

Frequency-constrained Economic Dispatch via Physics-informed Piecewise Linearization Under $N-1$ Contingencies

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Abstract—This paper presents a frequency constrained economic dispatch (FCED) method designed to prevent frequency violations under $N-1$ contingencies, encompassing both generator outages and high-voltage direct current (HVDC) infeed losses. Firstly, the coordinated frequency dynamics of synchronous generators, wind turbines, and HVDC lines are modeled, and the resulting frequency constraints are derived. To address the challenging frequency nadir (FN) constraint, a physics-informed piecewise linearization (PI-PWL) model is introduced to learn the FN security boundary. The formulation of the proposed FCED method is then presented, incorporating frequency constraints under $N-1$ contingencies. Additionally, an error-aware constraint cut (EACC) is proposed to eliminate errors introduced by data-driven methods and accelerate solution convergence. Case studies on the modified IEEE 39-bus and IEEE 118-bus test systems validate the effectiveness of the proposed FCED method. Results indicate that the proposed PI-PWL model more accurately captures the FN security boundary than the traditional PWL (T-PWL) model, particularly in scenarios with limited unstable samples. Moreover, the proposed FCED method ensures frequency security under $N-1$ contingencies, while the EACC effectively eliminates approximation errors and accelerates convergence compared with fixed-step constraint cuts.

Index Terms—Data-driven, frequency-constrained economic dispatch (FCED), frequency nadir, frequency security, $N-1$ contingency, piecewise linearization, error-aware constraint cut (EACC).

I. INTRODUCTION

THE integration of high-voltage direct current (HVDC) transmission and high shares of renewable energy in power system has enhanced its economic and low-carbon operation [1]. However, this also poses a threat to power sys-

tem stability, particularly with regard to frequency security issues due to reduced rotational inertia and insufficient primary frequency response (PFR) reserve [2], [3]. Addressing this challenge can be approached from two levels: system-level operation and device-level control. At the system level, the frequency security criteria under $N-1$ contingencies are incorporated as additional constraints in the power system dispatch problem, aiming to minimize economic (and environmental) costs while coordinating the optimization of generation, reserve, and PFR parameters of inverter-based resources (virtual inertia, droop, etc.) [4]–[8]. At the device level, advanced control algorithms are designed to provide faster and more efficient frequency support following contingencies. This paper focuses on the system level and proposes a frequency constrained economic dispatch (FCED) method.

In the field of frequency constrained power system dispatch, efficiently embedding the highly complex frequency nadir (FN) constraint has long been an open issue. In recent years, researchers have primarily tackled this challenge through two main approaches: model-driven and data-driven approaches. The model-driven derivation of FN constraints often results in convex formulations. A stochastic scheduling approach with inertia-dependent fast frequency response requirements is proposed in [9]. In [10], the PFR support of wind turbines (WTs) is considered, but their PFR delivery time is assumed to be the same as that of synchronous generators (SGs), which clearly underestimates the fast response capability of WTs. A frequency response model with varying PFR dead zones and delivery time is established in [11], leading to the FN constraint in second-order cone form, but system damping is overlooked. Overall, model-driven approaches often require approximations and simplifications in order to derive convex FN constraints, limiting their ability to fully capture frequency dynamics.

In contrast, data-driven approaches enable the incorporation of more complex frequency response models through powerful machine learning models for end-to-end regression or classification. Support vector machine (SVM) model is employed in [12] to extract the relationship among system inertia, damping, SG parameters, and FN security label. However, at low inertia levels, the FN security boundary becomes highly nonlinear, which significantly reduces the effectiveness of SVM. In our previous work [13], a tool called the alternate SVM decision tree [14] is used, but the intro-

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duction of excessive binary variables makes dispatch model solving quite challenging. Recently, deep neural networks (DNNs) are also used to extract power system stability constraints [15]. However, the ReLU activation function leads to an explosion of binary variables, further diminishing their practicality. Piecewise linearization (PWL), a simple and practical model, fits the security boundary using multiple hyperplanes trained from data and remains the most favored approach to date.

The traditional PWL (T-PWL) model employs the mean squared fitting error as its loss function and can be mathematically reformulated as a mixed-integer quadratic programming (MIQP) problem. In [16], the T-PWL model is applied to the frequency-constrained unit commitment problem. Furthermore, to improve the robustness of T-PWL, a fitting criterion is introduced, where the fitted values are constrained to be less than the actual values [17]. To reduce fitting errors in low-inertia condition, an iterative “dispatch model solving–parameter range updating–PWL training” approach is adopted in [18]. In [19], a heuristic method is designed to determine the optimal number of segments. These studies have made significant contributions to the improvement in accuracy of T-PWL model. However, they do not account for the physical laws reflecting the frequency dynamics, particularly the swing equation, during the training process. The integration of physical information into data-driven models has gained widespread attention and research in recent years, such as physics-informed neural networks (PINN) [20]. To the best of our knowledge, embedding physical information that captures frequency dynamics into the PWL model remains an unexplored area. This requires identifying the physical laws corresponding to FN while ensuring that the PWL model, after incorporating physical information, remains solvable by convex optimization algorithms.

On the other hand, the majority of frequency-constrained power system dispatch studies currently assume that power imbalances are known a priori when modeling frequency constraints such as maximum generator capacity [21], [22], a practice typically applied to high-inertia power systems. As the inertia level decreases, the impact of lost rotational inertia following an $N-1$ contingency becomes more significant on frequency dynamics. A bottleneck generator identification method is proposed in [23], but this study does not account for the effects of HVDC infeed loss and the support provided by virtual inertia and droop coefficient. In our previous work [13], frequency constraints under HVDC infeed loss contingencies are considered, however, the transmission power of HVDC is assumed given rather than optimized, and the effect of HVDC in providing frequency support is overlooked.

Based on the literature review and identified research gaps, we propose an FCED method to ensure frequency security under $N-1$ contingencies. The contributions are as follows:

1) A physics-informed PWL (PI-PWL) model is introduced, where the swing equation at the FN is embedded as a physical regularization term during the PI-PWL model training process.

Compared with T-PWL model, the proposed PI-PWL model demonstrates superior classification performance and generalization capability.

2) Unlike studies that ignore the loss of frequency regulation capability under $N-1$ contingencies [21], [22], we account for the combined impacts of power imbalance, inertia loss, and PFR reserve loss on frequency dynamics following SG and HVDC contingencies. Existing studies [23] that consider inertia and reserve losses often assume fixed inertia and damping, overlooking the schedulable flexibility of inverter-based resources. In contrast, we co-optimize HVDC virtual inertia and droop coefficient, enabling optimal settings that balance frequency support with post-contingency risk.

3) An error-aware constraint cut (EACC) is designed to eliminate errors introduced by the proposed PI-PWL model. In comparison to fixed-step constraint cuts, the proposed EACC strikes an optimal balance between convergence speed and economic efficiency.

The remainder of this paper is organized as follows. Section II introduces the proposed PI-PWL model. Section III presents the formulation of proposed FCED method. The EACC is proposed in Section IV. Section V provides the case study and Section VI draws the conclusion.

II. PROPOSED PI-PWL MODEL

A. Modeling of Frequency Dynamics

Following the approach adopted in the majority of studies, we focus on the center of inertia frequency as the analysis subject. After the occurrence of a contingency, the frequency dynamics are described as:

$$\frac{2H^{\text{sys}}}{f_0} \frac{d\Delta f(\tau)}{d\tau} + D^{\text{sys}} \Delta f(\tau) = \Delta P_{\text{PFR}}(\tau) - \Delta P^{\text{imb}} \quad (1)$$

where H^{sys} is the system inertia, which is considered to be composed of the inertia from SGs and the virtual inertia of HVDC [4]; f_0 is the nominal frequency; τ is the index of frequency response time; $\Delta f(\tau)$ is the frequency deviation; D^{sys} is the system damping, which is considered to be composed of load damping and HVDC droop; ΔP^{imb} is the power imbalance; and $\Delta P_{\text{PFR}}(\tau)$ is the total PFR power output, which is piecewise linearized [11], [24] as:

$$\Delta P_{\text{PFR}}(\tau) = \begin{cases} 0 & 0 < \tau \leq t_w \\ \frac{R^w}{T_w}(\tau - t_w) & t_w < \tau \leq t_G \\ \frac{R^w}{T_w}(\tau - t_w) + \frac{R^G}{T_G}(\tau - t_G) & t_G < \tau \leq t_w + T_w \\ R^w + \frac{R^G}{T_G}(\tau - t_G) & t_w + T_w < \tau \leq t_G + T_G \\ R^w + R^G & t_G + T_G < \tau \end{cases} \quad (2)$$

where t_w and T_w are the PFR deadband and delivery time for WTs, respectively; and t_G and T_G are the PFR deadband and delivery time for SGs, respectively, with R^w and R^G denoting their respective PFR reserves.

By substituting (2) into (1), $\Delta f(\tau)$ can be represented as:

$$\Delta f(\tau) = \begin{cases} \frac{c_1}{b} \left(1 - e^{-\frac{b}{a}\tau}\right) & 0 < \tau \leq t_W \\ \lambda_2 e^{-\frac{b}{a}\tau} + \frac{k_2}{b} \tau + \frac{c_2 b - a k_2}{b^2} & t_W < \tau \leq t_G \\ \lambda_3 e^{-\frac{b}{a}\tau} + \frac{k_3}{b} \tau + \frac{c_3 b - a k_3}{b^2} & t_G < \tau \leq t_W + T_W \\ \lambda_4 e^{-\frac{b}{a}\tau} + \frac{k_4}{b} \tau + \frac{c_4 b - a k_4}{b^2} & t_W + T_W < \tau \leq t_G + T_G \\ \lambda_5 e^{-\frac{b}{a}\tau} + \frac{c_5}{b} & t_G + T_G < \tau \end{cases} \quad (3)$$

where $a, b, c_1-c_5, \lambda_1-\lambda_5$, and k_2-k_4 are the intermediate parameters. The specific expressions of these parameters in (3) are provided in [25].

In the frequency dynamics of power system, the initial rate of change of frequency (RoCoF), FN, and quasi-steady-state frequency deviation (QSSFD) are three key metrics. The corresponding constraints can be expressed as:

$$\frac{\Delta P^{imb} f_0}{2H^{sys}} \leq \overline{RoCoF} \quad (4)$$

$$f_{nadir} \leq \varphi(\Delta P^{imb}, H^{sys}, D_n^{sys}, R_n^W, R_n^G) \quad (5)$$

$$\Delta P^{imb} \leq R_n^W + R_n^G + D_n^{sys} \overline{\Delta f_{qss}} \quad (6)$$

where $\overline{RoCoF}$, f_{nadir} , and $\overline{\Delta f_{qss}}$ are the limits of RoCoF, FN, and QSSFD, respectively; and $\varphi(\cdot)$ is the FN expression, which is determined by five variables.

It is worth noting that this paper mainly addresses low-frequency issues, as they are generally more critical and cannot be mitigated by disconnecting generators as is done during high-frequency events. Nevertheless, the proposed FCED method is equally applicable to high-frequency problems through symmetric modeling, without affecting its extensibility.

B. Embedding of FN Constraint via PI-PWL

The RoCoF and QSSFD constraints are linear and can be directly embedded into the convex optimization model. However, the FN expression is highly complex and difficult to convexify using traditional mathematical methods. Therefore, we propose employing a data-driven approach to tackle it. First, we review the training process of T-PWL, as shown in (7).

$$\begin{cases} \min \left[\sum_{n=1}^N \left(\varphi(X_n) - \tilde{\varphi}(X_n) \right)^2 \right] \\ \text{s.t. } \tilde{\varphi}(X_n) \geq a_k^T X_n + b_k \quad \forall k \\ \tilde{\varphi}(X_n) \leq a_k^T X_n + b_k + (1 - v_k) M \quad \forall k \\ \sum_{k=1}^K v_k = 1 \end{cases} \quad (7)$$

where N and K are the numbers of samples and segments, respectively; the subscripts n and k denote the n^{th} sample and k^{th} segment, respectively; X_n is the feature vector, given by

$X_n = [P_n^{imb}, H_n^{sys}, D_n^{sys}, R_n^W, R_n^G]^T$; $\tilde{\varphi}(\cdot)$ is the fitted value of $\varphi(\cdot)$; v_k is a binary variable; M is a large positive constant; and a_k and b_k are the slope and intercept, respectively, which are optimized variables.

The problem in (6) is essentially an MIQP problem, which, theoretically, provides the optimal fitting performance for the given set of samples compared to other regression techniques. However, it lacks generalizability and neglects the physical information reflected in the data.

In conventional physics-informed model training such as PINN, the objective typically involves fitting the time-domain solution of differential equations (e.g., generator rotor angle equations) [20] by incorporating the differential residuals as a regularization term. In contrast, this study focuses on learning the numerical value of the FN, for which no differential process is involved. Consequently, instead of using differential residuals, we embed the residual of the algebraic physical equation corresponding to the FN into the loss function. This enables the enforcement of physical consistency in the absence of differential terms:

$$\min \left[\sum_{n=1}^N \left(\varphi(X_n) - \tilde{\varphi}(X_n) \right)^2 + \lambda L_{sw} \right] \quad (8)$$

where L_{sw} is the physical regularization term corresponding to the swing equation; and λ is a hyperparameter controlling the weight of the physical regularization term in the optimization problem.

We focus only on the frequency dynamics when the frequency reaches the nadir, denoted as τ^* . At τ^* , the regularization term L_{sw} becomes:

$$L_{sw} = \sum_{n=1}^N \left(\Delta P_n^{imb} - \Delta P_{PFRn} - D_n^{sys} \tilde{\varphi}(X_n) \right)^2 \quad (9)$$

ΔP_{PFRn} comprises the combined contributions from SGs and WTs, which is expressed as:

$$\Delta P_{PFRn} = \min \left(\frac{R_n^G}{T_G} \left(\max(\tau_n^*, t_G) - t_G \right), R_n^G \right) + \min \left(\frac{R_n^W}{T_W} \left(\max(\tau_n^*, t_W) - t_W \right), R_n^W \right) \quad (10)$$

In (9), the summation term represents the cumulative violation of the swing equation at the instant when the FN is reached, across all samples. At this moment, the derivative of the frequency deviation is zero, implying that the first term on the left-hand side of (1) vanishes, which yields the corresponding algebraic equation (9). Physically, this condition reflects energy conservation in the frequency dynamics, where the sum of the PFR power output ΔP_{PFRn} and the system damping power $D_n^{sys} \tilde{\varphi}(X_n)$ equals the power imbalance ΔP_n^{imb} . Incorporating the regularization term L_{sw} enforces physical consistency in the learned FN constraint, improves generalization when training data are limited, and prevents the model from capturing non-physical relationships.

Finally, the proposed PI-PWL model can be formulated as the following multi-objective MIQP problem:

$$\begin{cases} \min \left\{ \sum_{n=1}^N \left[\left(\varphi(X_n) - \tilde{\varphi}(X_n) \right)^2 + \right. \right. \\ \quad \left. \left. \lambda \left(\Delta P_n^{imb} - \Delta P_{PFRn} - D_n^{sys} \tilde{\varphi}(X_n) \right)^2 \right] \right\} \\ \text{s.t. } \tilde{\varphi}(X_n) \geq \mathbf{a}_k^T X_n + b_k \quad \forall k \\ \tilde{\varphi}(X_n) \leq \mathbf{a}_k^T X_n + b_k + (1 - v_k)M \quad \forall k \\ \sum_{k=1}^K v_k = 1 \end{cases} \quad (11)$$

After solving problem (11), the FN security boundary is piecewise linearly approximated. At this point, the FN constraint can be restated as:

$$\mathbf{a}_k^T \mathbf{X} + b_k \leq \Delta f_{\max} \quad \forall k \quad (12)$$

where $\Delta f_{\max} = f_0 - f_{\text{nadir}}$; and $\mathbf{X}$ is the feature vector.

Remark 1: we choose to linearize PFR power output because system operators typically do not rely on detailed generator dynamic parameters for scheduling [23]. However, the proposed PI-PWL model can learn FN constraints based on more detailed frequency dynamic models [26]. Moreover, in existing studies on frequency-constrained scheduling, this assumption has been widely adopted and has been shown to be approximately conservative compared with detailed dynamic models, which is beneficial for making more secure scheduling decisions [9]-[11].

Remark 2: although the training of the PWL model is formulated as a regression problem, the objective is to approximate the FN security boundary rather than the entire FN value range. Therefore, the performance of the PWL model is evaluated using classification metrics such as accuracy rather than regression metrics such as mean squared error.

III. FORMULATION OF PROPOSED FCED METHOD

A. Objective Function

The proposed FCED method aims to minimize the sum of the operation costs of SGs (including generation and PFR reserve costs), power infeed and PFR reserve costs of HVDC, deloading costs of WTs, and load shedding costs, as shown in (13).

$$\begin{aligned} \min & \left[\sum_{t \in T} \sum_{g \in G} \left(c_g^{GEN} P_{g,t}^{GEN} + c_g^{PFR} R_{g,t} \right) + \right. \\ & \sum_{t \in T} \sum_{h \in H} \left(c_{h,t}^{PE} P_{h,t}^{DC} + c_{h,t}^{PFR} R_{h,t} \right) + \\ & \left. \sum_{t \in T} \sum_{w \in W} c_w^{Del} P_{w,t}^{Del} + \sum_{t \in T} \sum_{d \in D} c_{d,t}^{Shed} P_{d,t}^{Shed} \right] \end{aligned} \quad (13)$$

where T , G , H , W , and D are the sets of time periods, SGs, HVDC lines, WTs, and loads, respectively; c_g^{GEN} and c_g^{PFR} are the generation and PFR reserve prices of SG g , respectively; $P_{g,t}^{GEN}$ and $R_{g,t}$ are the generation and PFR reserve power of SG g during time period t , respectively; $c_{h,t}^{PE}$ and $c_{h,t}^{PFR}$ are the power infeed price and PFR reserve price of HVDC line h during time period t , respectively; $P_{h,t}^{DC}$ and $R_{h,t}$ are the infeed power and PFR reserve power of HVDC line h during

time period t , respectively; c_w^{Del} and $P_{w,t}^{Del}$ are the deloading price and deloading power of WT w during time period t , respectively; and $c_{d,t}^{Shed}$ and $P_{d,t}^{Shed}$ are the shedding price and shedding power of load d during time period t , respectively.

B. Constraints

1) SG Constraints

The SG constraints include the maximum and minimum power output constraints (14) and (15), as well as ramp constraints (16) and (17).

$$P_{g,t}^{GEN} + R_{g,t} \leq P_g^{\max} \quad \forall g \in G, \forall t \in T \quad (14)$$

$$P_g^{\min} \leq P_{g,t}^{GEN} \quad \forall g \in G, \forall t \in T \quad (15)$$

$$P_{g,t}^{GEN} + R_{g,t} - P_{g,t-1}^{GEN} \leq RU_g \quad \forall g \in G, \forall t \in T \quad (16)$$

$$P_{g,t-1}^{GEN} + R_{g,t-1} - P_{g,t}^{GEN} \leq RD_g \quad \forall g \in G, \forall t \in T \quad (17)$$

where $P_g^{\max}$ and $P_g^{\min}$ are the maximum and minimum power output limits of SG g , respectively; and RU_g and RD_g are the maximum upward and downward ramp rate limits of SG g , respectively.

2) WT Constraints

WTs can operate in a deloading mode to provide PFR reserve:

$$P_{w,t}^{GEN} + P_{w,t}^{Del} = P_{w,t}^{Fore} \quad \forall w \in W, \forall t \in T \quad (18)$$

$$R_{w,t} \leq P_{w,t}^{Del} \quad \forall w \in W, \forall t \in T \quad (19)$$

where $P_{w,t}^{Fore}$ and $R_{w,t}$ are the forecast power and PFR reserve of WT w during time period t , respectively.

3) HVDC Line Constraints

This paper considers the case where HVDC injects power into the receiving grid. The steady-state operation constraints for HVDC line mainly include the maximum and minimum power transmission limit (20), the maximum and minimum limits of upward power adjustment (21) and downward power adjustment (22), mutually exclusive adjustment direction constraint (23), mutual exclusion constraints for adjustment direction during adjacent periods (24) and (25), and the upper limit of the number of power adjustment actions (26):

$$P_h^{\min} \leq P_{h,t}^{DC} \leq P_h^{\max} \quad \forall h \in H, \forall t \in T \quad (20)$$

$$\begin{aligned} -Mx_{h,t}^- + x_{h,t}^+ r_{h,\min}^+ & \leq P_{h,t}^{DC} - P_{h,t-1}^{DC} \leq \\ Mx_{h,t}^- + x_{h,t}^+ r_{h,\max}^+ & \quad \forall h \in H, \forall t \in T \end{aligned} \quad (21)$$

$$\begin{aligned} -Mx_{h,t}^+ + x_{h,t}^- r_{h,\min}^- & \leq P_{h,t}^{DC} - P_{h,t-1}^{DC} \leq \\ Mx_{h,t}^+ + x_{h,t}^- r_{h,\max}^- & \quad \forall h \in H, \forall t \in T \end{aligned} \quad (22)$$

$$x_{h,t}^+ + x_{h,t}^- \leq 1 \quad \forall h \in H, \forall t \in T \quad (23)$$

$$x_{h,t}^+ + x_{h,t-1}^- \leq 1 \quad \forall h \in H, \forall t \in T \quad (24)$$

$$x_{h,t-1}^+ + x_{h,t}^- \leq 1 \quad \forall h \in H, \forall t \in T \quad (25)$$

$$\sum_{t \in T} (x_{h,t}^+ + x_{h,t}^-) \leq X_{h,\max}^{DC} \quad \forall h \in H \quad (26)$$

where $P_h^{\min}$ and $P_h^{\max}$ are the minimum and maximum transmission power of HVDC line h , respectively; $X_{h,\max}^{DC}$ is the upper limit of the number of power adjustment actions of HVDC line h ; $x_{h,t}^+$ and $x_{h,t}^-$ are the binary variables that equal 1 when HVDC line h increases and decreases its power dur-

ing time period t , respectively; $r_{h,\max}^+$ and $r_{h,\max}^-$ are the maximum limits of upward and downward power adjustments of HVDC line h , respectively; and $r_{h,\min}^+$ and $r_{h,\min}^-$ are the minimum limits of the upward and downward power adjustments of HVDC line h , respectively.

In the HVDC transient frequency support modeling part, we consider its participation in the power system frequency response through virtual inertia and droop control [27]. To ensure the availability of energy for frequency support after a contingency, a certain amount of PFR capacity should be reserved. According to [28], the PFR reserve constraint can be conservatively approximated as:

$$\frac{2H_{h,t}P_h^{\max}\overline{RoCoF}}{f_0} + \kappa_{h,t}P_h^{\max}\Delta f_{\max} \leq R_{h,t} \quad \forall h \in H, \forall t \in T \quad (27)$$

where $H_{h,t}$ and $\kappa_{h,t}$ are the virtual inertia and droop coefficient set by HVDC line h during time period t , respectively.

Given that HVDC is allowed to overload power for a short duration when participating in frequency response, we have:

$$P_{h,t}^{DC} + \frac{2H_{h,t}P_h^{\max}\overline{RoCoF}}{f_0} + \kappa_{h,t}P_h^{\max}\Delta f_{\max} \leq (1+\rho_h)P_h^{\max} \quad \forall h \in H, \forall t \in T \quad (28)$$

where ρ_h is the overload factor of HVDC line h .

4) System Constraints

The system constraints include power balance and AC transmission line capacity constraints, as shown in (29) and (30), respectively.

$$\sum_{g \in G} P_{g,t}^{GEN} + \sum_{w \in W} P_{w,t}^{GEN} + \sum_{h \in H} P_{h,t}^{DC} = \sum_{d \in D} (P_{d,t}^L - P_{d,t}^{Shed}) \quad \forall t \in T \quad (29)$$

$$-P_l^{\max} \leq \sum_{g \in G} \pi_{g,l} P_{g,t}^{GEN} + \sum_{w \in W} \pi_{w,l} P_{w,t}^{GEN} + \sum_{h \in H} \pi_{h,l} P_{h,t}^{DC} - \sum_{d \in D} \pi_{d,l} (P_{d,t}^L - P_{d,t}^{Shed}) \leq P_l^{\max} \quad \forall l \in E, \forall t \in T \quad (30)$$

where E is the set of AC transmission lines; $P_{d,t}^L$ is the power demand of load d during time period t ; $\pi_{g,l}$, $\pi_{w,l}$, $\pi_{h,l}$ and $\pi_{d,l}$ are the power transfer distribution factors of SG g , WT w , HVDC line h , and load d on AC transmission line l , respectively; and $P_l^{\max}$ is the transmission capacity of AC transmission line l .

5) Frequency Constraints

Based on the three frequency constraints derived in Section II, we extend them to any $N-1$ contingency scenario during all time periods:

$$\frac{\Delta P_{s,t}^{imb} f_0}{2H_{s,t}^{sys}} \leq \overline{RoCoF} \quad \forall s \in S, \forall t \in T \quad (31)$$

$$\max_{k=1,2,\dots,K} (a_k^T X_{s,t} + b_k) \leq \Delta f_{\max} \quad \forall s \in S, \forall t \in T \quad (32)$$

$$\Delta P_{s,t}^{imb} \leq R_{s,t}^W + R_{s,t}^G + D_{s,t}^{sys} \overline{\Delta f_{qss}} \quad \forall s \in S, \forall t \in T \quad (33)$$

where S is the set of $N-1$ contingency scenarios, which include both generator outage and HVDC infeed loss contingency, represented by the sets S_{SG} and S_{DC} , respectively, and $S_{SG} \cup S_{DC} = S$; and the subscript s denotes the s^{th} scenario.

When the generator outage occurs, we have:

$$\begin{cases} \Delta P_{s,t}^{imb} = P_{s,t}^{GEN} \\ H_{s,t}^{sys} = \sum_{g \in G, g \neq s} P_g^{\max} H_g + \sum_{h \in H} P_h^{\max} H_{h,t} \\ D_{s,t}^{sys} = \sum_{d \in D} k_D (P_{d,t}^L - P_{d,t}^{Shed}) + \sum_{h \in H} P_h^{\max} \kappa_{h,t} \\ R_{s,t}^W = \sum_{w \in W} R_{w,t} \\ R_{s,t}^G = \sum_{g \in G, g \neq s} R_{g,t} \end{cases} \quad \forall s \in S_{SG} \quad (34)$$

When the HVDC infeed loss contingency occurs, we have:

$$\begin{cases} \Delta P_{s,t}^{imb} = P_{s,t}^{DC} \\ H_{s,t}^{sys} = \sum_{g \in G} P_g^{\max} H_g + \sum_{h \in H, h \neq s} P_h^{\max} H_{h,t} \\ D_{s,t}^{sys} = \sum_{d \in D} k_D (P_{d,t}^L - P_{d,t}^{Shed}) + \sum_{h \in H, h \neq s} P_h^{\max} \kappa_{h,t} \\ R_{s,t}^W = \sum_{w \in W} R_{w,t} \\ R_{s,t}^G = \sum_{g \in G} R_{g,t} \end{cases} \quad \forall s \in S_{DC} \quad (35)$$

where k_D is the load damping rate.

It is important to clarify that the $N-1$ contingency scenarios considered in this paper only include the sudden loss of SGs and DC injection. The frequency constraints ((31)-(33)) ensure the deliverability of reserves for each contingency scenario s . Since the frequency analyzed in this study is the center-of-inertia (COI) frequency, the localized frequency deviations caused by AC transmission line outages are beyond the scope of this study and are left for future research.

C. Full Formulation of Proposed FCED Method

Based on the previous introduction in this section, the full formulation of the proposed FCED method can be expressed as the following mixed-integer linear programming (MILP) problem:

$$\begin{cases} \min_{\theta} & (13) \\ \text{s.t.} & (14)-(33) \end{cases} \quad (36)$$

$$\Theta = \{P_{g,t}^{GEN}, R_{g,t}, P_{h,t}^{DC}, R_{h,t}, H_{h,t}, \kappa_{h,t}, x_{h,t}^+, x_{h,t}^-, P_{w,t}^{Del}, R_{w,t}, P_{d,t}^{Shed}\} \quad (37)$$

Remark 3: to emphasize the main contributions of this paper, we do not consider wind power uncertainty. However, the proposed FCED method can be effectively integrated with stochastic optimization, robust optimization, and distributionally robust optimization methods.

IV. PROPOSED EACC

The direct solution of the proposed FCED method (36) cannot strictly guarantee that the FN remains above the specified threshold under any $N-1$ contingency due to the inherent inaccuracies of data-driven methods. In this section, we propose the EACC, which aims to eliminate unsafe scenarios.

The design of the proposed EACC is based on the premise that the security boundary of a power system is typically close to convex [14]. This implies that by continually reducing $a_k^T X + b_k$, we can approximate the FN security boundary as follows:

$$\mathbf{a}_k^T \mathbf{X} + b_k \leq \zeta_i \cdot \max_{k=1,2,\dots,K} (\mathbf{a}_k^T \mathbf{X}_i^* + b_k) \quad \forall k \quad (38)$$

where ζ_i is the reduction step size of the i^{th} iteration, controlling the convergence speed and economic cost; and $\mathbf{X}_i^*$ is the result obtained from the i^{th} iteration.

On one hand, the degree of reduction has traditionally been designed using a fixed step size [29], which can lead to slow convergence in scenarios with large errors. On the other hand, it can result in overly conservative outcomes, causing excessively high economic costs or even making the scheduling infeasible.

Our goal is to choose an appropriate value for ζ_i . This ensures that when the FN calculated after the i^{th} iteration is far from the threshold, a larger ζ_i is used to converge quickly toward the boundary. Conversely, when approaching the boundary, a smaller ζ_i is applied to prevent excessive step sizes, which could lead to higher economic costs. To achieve this, we propose the EACC. Specifically, we express ζ_i as a function related to the FN error:

$$\zeta_i = \max \left(\min \left(\zeta^{\max} e^{-\beta \frac{FN_i - f_{\text{nadir}}}{f_{\text{nadir}}}}, \zeta^{\max} \right), \zeta^{\min} \right) \quad (39)$$

where FN_i is the FN calculated after the i^{th} iteration; $\zeta^{\max}$ and $\zeta^{\min}$ are the upper and lower bounds of ζ_i , respectively; and β is a positive constant that controls the rate of decay.

Through (39), ζ_i can be adjusted freely within its upper and lower bounds based on the current FN value. This provides flexibility for constraint cutting generation in different error scenarios. The presence of ζ_i drives the optimization decision towards the security boundary by increasing the PFR reserves, system inertia, and system damping in the feature vector $\mathbf{X}$, or by reducing the power imbalance. Although this is a heuristic design, it is consistent with the fact that the frequency security boundary in power systems is typically close to convex [14]. Hence, more reserves, inertia, and damping, or a smaller power imbalance, will generally lead to a smaller FN, and the process will converge after a finite number of iterations due to the monotonic movement of the solution towards the security boundary.

In summary, the solution framework of the proposed FCED method can be divided into three parts: optimization solution, FN verification, and EACC generation, as shown in Fig. 1.

Remark 4: for practical implementation of the proposed FCED method, system operators are advised to proceed as follows.

1) Collect all fundamental parameters, including forecasted wind power and load profiles, SG parameters (e.g., generation costs, inertia constants), and HVDC parameters (e.g., transfer capacities).

2) Based on the collected data, determine the upper and lower bounds of the feature vector $\mathbf{X}$ in the proposed PI-PWL model. For example, the upper and lower bounds of the power imbalance ΔP^{imb} can be set to the maximum and minimum capacities among all relevant devices. Within these bounds, generate samples for training the proposed PI-PWL model (11) and obtain the linear surrogate (12) for the complex FN constraint.

3) Formulate the remaining constraints and the objective function to establish the optimization model (36).

4) Solve the proposed PI-PWL model using the solution framework illustrated in Fig. 1.

5) Output the final dispatch results.

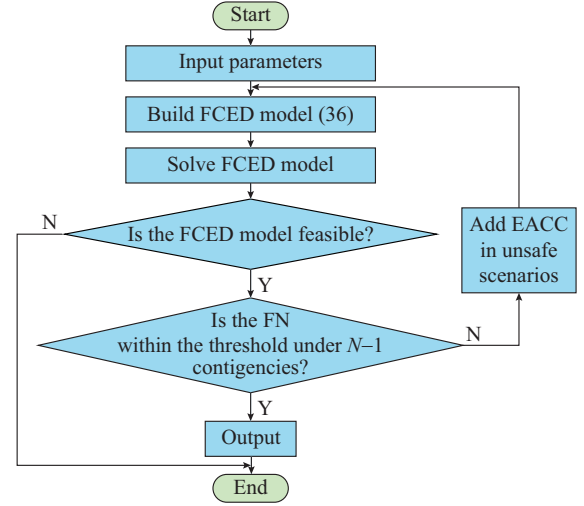


Fig. 1. Solution framework of proposed FCED method.

V. CASE STUDY

In this section, we perform case studies on the modified IEEE 39-bus and IEEE 118-bus test systems. The common parameter information for both systems is as follows: the dispatch horizon spans 24 hours with a 15-min time resolution, resulting in a total of 96 dispatch periods, the nominal frequency is 50 Hz, $RoCoF = 0.5$ Hz/s, $f_{\text{nadir}} = 49.2$ Hz, $\Delta f_{qss} = 0.25$ Hz, $t_w = 0.2$ s, $t_G = 1$ s, $T_w = 2$ s, $T_G = 10$ s, $k_D = 0.03$, and $M = 1 \times 10^5$. The number of segments is $K = 3$.

For comparison purposes, we define two methods.

1) M1: the FCED method with $N-1$ FN constraints.

2) M2: the FCED method without $N-1$ FN constraints, i.e., the method proposed in this paper.

The training of the proposed PI-PWL model and the solving of M1 and M2 are performed on a computer with a 3.00 GHz Intel Core i5-8500 CPU and 16 GB of RAM, using Gurobi 11.0.0 as the solver and Python 3.10.6 as the programming language. The mixed integer programming gap (MIP-Gap) is set to be 0.1%.

A. Validation of Proposed PI-PWL Model

1) Sample Generation

Using the Latin hypercube sampling method, 20000 samples are drawn within the parameter ranges of the characteristic variables listed in Table I, and the corresponding FN values are computed. After applying our previously proposed invalid sample filtering method [4], [13], 17873 samples remain, with approximately 25% of them violating the FN constraint, labeled as 1. This proportion aligns with the reality that insecure operating states are relatively rare in actual power systems [30]. Since our objective is not to fit the entire dataset but to identify a linear surrogate for the FN security boundary, the problem is fundamentally a classification task implemented through regression. Therefore, only sam-

ples within the range of 49.19 Hz to 49.21 Hz are selected to construct the T-PWL and proposed PI-PWL models. Additionally, FN values are scaled by a factor of 1000 to mitigate numerical instability. Based on multiple tests, the hyperparameter λ is set to be 250.

TABLE I
PARAMETER RANGES OF CHARACTERISTIC VARIABLES

Variable	The minimum value	The maximum value
P^{imb} (p.u.)	0	0.15
H^{sys} (s)	3	8
D^{sys} (%)	0.5	5.0
R^W (p.u.)	0	0.1
R^G (p.u.)	0	0.3

2) Classification Performance Comparison

As shown in Table II, the classification performance of the T-PWL and proposed PI-PWL models exhibits significant differences. Accuracy serves as a measure of overall classification performance, while precision evaluates the proportion of correctly identified unsafe samples among those predicted as unsafe (label 1). Recall indicates the proportion of actual unsafe samples correctly identified by the model. The F1 score provides a balanced measure of precision and recall. The results demonstrate that the proposed PI-PWL model outperforms the T-PWL model across all metrics, with particularly notable improvements in precision and F1 score, indicating a more reliable classification capability for unsafe states. Although the T-PWL model achieves a relatively high recall, its precision is considerably lower, suggesting a tendency to misclassify safe states as unsafe (higher false positives). The superior overall performance of the proposed PI-PWL model, especially in accurately identifying unsafe states, makes it a more effective choice when security classification is a critical concern.

TABLE II
CLASSIFICATION PERFORMANCE COMPARISON OF T-PWL AND PROPOSED PI-PWL MODELS

Model	Accuracy	Precision	Recall	F1 score
T-PWL	0.9330	0.8404	0.9115	0.8745
Proposed PI-PWL	0.9725	0.9439	0.9489	0.9464

3) Sensitivity Analysis

By modifying the range of power imbalance, we generate seven datasets with different unsafe sample proportions. We analyze the classification performance of the proposed PI-PWL model under varying proportions and compare its performance with the T-PWL model, as shown in Table III and Fig. 2, where the ratio denotes the relative performance of the proposed PI-PWL model with respect to the T-PWL model across the four evaluation metrics. As shown in Table III, when the proportion of unsafe samples is low (8%-15%), the proposed PI-PWL model exhibits relatively low precision, while recall remains consistently high. This indicates that proposed PI-PWL model effectively identifies un-

safe samples in imbalanced datasets but has a higher false positive rate. As the proportion of unsafe samples increases ($\geq 40\%$), the precision improves significantly, classification performance stabilizes, and the F1 score increases accordingly, reflecting an overall enhancement in recognition capability. Furthermore, at higher proportions (56%-83%), all metrics remain at high levels, demonstrating that the proposed PI-PWL model generalizes well when sufficient unsafe samples are available. These results suggest that under low proportions, the proposed PI-PWL model may be affected by data imbalance, leading to a higher false positive rate, whereas with a higher proportion of unsafe samples, its classification performance becomes more stable and unsafe states can be accurately identified.

TABLE III
CLASSIFICATION PERFORMANCE OF PROPOSED PI-PWL MODEL UNDER VARYING PROPORTIONS

Proportion (%)	Accuracy	Precision	Recall	F1 score
8	0.9776	0.8114	0.9178	0.8613
15	0.9651	0.8679	0.9095	0.8882
28	0.9582	0.9308	0.9193	0.9250
40	0.9628	0.9589	0.9474	0.9531
56	0.9472	0.9783	0.9257	0.9512
69	0.9549	0.9829	0.9509	0.9666
83	0.9714	0.9916	0.9739	0.9827

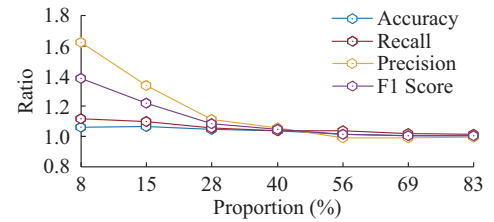


Fig. 2. Comparison of proposed PI-PWL model with T-PWL model in various metrics.

As shown in Fig. 2, when the proportion is low, the proposed PI-PWL model exhibits a more significant advantage, particularly in terms of precision. This indicates that the proposed PI-PWL model more accurately captures the intrinsic characteristics of unsafe states rather than merely relying on data-driven pattern recognition. As the proportion increases, the performance of both models converges. This demonstrates that the proposed PI-PWL model, by incorporating physical information, has a stronger capability in identifying unsafe labels.

B. Modified IEEE 39-bus Test System

In the modified IEEE 39-bus test system, buses 30, 31, 34, 37, and 38 are each connected to one SG (G1-G5), with a total installed capacity of 1242 MW. Buses 36 and 37 are each connected to a 300 MW wind farm (W1 and W2). Buses 18 and 19 are the connection points for two HVDC lines (D1 and D2), each with a transmission capacity of 300 MW. The load demand ranges from 855 MW to 1310 MW. For more detailed parameters, please refer to [25].

1) Economic Cost Comparison of M1 and M2

The economic cost comparison of M1 and M2 in modified IEEE 39-bus test system is shown in Table IV. As observed, due to the inclusion of the FN constraint under $N-1$ contingencies, the total cost increases by 8.6%, with the largest increase in load shedding. The load shedding cost for M2 is 4.27 times that of M1, while the power infeed and PFR reserve costs of SGs decrease relative to those of M1. This outcome occurs because the inclusion of the FN constraint further increases the requirements for system inertia, droop, and PFR reserve. It also necessitates that the power output of each SG and HVDC should not be too high to prevent a large power imbalance following unexpected events. Additionally, the reduction in the PFR reserve cost of SGs comes at the expense of scheduling more WTs and HVDC PFR reserves, as SGs with slower delivery time contribute less to improving FN compared to power electronic devices. Under the combined effects of reduced power output and increased PFR reserve demand, the load demand becomes more difficult to meet, leading to an increase in load shedding.

TABLE IV
ECONOMIC COST COMPARISON OF DIFFERENT METHODS IN MODIFIED
IEEE 39-BUS TEST SYSTEM

Cost component	Cost (k\$)	
	M1	M2
Generation	1018.66	1073.84
Power infeed	930.77	861.19
PFR reserve of SGs	85.15	71.01
PFR reserve of HVDC	297.39	421.76
WT deloading	10.21	11.92
Load shedding	32.69	139.45
Total	2374.87	2579.18

2) Generation Dispatch Results of M1 and M2

Figure 3 presents the generation dispatch results of M1 and M2, where V_{oll} denotes the amount of load shedding. Compared to M1, M2 sheds a portion of the load during the early morning (periods 0-24), whereas M1 does not. In the afternoon (periods 48-72), both methods implement load shedding, but M2 exhibits a significantly higher amount. Another notable difference is observed in HVDC power transmission. Around midday, M2 reduces the power transfer of D2 while slightly increasing that of D1 compared to M1.

This phenomenon can be explained by examining the HVDC virtual inertia and droop coefficient dispatch results shown in Figs. 4 and 5. It is evident that D2 is assigned a higher level of virtual inertia and droop coefficients than D1. This allocation necessitates a greater PFR reserve for D2, thereby constraining its power transmission capacity. Moreover, in the event of a contingency involving D2, the system experiences a larger loss of virtual inertia and droop coefficients. To mitigate further frequency deterioration, preventing excessive power imbalance becomes crucial, leading to a reduction in power transmission of D2.

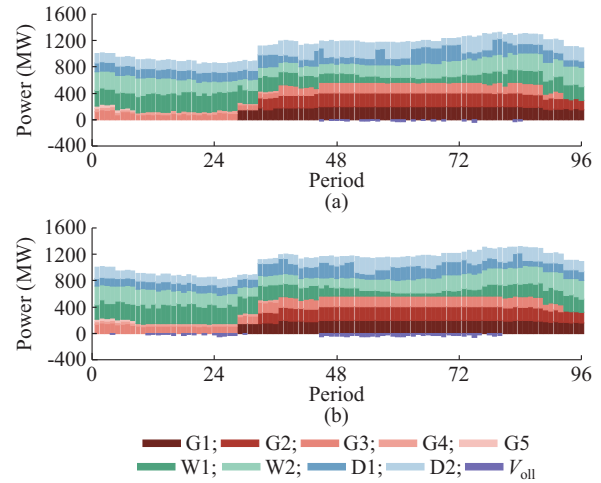


Fig. 3. Generation dispatch results of M1 and M2. (a) M1. (b) M2.

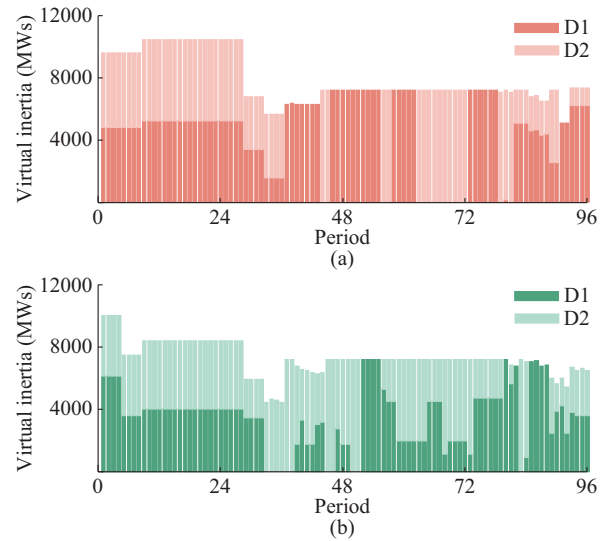


Fig. 4. HVDC virtual inertia dispatch results of M1 and M2. (a) M1. (b) M2.

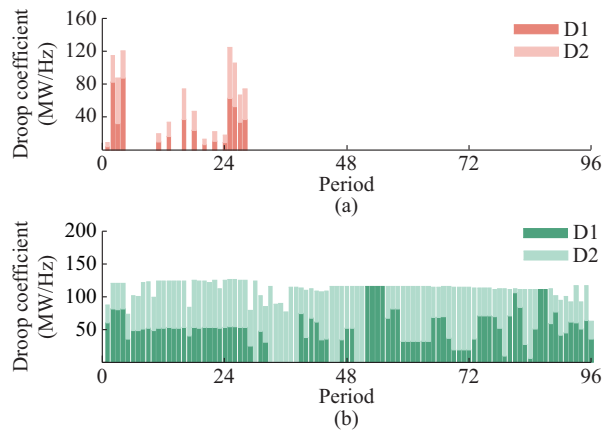


Fig. 5. HVDC droop coefficient dispatch results of M1 and M2. (a) M1. (b) M2.

3) PFR Reserve Dispatch Results of M1 and M2

Figure 6 presents the PFR reserve dispatch results and total PFR reserve composition of M1 and M2. It can be ob-

served that in terms of total PFR reserve composition, the proportion of PFR reserves provided by SGs in M2 decreases significantly, while the contributions from HVDC and WTs increase. Regarding the total PFR reserve composition, M2 exhibits slightly higher values than M1 during certain periods, though the overall difference remains small in most cases. These findings indicate that incorporating the FN constraint primarily affects the composition of PFR reserves rather than their total amount, which remains largely constrained by QSSFD. Additionally, the inclusion of the FN constraint increases the system reliance on PFR reserves from power electronic devices while reducing the PFR reserve dispatch from SGs for economic efficiency. This shift occurs because power electronic-based resources provide faster response, enabling them to rapidly supply PFR power to mitigate power imbalances following $N-1$ contingencies, making them a more favorable choice.

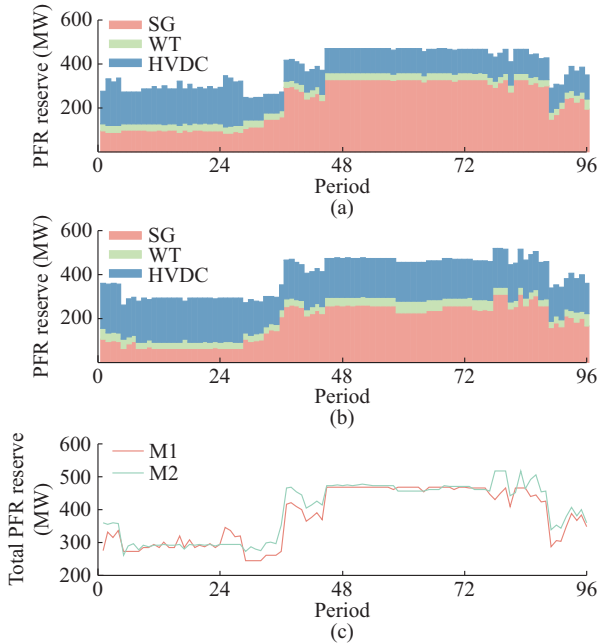


Fig. 6. PFR reserve dispatch results and total PFR reserve composition of M1 and M2. (a) PFR reserve dispatch results of M1. (b) PFR reserve dispatch results of M2. (c) Total PFR reserve composition of M1 and M2.

4) Frequency Security Comparison of M1 and M2

Figure 7 presents the comparison of FN results of M1 and M2 following $N-1$ contingencies.

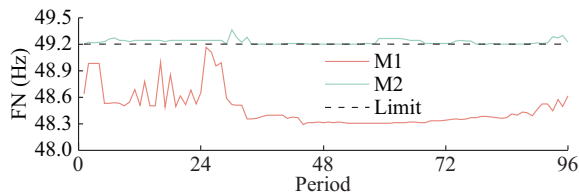


Fig. 7. Comparison of FN results of M1 and M2.

These FNs are calculated by substituting the dispatch results into (2). It can be observed that M2 strictly maintains FN above 49.2 Hz, with most periods being very close to

this threshold, ensuring both system security and economic efficiency. Periods when FN is significantly higher than 49.2 Hz may be attributed to errors in the data-driven approach or constraints imposed by temporal coupling. In contrast, the FN of M1 drops as low as 48.3 Hz, posing a severe frequency security risk to the power system in the event of an $N-1$ contingency. This highlights the effectiveness of M2 in maintaining frequency security while optimizing economic operation.

5) Validation of Importance of Considering $N-1$ Contingencies

We introduce the method M2-1, which represents an FCED method that considers FN constraints only under the maximum power loss contingency. This differs from the proposed FCED method, which additionally accounts for the loss of rotational inertia and other frequency support parameters. The total economic cost of M2-1 is \$2442860, which is 5.58% lower than that of M2. The FNs of M1, M2, and M2-1 under all $N-1$ contingencies are presented in Fig. 8. In Fig. 8(a) and (c), the red color indicates the violations of the FN constraint, while the green color represents the compliance. In Fig. 8(b), lighter green shades indicate FN values closer to 49.2 Hz. The gray areas in Fig. 8 denote periods when SGs are not in operation. Overall, it can be observed that outages of G1 and G2 result in the most severe frequency security issues among all SGs, as they have the highest inertia and power output. Additionally, infeed loss of the two HVDC lines leads to a loss of virtual inertia and droop coefficient, while also causing a significant power imbalance. In contrast, outages of smaller-capacity generators such as G4 and G5 have a negligible impact on system frequency, even in the absence of FN constraints. When comparing the three methods, M2-1 still results in a high proportion of FN violations. In contrast, M2 accounts for not only the power imbalance caused by equipment loss but also the accompanying losses in inertia, droop coefficient, and PFR reserves. This enables the scheduling decisions to better align with actual system operation conditions.

6) Comparison of Proposed FCED Method with Different PWL Models

We extract FN constraints using both the T-PWL model and the proposed PI-PWL model and embed them into the proposed FCED method. When the FN constraint derived from the T-PWL model is adopted, the total dispatch cost reaches \$3074490, which is 1.19 times that obtained using the proposed PI-PWL model. This is because the FN security boundary learned by the T-PWL model is more conservative. As shown in Fig. 9, the FNs obtained using the proposed PI-PWL model are more frequently closer to 49.2 Hz, whereas the T-PWL model results in a higher concentration around 49.4 Hz. A larger deviation of FN from 49.2 Hz in the distribution indicates that the system allocates more frequency support resources such as PFR reserves to ensure frequency security, which in turn leads to excessive redundancy. This indicates that the proposed PI-PWL model learns a more accurate FN security boundary without incurring excessive economic costs.

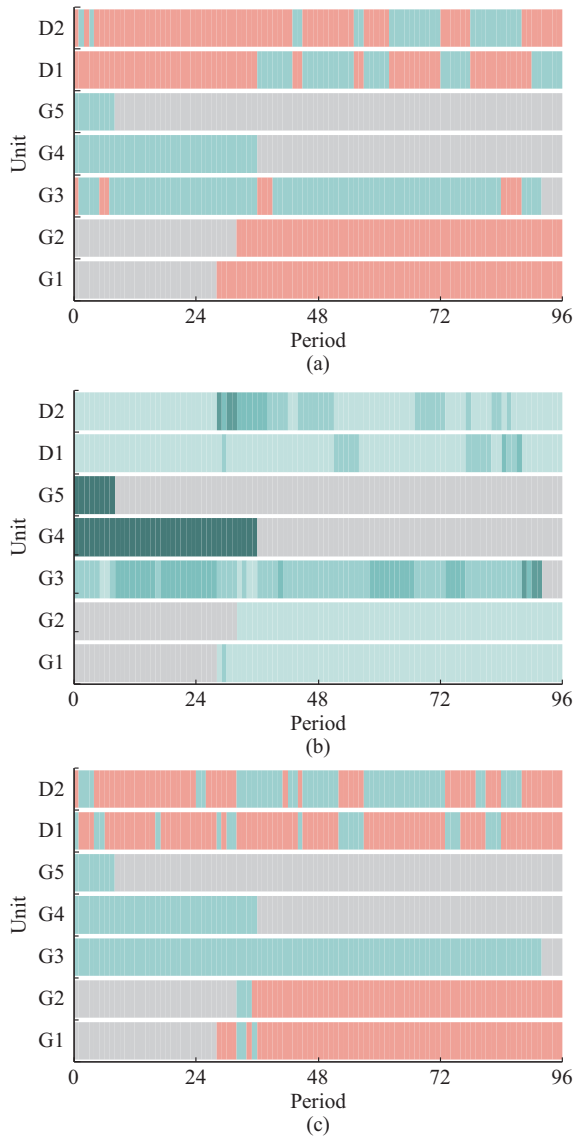


Fig. 8. Comparison of FNs of M1, M2, and M2-1 under all $N-1$ contingencies. (a) M1. (b) M2. (c) M2-1.

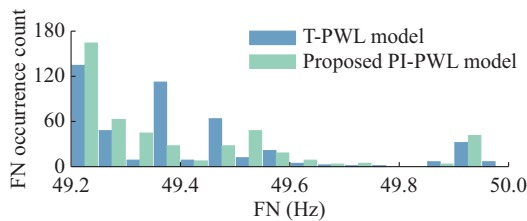


Fig. 9. FN distribution obtained using T-PWL model and proposed PI-PWL model.

7) Validation of Proposed EACC

In this case study, the step size is constrained within the range of 0.99 to 0.999. Based on the proposed EACC, the constraint cuts generated in each iteration dynamically adjust the step size within this range according to the relative error. Figure 10 illustrates the iterative process in three scenarios: scenario 1, with a fixed step size of 0.99; scenario 2, with a fixed step size of 0.999; and scenario 3, with an adaptive step size controlled by the proposed EACC. It can be ob-

served that a rapid cost increase is achieved in scenario 3 in the early iterations, followed by a slower growth phase. This is because the initial error is relatively large, allowing for more aggressive step sizes, whereas fixing the step size at 0.999 results in overly cautious updates, leading to excessive iterations. Conversely, a fixed step size of 0.99 progresses too aggressively, achieving fast convergence but at the cost of significantly higher overall expenses.

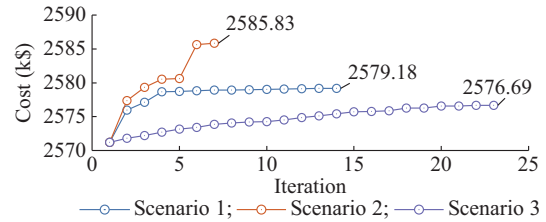


Fig. 10. Iterative process in different scenarios.

8) Error Analysis of FN Calculation

In this part, the FNs calculated using (2) are compared with those obtained from the average system frequency response model adopted in [10]. The relative error remains within 5% across all scenarios, indicating a satisfactory approximation accuracy. It is noteworthy that, for all scenarios, the FNs calculated via (2) are consistently lower than those obtained from time-domain simulations. This reflects the robustness of the linear PFR approximation, as it yields conservatively lower FN estimates, thereby enhancing frequency security in scheduling decisions.

C. Modified IEEE 118-bus Test System

In the modified IEEE 118-bus test system, a total of 54 SGs are installed with a combined capacity of 10370 MW. Wind farms with a total capacity of 2500 MW are connected at buses 3, 15, 54, 65, 110, and 112. Additionally, five HVDC lines connect to buses 2, 27, 66, 74, and 112, each with a transmission capacity ranging from 500 MW to 800 MW. The system load demand varies between 8885 MW and 11358 MW. For more detailed parameters, please refer to [25].

Table V presents the economic cost comparison of M1 and M2 in modified IEEE 118-bus test system. Incorporating the FN constraint increases the total cost by 5.39%, while HVDC power transmission is suppressed, and a greater share of load demand is met by SGs. This occurs because, in this test system, severe frequency security events are primarily triggered by HVDC infeed loss. As shown in Fig. 11, when FN constraints are not considered, all HVDC lines (D1-D5), except D5, lead to violations of the FN constraint following an infeed loss. In Fig. 11(a), the red color indicates the violations of the FN constraint, while the green color represents the compliance. In Fig. 11(b), lighter green shades indicate FN values closer to 49.2 Hz, whereas darker green shades correspond to larger deviations from 49.2 Hz, implying higher frequency security. Consequently, it is necessary to limit HVDC power transmission to prevent excessive reliance on external grid power exchanges, which could reduce the inherent frequency regulation capability of the sys-

tem. Furthermore, the HVDC power transmission dispatch results of M1 and M2 are illustrated in Fig. 12.

TABLE V
ECONOMIC COST COMPARISON OF DIFFERENT METHODS IN MODIFIED
IEEE 118-BUS TEST SYSTEM

Cost component	Cost (M\$)	
	M1	M2
Generation	29.01	29.18
Power infeed	13.02	12.95
PFR reserve of SGs	0.14	0.12
PFR reserve of HVDC	0.04	0.09
WT deloading	0.00	0.00
Load shedding	2.16	2.14
Total	44.36	44.48

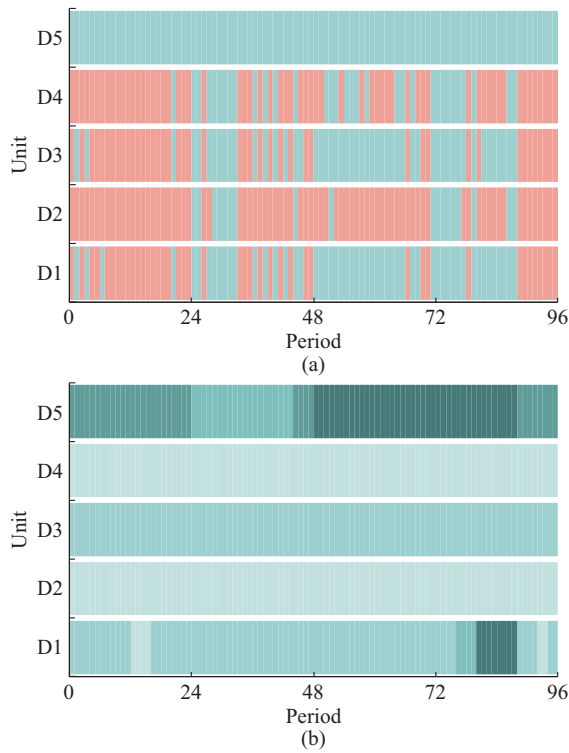


Fig. 11. Comparison of FNs of M1 and M2 under all HVDC infeed loss contingencies. (a) M1. (b) M2.

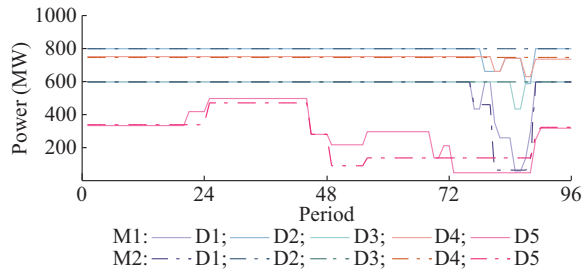


Fig. 12. HVDC power transmission dispatch results of M1 and M2.

VI. CONCLUSION

This paper proposes an FCED method under $N-1$ contingencies. The FN constraint is linearized using the proposed

PI-PWL model, while the associated modeling inaccuracies are mitigated through the proposed EACC. Based on the case study analysis, the following conclusions are drawn.

1) The proposed FCED method effectively manages frequency security under $N-1$ contingencies, ensuring that FN remains above 49.2 Hz while minimizing economic costs. Additionally, incorporating $N-1$ frequency security constraints both necessitates more frequency support resources and places limits on generation output.

2) The proposed PI-PWL model explicitly integrates dynamic equations into the training process, enhancing its generalization capability. This improvement is particularly significant in datasets where unsafe samples are relatively scarce. Compared to the T-PWL model, the proposed PI-PWL model achieves performance improvements of up to 1.05-1.65 times across various metrics.

3) The proposed EACC effectively eliminates the inherent inaccuracies of data-driven methods, ensuring that the dispatch results strictly satisfy frequency constraints. Moreover, it adaptively updates the step size within the specified maximum-minimum range based on FN error, enabling convergence within fewer iterations without causing significant increases in economic costs.

In the future, we will further explore the integration of physical information into more complex machine learning models such as DNNs. Additionally, beyond frequency stability, other stability constraints, including voltage and rotor angle stability, should be considered together to develop a model-data joint-driven stability-constrained optimization framework.

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