.5 Hz (the oscillation period of 0.105 s) is consistent with the results in Fig. 7(a). However, the decrease of K_n not only induces LFO issues around 20 Hz (the oscillation period of 0.05 s) in Supplementary Material A Fig. SA8, but also deteriorates the damping of the system around synchronous frequency, which is represented by the oscillation with a higher frequency near 50 Hz (the oscillation period of 0.02 s) starting at 1.5 s and 2 s. This also matches with a resonance peak of 0.7 at 20.6 Hz and the other resonance peak 0.75 at 50 Hz in Fig. 7(b). The decrease of load leads to slightly reduced low-frequency damping in Supplementary Material A Fig. SA9, showing a similar oscillation frequency of 17.5 Hz (the oscillation period of 0.057 s) compared with the default case, which validates the performed analysis.

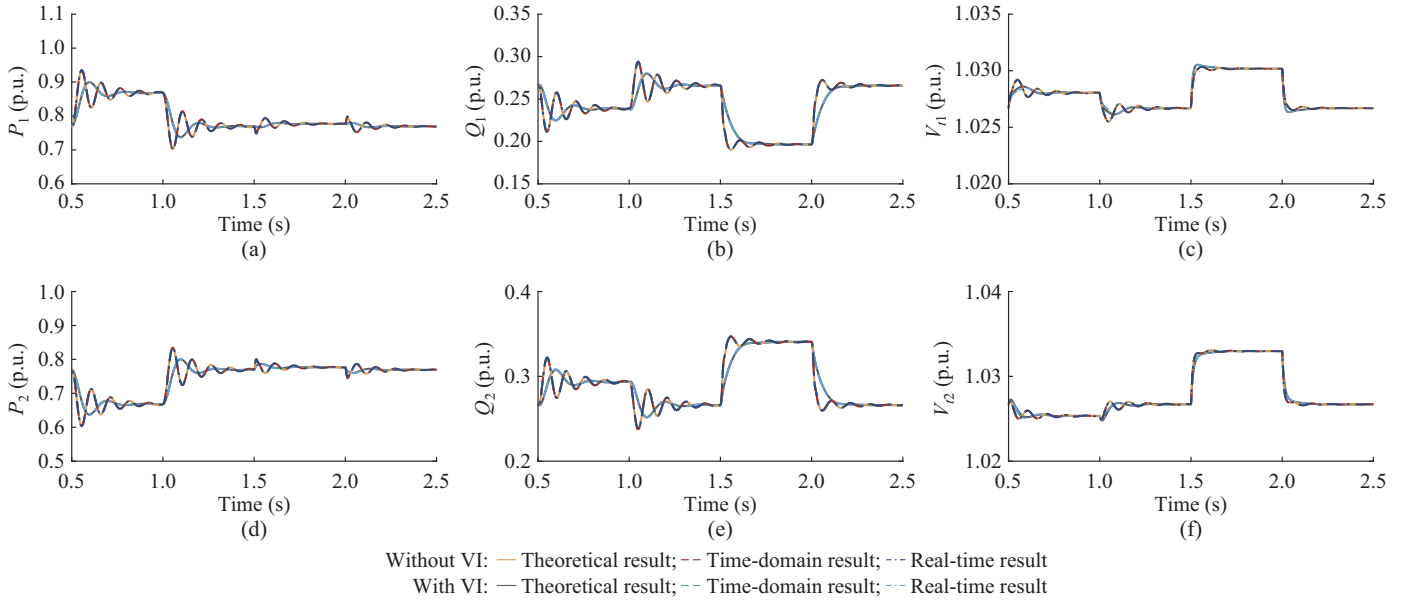


Fig. 9. Results with $J = 4J_n$. (a) Active power P_1 . (b) Reactive power Q_1 . (c) AC voltage V_{n1} . (d) Active power P_2 . (e) Reactive power Q_2 . (f) AC voltage V_{n2} .

The worst cases are reevaluated when the VI is implemented, with the configurations obtained in Section V. Obviously, the oscillation issues due to diverse parameter variations can be effectively damped, which demonstrates the benefit of implementing the VI on system stability. For example, the settling time responding to the step change at 1 s in Fig. 9 and Supplementary Material A Figs. SA8 and SA9 decreases from 0.4 s, 0.15 s, 0.15 s to 0.2 s, 0.1 s, 0.1 s when the VI is implemented, which correspond to 50%, 30%, 30% reductions, respectively. The reason of the similarity between

oscillation response with and without VI in Fig. 9 can be explained as follows. The employed VI parameters are optimized with respect to a set of uncertainties, ensuring that the closed-loop system maintains an adequate level of robustness. Under a specific worst-case operating condition ($J = 4J_n$), a certain conservatism in performance improvement is unavoidable. Nevertheless, the proposed robust design by structured μ synthesis is still applicable when only the dynamics under this condition are considered, which will not be discussed further in this paper.

VII. CONCLUSION

In this paper, robust stability analysis using SSV approach and design of parallel GFM converters in islanded MG by structured μ synthesis are presented. A modular state-space model for small-signal stability study is established with CCM, where an arbitrary number of parallel GFM converters can be integrated. Based on this, robust stability analysis using SSV approach is conducted. Additional GFM converters operating nearby deteriorate the robustness with respect to potential uncertainties. Besides, it is observed that control parameters not only affect the robust stability margin of the system, but also change the oscillation frequency, while load changes only take effect on the former. Robust design by structured μ synthesis for VI tuning is proposed to damp the resonance peaks, while robust stability is enhanced under the same uncertainty. Simulation results validate the accuracy, as well as the effectiveness of implementing the VI.

REFERENCES

- [1] M. Chen, D. Zhou, and F. Blaabjerg, "Modelling, implementation, and assessment of virtual synchronous generator in power systems," *Journal of Modern Power Systems and Clean Energy*, vol. 8, no. 3, pp. 399-411, May 2020.
- [2] J. Wang, Z. Liu, Y. Shi *et al.*, "A communication-less secondary control method for parallel grid-forming inverters in islanded microgrids based on mode switching," *IEEE Transactions on Power Electronics*, vol. 39, no. 3, pp. 3683-3701, Mar. 2024.
- [3] A. González-Cajigas, J. Roldán-Pérez, and E. J. Bueno, "Design and analysis of parallel-connected grid-forming virtual synchronous machines for island and grid-connected applications," *IEEE Transactions on Power Electronics*, vol. 37, no. 5, pp. 5107-5121, May 2022.
- [4] S. Fu, Y. Sun, L. Li *et al.*, "Power oscillation suppression of multi-VSG grid via decentralized mutual damping control," *IEEE Transactions on Industrial Electronics*, vol. 69, no. 10, pp. 10202-10214, Oct. 2022.
- [5] X. Wang, M. G. Taul, H. Wu *et al.*, "Grid-synchronization stability of converter-based resources – an overview," *IEEE Open Journal of Industry Applications*, vol. 1, pp. 115-134, Aug. 2020.
- [6] Q. Zhong and G. Weiss, "Synchronverters: inverters that mimic synchronous generators," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 4, pp. 1259-1267, Apr. 2011.
- [7] R. H. Lasseter, Z. Chen, and D. Pattabiraman, "Grid-forming inverters: a critical asset for the power grid," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 8, no. 2, pp. 925-935, Jun. 2020.
- [8] F. Zhao, T. Zhu, Z. Li *et al.*, "Low-frequency resonances in grid-forming converters: causes and damping control," *IEEE Transactions on Power Electronics*, vol. 39, no. 11, pp. 14430-14447, Nov. 2024.
- [9] F. Zhao, T. Zhu, L. Harnefors *et al.*, "Closed-form solutions for grid-forming converters: a design-oriented study," *IEEE Open Journal of Power Electronics*, vol. 5, pp. 186-200, Feb. 2024.
- [10] R. Liu, H. Zhang, C. Xue *et al.*, "Distributed low-frequency oscillation damping in low-voltage islanded multi-bus microgrids with virtual synchronous generators," *IEEE Transactions on Smart Grid*, vol. 15, no. 6, pp. 5331-5345, Nov. 2024.
- [11] J. Liu, Y. Miura, and T. Ise, "Comparison of dynamic characteristics between virtual synchronous generator and droop control in inverter-based distributed generators," *IEEE Transactions on Power Electronics*, vol. 31, no. 5, pp. 3600-3611, May 2016.
- [12] Z. Shuai, W. Huang, Z. J. Shen *et al.*, "Active power oscillation and suppression techniques between two parallel synchronverters during load fluctuations," *IEEE Transactions on Power Electronics*, vol. 35, no. 4, pp. 4127-4142, Apr. 2020.
- [13] M. Chen, D. Zhou, C. Wu *et al.*, "Characteristics of parallel inverters applying virtual synchronous generator control," *IEEE Transactions on Smart Grid*, vol. 12, no. 6, pp. 4690-4701, Nov. 2021.
- [14] B. Yang, H. Li, S. Xu *et al.*, "Systematic methods to eliminate the transient circulating powers in the multi-VSGs system," *IEEE Transactions on Smart Grid*, vol. 15, no. 1, pp. 179-190, Jan. 2024.
- [15] S. Chen, H. Han, Z. Wu *et al.*, "Distributed frequency interactive damping control for multiple VSGs in islanded microgrids," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 71, no. 8, pp. 3867-3879, Aug. 2024.
- [16] X. Li, S. Huang, Z. Wang *et al.*, "Decentralized frequency restoration and power oscillation damping control for islanded microgrids with multiple VSGs," *Journal of Modern Power Systems and Clean Energy*, vol. 13, no. 2, pp. 564-573, Mar. 2025.
- [17] M. Yang, Y. Wang, and J. M. Guerrero, "Impact of reactive power control on low-frequency oscillation of virtual synchronous generators," *IEEE Transactions on Power Systems*, vol. 40, no. 4, pp. 3604-3607, Jul. 2025.
- [18] N. Pogaku, M. Prodanovic, and T. C. Green, "Modeling, analysis and testing of autonomous operation of an inverter-based microgrid," *IEEE Transactions on Power Electronics*, vol. 22, no. 2, pp. 613-625, Mar. 2007.
- [19] M. Li, Y. Wang, W. Hu *et al.*, "Unified modeling and analysis of dynamic power coupling for grid-forming converters," *IEEE Transactions on Power Electronics*, vol. 37, no. 2, pp. 2321-2337, Feb. 2022.
- [20] S. Fu, Y. Sun, J. Lin *et al.*, " $P/Q-\omega/V$ admittance modeling and oscillation analysis for multi-VSG grid-connected system," *IEEE Transactions on Power Systems*, vol. 38, no. 6, pp. 5849-5859, Nov. 2023.
- [21] R. Wang, Y. Wang, P. Zhang *et al.*, "System modeling and robust stability region analysis for multi-inverters based on VSG," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 12, no. 6, pp. 6042-6052, Dec. 2024.
- [22] Z. Shuai, Y. Hu, Y. Peng *et al.*, "Dynamic stability analysis of synchronverter-dominated microgrid based on bifurcation theory," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 9, pp. 7467-7477, Sept. 2017.
- [23] J. Yu, S. Wang, Z. Liu *et al.*, "Accurate small-signal terminal characteristic model and SISO stability analysis approach for parallel grid-forming inverters in islanded microgrids," *IEEE Transactions on Power Electronics*, vol. 38, no. 5, pp. 6597-6612, May 2023.
- [24] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*. Hoboken: Wiley, 2001.
- [25] K. Zhou and J. C. Doyle, *Essentials of Robust Control*. Upper Saddle River: Prentice-Hall, 1998.
- [26] R. Rosso, S. Engelken, and M. Liserre, "Robust stability analysis of synchronverters operating in parallel," *IEEE Transactions on Power Electronics*, vol. 34, no. 11, pp. 11309-11319, Nov. 2019.
- [27] D. B. Rathnayake, S. P. Me, R. Razzaghi *et al.*, " H_∞ -based control design for grid-forming inverters with enhanced damping and virtual inertia," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 11, no. 2, pp. 2311-2325, Apr. 2023.
- [28] A. Singh, V. Debusschere, N. Hadjsaid *et al.*, "Slow-interaction converter-driven stability in the distribution grid: small-signal stability analysis with grid-following and grid-forming inverters," *IEEE Transactions on Power Systems*, vol. 39, no. 2, pp. 4521-4536, Mar. 2024.
- [29] M. A. Guerrero, B. Lakshminarayanan, and C. R. Rojas, "Data-driven estimation of structured singular values," *IEEE Control Systems Letters*, vol. 9, pp. 1976-1981, Jun. 2025.
- [30] Y. Chen, S. Pan, M. Huang *et al.*, "Robust stability assessment of single-phase inverter with multiparameter distributions," *IEEE Transactions on Power Electronics*, vol. 37, no. 5, pp. 6062-6073, May 2022.
- [31] S. Sumsurooah, M. Odavic, and S. Bozhko, " μ approach to robust stability domains in the space of parametric uncertainties for a power system with ideal CPL," *IEEE Transactions on Power Electronics*, vol. 33, no. 1, pp. 833-844, Jan. 2018.
- [32] M. Naderi, Y. Khayat, Q. Shafiee *et al.*, "Robust synchronization of multiple converter-based weak microgrids for smooth interconnection," *IEEE Transactions on Power Systems*, vol. 39, no. 2, pp. 2763-2774, Mar. 2024.
- [33] S. Chakraborty, M. T. Rana, and M. V. Salapaka, "A robust power hardware-in-the-loop interface under uncertain software and hardware system," *IEEE Transactions on Industrial Electronics*, vol. 72, no. 6, pp. 6088-6102, Jun. 2025.
- [34] Mathworks Inc. (2025, Mar.). Matlab robust control toolbox user's guide. [Online]. Available: http://www.mathworks.com/help/pdf_doc/robust/robust Ug.pdf
- [35] P. Gahinet and P. Apkarian, "Structured H_∞ synthesis in MATLAB," in *Proceedings of the 18th World Congress of the International Federation of Automatic Control*, Milano, Italy, Aug.-Sept. 2011, pp. 1435-1440.
- [36] J. Zhao, J. Dong, H. Cheng *et al.*, "Power oscillation analysis and suppression for paralleled system of diesel generator and virtual synchronous generator integrated with periodic pulsed load," *Journal of Modern Power Systems and Clean Energy*, vol. 14, no. 2, pp. 416-429,

Mar. 2026.

- [37] M. Ghafouri, U. Karaagac, H. Karimi *et al.*, “Robust subsynchronous interaction damping controller for DFIG-based wind farms,” *Journal of Modern Power Systems and Clean Energy*, vol. 7, no. 6, pp. 1663-1674, Nov. 2019.

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