

Assessment of Equivalent Inertia and Primary Frequency Regulation of Power Systems Considering Power Reserve Limits of Inverter-based Resources

Yu Shan, Huisheng Gao, Jialiang Wu, Linbin Huang, Zhen Wang, and Huanhai Xin

Abstract—Assessments of equivalent inertia and primary frequency regulation (PFR) are critical for evaluating frequency stability in modern power systems. Traditional methods typically aggregate the contributions of all generation units or rely on system identification during disturbances. However, these methods may yield inaccurate or incomplete results in systems with a high penetration of inverter-based resources (IBRs), as IBRs may reach their power reserve limits (PRLs) and fail to provide the expected frequency support. To overcome this limitation, this paper proposes an assessment method of equivalent inertia and PFR of power systems considering PRLs of IBRs. In addition, dead zones and control delays are incorporated to further enhance the accuracy of the assessment. Based on the assessment results, an optimal parameter-setting approach for equivalent inertia and PFR is then developed. Notably, the theoretical optimal settings are generally difficult to achieve in practice, implying that conventional parameter settings may lead to significantly lower equivalent inertia than expected. The effectiveness of the proposed method is validated through time-domain simulations.

Index Terms—Equivalent inertia, primary frequency regulation (PFR), power reserve limit (PRL), parameter setting, stability, frequency support, inverter-based resource.

I. INTRODUCTION

MODERN power systems have experienced a rapid expansion of renewable energy in recent years. Most renewable energy sources such as photovoltaic (PV) and wind turbine generators (WTGs) are integrated into the power grid through power electronic converters. Unlike synchronous generators (SGs), these inverter-based resources (IBRs) typi-

cally lack physical inertia and strong regulation capabilities [1]. Consequently, the large-scale replacement of SGs with IBRs has deteriorated system frequency response characteristics, leading to higher rate of change of frequency (RoCoF) and increasing the risk of frequency-related blackouts [2], [3].

To enhance the frequency stability of power systems, various regulation methods for IBRs such as virtual inertia and droop control, i.e., primary frequency regulation (PFR), have been widely investigated [4], [5]. A common implementation strategy is to modify the power reference based on the frequency deviations [6]. Furthermore, extensive research has been devoted to assessing inertia and PFR of power systems, providing theoretical guidance for their enhancement [7], [8].

Currently, assessment methods of frequency regulation are generally categorized into two types: aggregation-based and identification-based methods. Aggregation-based method uses the frequency regulation models and parameters of individual devices and aggregate them to obtain the overall inertia and PFR parameters of power systems [9], [10]. However, the heterogeneity of regulation dynamics such as PFRs with different time constants can result in excessively high-order aggregated models. To mitigate this issue, model reduction techniques are often employed. For instance, [10] proposes the use of averaged parameters, e.g., averaged PFR time constants, to approximate the aggregated model.

In contrast, identification-based method assesses the frequency regulation dynamics of the entire system by analyzing disturbed frequency and power trajectories, typically collected via phasor measurement units (PMUs) [11], [12]. The method treats the system as a whole for identification and generally requires large external disturbances to excite its overall frequency response [13], [14]. It is worth noting that some studies identify the regulation models of individual devices using ambient signals [15]. Nevertheless, since aggregation is still necessary for a system-level assessment, these methods can be classified under the first category.

While the above methods can accurately assess the frequency regulation capacities of power systems in test scenarios, they generally overlook the power reserve limits (PRLs) of IBRs, potentially leading to inaccurate or incomplete re-

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Y. Shan, H. Gao (corresponding author), J. Wu, L. Huang, Z. Wang, and H. Xin are with the College of Electrical Engineering, Zhejiang University, Hangzhou, China (e-mail: yshan@zju.edu.cn; gaohuisheng@zju.edu.cn; 11810035@zju.edu.cn; hlinbin@zju.edu.cn; eezwang@ieee.org; xinh@zju.edu.cn).

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sults in extreme scenarios.

Specifically, IBRs, especially the grid-following (GFL) type, need power reserves to support frequency [16]. These reserves are often unevenly distributed across the power system [17], and fluctuate due to weather conditions. In systems with a high penetration of IBRs, these reserves may rapidly decrease if weather conditions change, causing some IBRs to reach low reserve levels. The IBRs are likely to hit their limits during large disturbances, preventing them from providing expected inertia and PFR responses. As a result, the equivalent inertia and PFR parameters of the power system may decrease as the disturbance magnitude increases. In such scenarios, conventional inertia and PFR assessment methods that do not account for the PRLs cannot provide a comprehensive result. It is worth noting that insufficient power reserves have been observed in real-world IBR-integrated power systems such as during the 2019 blackout in Great Britain [18].

Previous studies have conducted some investigations into the PRLs of generators. In terms of assessment, [19] evaluates frequency security using a data-driven multivariate classification method that incorporates power saturation. Similarly, [20] derives nonlinear frequency constraints for system parameters such as PRLs through a sparse weighted oblique decision tree. From an operational scheduling perspective, [17] proposes an optimization-based scheduling framework that accounts for WTG power limits to enhance frequency security. Likewise, [21] introduces a frequency-constrained optimization framework that allocates equivalent inertia and PFR parameters in a rolling manner, with reserve requirements determined by tuned parameters. However, these studies generally rely on complex numerical algorithms, which makes it difficult to uncover the underlying mechanisms behind the outcomes.

To address these limitations, this paper analytically investigates how the equivalent inertia and PFR parameters vary with the magnitude of disturbances, explicitly considering the PRLs of IBRs. Based on these insights, an optimal parameter-setting approach for the equivalent inertia and PFR of IBRs is proposed. The main contributions are summarized as follows.

- 1) An assessment method is proposed to assess the equivalent inertia and PFR of power systems, establishing a quantitative relationship among the equivalent inertia and PFR parameters, the disturbance magnitude, and the PRLs of IBRs.
- 2) Besides PRLs, the effects of dead zones and control delays are also examined to improve the assessment accuracy.
- 3) An optimal parameter-setting approach for equivalent inertia and PFR is then developed. Nevertheless, such theoretically optimal settings are difficult to achieve in practice, implying that the equivalent inertia of power systems may be significantly lower than expected.

The rest of this paper is organized as follows. Section II presents system modeling and problem formulation. Section III presents the assessment method of equivalent inertia and PFR. Section IV proposes an optimal parameter-setting approach for equivalent inertia and PFR. Section V presents

case studies. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODELING AND PROBLEM FORMULATION

A. Modeling of Generation Devices

A multi-device power system is shown in Fig. 1, which consists of n GFL IBRs (denoted by GFL1-GFL n), m_M GFM IBRs (denoted by GFM1-GFM m_M) and m_S SGs (denoted by SG1-SG m_S). Let $m = m_M + m_S$. Their frequency control models are illustrated in Supplementary Material A Fig. SA1. In this paper, particular attention is paid to the PRLs, dead zones, and control delays of these devices, as these characteristics directly affect the equivalent inertia and PFR of power systems. A summary of these features is given in Table I, with further explanations provided below.

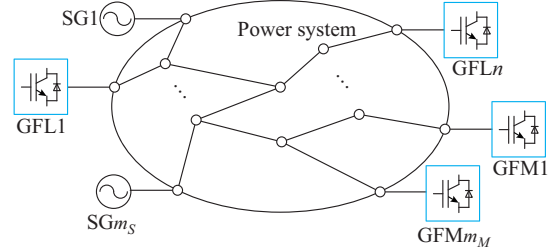


Fig. 1. Demonstration of a multi-device power system.

TABLE I
PRLs, DEAD ZONES, AND CONTROL DELAYS OF DEVICES

Device	Feature	PRL	Dead zone	Control delay
SG	Inertia	$\approx \infty$	0	No
	Damping	$\approx \infty$	0	No
	PFR	$< \infty$	> 0	Yes ¹
GFL IBR	Inertia	$< \infty$	> 0	Yes ²
	Damping	$< \infty$	> 0	No
GFM IBR	Inertia	$\approx \infty$	0	No ²
	Damping	Depend on situations ³	0	No

Note: 1 denotes that these delays do not impact the steady-state PFR assessment; 2 denotes that the coupling with damping may need to be considered; and 3 denotes that with sufficient energy storage, the reserve for GFM IBR damping can be considered infinite, otherwise, it is limited.

1) SG

The linearized frequency regulation dynamic of SG can be typically formulated as [22]:

$$G^{SG}(s) = J^{SG}s + D^{SG} + \frac{K^{SG}}{1 + T^{SG}s} \quad (1)$$

where J^{SG} and D^{SG} are the inertia and damping parameter of SG, respectively; K^{SG} is the PFR parameter; and T^{SG} is the control delay.

This paper focuses on assessing equivalent inertia and PFR, which mainly acts on a time scale of up to tens of seconds [2]. Therefore, the secondary frequency regulation, which typically operates on a longer time scale (e.g., tens of seconds to minutes), is not considered.

The inertia response of SGs stems from their physical rotors, and SGs generally have strong overload capabilities. Be-

sides, the damping of SGs primarily represents the behavior of frequency-sensitive loads [9]. Consequently, the inertia and damping of SGs are assumed to have no PRLs, dead zones, or control delays. In contrast, the PFR of SGs exhibits PRLs, dead zones, and control delay T^{SG} , which is similar to that of the GFL IBRs discussed later. Since this paper primarily focuses on IBRs, a detailed discussion on SGs is omitted here.

2) GFL IBR

The GFL IBR typically uses a phase-locked loop (PLL) to detect frequency variations and adjust the power reference accordingly. The linearized frequency regulation dynamic of GFL IBR is expressed as:

$$G^{GFL}(s) = \frac{J^{GFL}s}{1 + T^{GFL}s} + D^{GFL} \quad (2)$$

where J^{GFL} and D^{GFL} are the virtual inertia and damping parameter of GFL IBR, respectively; and T^{GFL} is the time constant of a low-pass filter (LPF) used to suppress measurement noise.

The frequency regulation response of GFL IBRs may also be influenced by other factors such as the power control loop or PLL (with low bandwidth). Their effects can be approximately modeled by increasing T^{GFL} or introducing additional LPFs. Such factors do not affect the steady-state PFR, and their impact on inertia assessment can be accounted for by analyzing the effective inertia. It is also worth noting that inertia assessment is generally conducted over a short period following a disturbance, rather than instantaneously. During this period, damping may also help to suppress RoCoF, and this effect can also be reflected by the effective inertia.

The power outputs of GFL IBRs are constrained by their reserves. Taking WTG as an example, its maximum power point tracking (MPPT) is shown in Supplementary Material A Fig. SA2, and is expressed as (3)-(5) [23].

$$P_M(v) = \begin{cases} 0 & v \leq v_{in} \text{ or } v > v_{out} \\ \sum_{i=0}^k \alpha_i v^i & v_{in} < v \leq v_0 \\ 1 & v_0 < v \leq v_{out} \end{cases} \quad (3)$$

$$P^0 = K_{dl} P_M \quad (4)$$

$$\Delta P_{Lmt} = (1 - K_{dl}) P_M \quad (5)$$

where P_M is the maximum wind power; v is the wind speed; v_{in} , v_{out} , and v_0 are the cut-in, cut-out, and rated wind speeds, respectively; α_i ($i=1, 2, \dots, k$) is the parameter of MPPT curve; P^0 is the deloaded power reference; K_{dl} is the deloading parameter; and ΔP_{Lmt} is the PRL. Only the upper PRL is considered, as K_{dl} is typically greater than 0.5 (e.g., 0.9) for economic reasons, making the upper limit more likely to be reached. Nevertheless, the proposed method is also applicable when the lower limits are relevant.

The power reserve of PV systems can be expressed as functions of solar irradiance [24], similar to WTGs. Besides, the PRLs of energy storage systems are not influenced by weather conditions, but are directly determined by the capacity limits of the converter. As this paper does not focus on these aspects, they are not discussed in detail.

In addition, dead zones are generally introduced to avoid unnecessarily frequent activations, as illustrated in Supplementary Material A Fig. SA3 and (6), where the dead zone in the inertia control is demonstrated, showing only the half of $\Delta\dot{\omega} < 0$. The dead zone in the damping of PFR ω_{dz} operates similarly.

$$\Delta P_J = \begin{cases} -J^{GFL}(\Delta\dot{\omega} + \dot{\omega}_{dz}) & \Delta\dot{\omega} < -\dot{\omega}_{dz} \\ 0 & |\Delta\dot{\omega}| \leq \dot{\omega}_{dz} \end{cases} \quad (6)$$

where ΔP_J is the power reference change due to virtual inertia; $\Delta\dot{\omega}$ is the filtered RoCoF; and $\dot{\omega}_{dz} > 0$ is the threshold of $\Delta\dot{\omega}$.

3) GFM IBR

Taking virtual synchronous generator (VSG) as an example, the frequency response model is:

$$G^{GFM}(s) = J^{GFM}s + D^{GFM} \quad (7)$$

where J^{GFM} and D^{GFM} are the inertia and damping parameters of the GFM IBR, respectively.

In GFM IBRs, inertia and damping are intrinsic to their synchronization mechanism, and providing such frequency support constitutes a fundamental function of voltage-source operation. To maintain stable GFM behavior, GFM IBRs are typically designed with strong overload capability and are often supplemented by energy storage, e.g., batteries or supercapacitors [25]. These storage units are generally sufficient for short-term inertia support (on the order of several seconds) but may be inadequate for sustained PFR, thereby requiring case-specific assessment. Furthermore, GFM IBRs usually have no dead zone in inertia or damping control to enhance synchronization performance. Finally, since GFM IBRs do not rely on measurement to generate frequency regulation commands, their response delays are shorter. Nevertheless, within the selected time window for inertia assessment, their relatively large damping may still exhibit coupling effects.

Based on the above analysis, it can be concluded that the inertia and damping responses of GFM IBRs exhibit similar characteristics to SGs. Therefore, in this paper, the PRLs of GFM IBRs are mainly disregarded. Nevertheless, we note that when GFM IBRs are unable to provide sustained PFR (e.g., due to insufficient stored energy), the proposed method remains applicable with only minor modifications. To be specific, they can be treated as GFL IBRs, and their outputs are also subject to PRLs. In these cases, overload mitigation control [26] is required to ensure their stable operation.

B. System Frequency Response and Problem Formulation

Combining SGs and IBRs, the system frequency response model is depicted in Fig. 2, where ΔP_{dis} is the external disturbance; J_{sum}^{SG} and D_{sum}^{SG} are the sum of inertia and the sum of damping of all SGs, respectively; $\Delta\omega$ is the frequency response; ΔP^{GFL} and ΔP^{GFM} are the active power responses of GFL IBR and GFM IBR, respectively; and J_{sum}^{GFM} and D_{sum}^{GFM} are the sum of inertia and the sum of damping of GFM IBRs, respectively.

Traditionally, aggregation is widely used to assess the system inertia or PFR parameters:

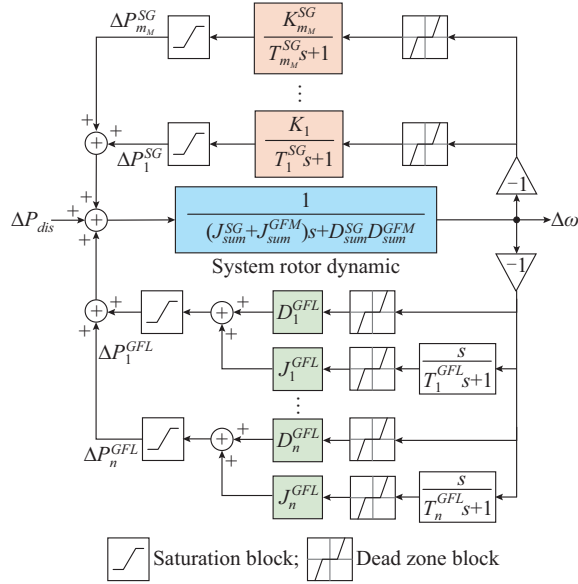


Fig. 2. System frequency response model.

$$\begin{cases} J_{sum} = J_{sum}^{SG} + J_{sum}^{GFM} + J_{sum}^{GFL} \\ D_{sum} = D_{sum}^{SG} + D_{sum}^{GFM} + D_{sum}^{GFL} + K_{sum}^{SG} \end{cases} \quad (8)$$

where J_{sum} and D_{sum} are the sum of equivalent inertia and PFR parameters, respectively; J_{sum}^{GFL} and D_{sum}^{GFL} are the sum of inertia and damping of GFL IBRs, respectively; and K_{sum}^{SG} is the total PFR parameter of SGs. Note that under quasi-steady-state conditions, the PFR and damping are equivalent.

However, the above aggregation method may yield inaccurate results when the disturbance magnitude is large. This is mainly because IBRs (particularly GFL IBRs) may saturate due to their PRLs, significantly constraining their power outputs, and thereby reducing the equivalent inertia and PFR. In addition, factors such as dead zones and control delays further degrade the regulation responses, making the aggregation results less reliable. These issues will be discussed in detail in the following section.

III. ASSESSMENT METHOD OF EQUIVALENT INERTIA AND PFR OF POWER SYSTEMS

A. Influence of PRL

It is well known that the total inertia is inversely proportional to the initial RoCoF after a disturbance. However, when IBRs reach their PRLs, the total inertia may no longer serve as a reliable metric. In this situation, the RoCoF can still be computed, then the equivalent inertia can be obtained accordingly. The detailed derivation is presented below.

Without loss of generality, it is assumed that the devices in the power system are arranged in the following way, as mathematically illustrated in (9). The first n devices are GFL IBRs with PRLs, while the remaining devices are GFM IBRs or SGs, whose inertia response is not subject to such limitations ($\Delta P_{Lmt,i} \approx \infty$), as previously assumed.

$$\frac{\Delta P_{Lmt,1}}{J_1} \leq \dots \leq \frac{\Delta P_{Lmt,n}}{J_n} < \frac{\Delta P_{Lmt,n+1}}{J_{n+1}} \leq \dots \leq \frac{\Delta P_{Lmt,n+m}}{J_{n+m}} \quad (9)$$

If the disturbance exceeds the PRLs of the whole system, i.e., $\Delta P_{dis} > \Delta P_{Lsum} = \sum_i \Delta P_{Lmt,i}$ (only occurs when there is no GFM IBR or SG), the system cannot maintain stable operation and its inertia is meaningless. Otherwise, the following proposition holds.

Proposition For the aforementioned power systems under disturbance $\Delta P_{dis} \leq \Delta P_{Lsum}$, during the inertia response period, the number of saturated GFL IBRs (denoted by $k \in [0, n]$) can be determined by (10). Let $\Delta P_{Lmt,n+1} = 0$ if $m = 0$; $\Delta P_{Lmt,0} = 0$; and the summation equals zero if its lower bound is larger than the upper bound, for simplicity.

$$\sum_{i=1}^{k-1} \Delta P_{Lmt,i} + \frac{\sum_{i=k}^{n+m} J_i}{J_k} \Delta P_{Lmt,k} \leq \Delta P_{dis} < \sum_{i=1}^k \Delta P_{Lmt,i} + \frac{\sum_{i=k+1}^{n+m} J_i}{J_{k+1}} \Delta P_{Lmt,k+1} \quad (10)$$

Then, the equivalent inertia is:

$$J_{eq}(\Delta P_{dis}) = \frac{\Delta P_{dis}}{\Delta P_{dis} - \sum_{i=1}^k \Delta P_{Lmt,i}} \sum_{i=k+1}^{n+m} J_i \quad (11)$$

Proof We begin by examining the sequence in which devices reach saturation, when the system is subjected to a series of independent disturbances of increasing magnitude. For small disturbances such that all devices are unsaturated, the disturbance is shared among them in proportion to their inertia. As the disturbance magnitude increases, the first device reaches its PRL (12) is satisfied.

$$\frac{J_i}{J_{sum}} \Delta P_{dis} = \Delta P_{Lmt,i} \quad i \in [1, n] \quad (12)$$

By combining (9) and (12), it can be inferred that the first device to become saturated is GFL1, since it has the smallest value of $\Delta P_{Lmt,i}/J_i$. For larger disturbances, GFL1 will remain saturated, and it can be viewed as a constant power source. Then, the system can be considered as consisting of $n-1$ GFL IBRs (excluding GFL1) and m GFM IBRs or SGs, subject to a disturbance of $\Delta P_{dis} - \Delta P_{Lmt,1}$. As the disturbance further increases, it can be deduced that the next device to become saturated is GFL2, followed by GFL3, until GFL n . When exactly k devices are saturated, it implies that the power output of GFL $k+1$ remains below its PRL while the GFL k is saturated:

$$\begin{aligned} \frac{J_{k+1}}{\sum_{i=k+1}^{n+m} J_i} \left(\Delta P_{dis} - \sum_{i=1}^k \Delta P_{Lmt,i} \right) < \\ \Delta P_{Lmt,k+1} \leq \frac{J_k}{\sum_{i=k}^{n+m} J_i} \left(\Delta P_{dis} - \sum_{i=1}^{k-1} \Delta P_{Lmt,i} \right) \geq \Delta P_{Lmt,k} \end{aligned} \quad (13)$$

Rearranging the above inequality results in (10).

Meanwhile, the RoCoF of the system can be calculated by (14). It is equivalent to a system where the disturbance is ΔP_{dis} and the equivalent inertia is shown in (11), which completes the proof.

$$\Delta\dot{\omega} = - \frac{\Delta P_{dis} - \sum_{i=1}^k \Delta P_{Lmt,i}}{\sum_{i=k+1}^{n+m} J_i} \quad (14)$$

We can plot the curve of $J_{eq}(\Delta P_{dis})$, which is necessarily monotonic and non-increasing. Because J_{eq} is segmentally continuous, and within each segment (i.e., for each k), the derivative of J_{eq} with respect to ΔP_{dis} is non-positive.

$$\frac{dJ_{eq}}{d\Delta P_{dis}} = \frac{-\sum_{i=1}^k \Delta P_{Lmt,i}}{\left(\Delta P_{dis} - \sum_{i=1}^k \Delta P_{Lmt,i}\right)^2} \sum_{i=k+1}^{n+m} J_i \leq 0 \quad (15)$$

The equality holds if and only if $k=0$, indicating that whenever a device becomes saturated, the equivalent inertia J_{eq} is less than its nominal value J_{sum} . When the disturbance increases, the equivalent inertia continues to decrease, approaching the limit of $J_{sum}^{SG} + J_{sum}^{GFM}$ (if there is at least one SG or GFM IBR). If no SG or GFM IBR exists, the equivalent inertia declines to zero, as ΔP_{dis} approaches ΔP_{Lsum} .

The proposed method is also applicable to assess the equivalent PFR parameter D_{eq} in the quasi-steady-state, where the disturbance power is distributed according to the PFR parameters, analogous to how inertia governs power sharing immediately after a disturbance. It should be noted that the damping of SG mainly originates from the frequency-sensitive load and is therefore unaffected by PRLs, whereas their PFRs are constrained by such limits. Therefore, the two parts should be treated separately. In addition, for GFM IBRs, if the stored energy cannot sustain PFR over a longer duration, they should be treated as GFL IBRs in this analysis.

B. Influence of Dead Zone

The basic idea for accounting for dead zones is to re-compute the RoCoF (or quasi-steady-state frequency deviation) while considering their effects, and then assess the equivalent inertia or PFR accordingly. In the following, the inertia assessment is used as an illustrative example. The effect of control delay is neglected and will be discussed later.

For simplicity, all GFL IBRs are assumed to share the same dead zone, i.e., $\dot{\omega}_{dz,i} = \dot{\omega}_{dz}$. Note that the inertia responses from SGs and GFM IBRs are not subject to dead zones.

If the RoCoF $\Delta\dot{\omega} (<0)$ remains below the dead zone $\dot{\omega}_{dz} (>0)$, i.e., $\Delta\dot{\omega} + \dot{\omega}_{dz} > 0$, the result is straightforward: $J_{eq} = J_{sum}^{SG} + J_{sum}^{GFM}$. When the RoCoF exceeds the dead zone and none of the GFL IBR outputs reaches their limits, we can obtain:

$$\begin{aligned} \Delta P_{dis} &= -J_{sum}^{GFL}(\Delta\dot{\omega} + \dot{\omega}_{dz}) - (J_{sum}^{SG} + J_{sum}^{GFM})\Delta\dot{\omega} \\ &\Rightarrow \Delta P'_{dis} = -(J_{sum}^{GFL} + J_{sum}^{SG} + J_{sum}^{GFM})\Delta\dot{\omega}' \end{aligned} \quad (16)$$

where $\Delta\dot{\omega}' = \Delta\dot{\omega} + \dot{\omega}_{dz}$ and $\Delta P'_{dis} = \Delta P_{dis} - (J_{sum}^{SG} + J_{sum}^{GFM})\dot{\omega}_{dz}$ are the equivalent RoCoF and equivalent disturbance, respectively.

From (16), the equivalent disturbance $\Delta P'_{dis}$ is distributed among GFL IBRs in proportion to their inertia, similar to how ΔP_{dis} is allocated when dead zones are zero. Conse-

quently, the sequence in which GFL IBRs reach saturation remains unchanged. When there are exactly k GFL IBRs saturated, (16) can be rewritten as:

$$\Delta P_{dis} = \sum_{i=1}^k \Delta P_{Lmt,i} - \sum_{i=k+1}^n J_i(\Delta\dot{\omega} + \dot{\omega}_{dz}) - (J_{sum}^{SG} + J_{sum}^{GFM})\Delta\dot{\omega} \quad (17)$$

Then, the RoCoF and equivalent inertia can be calculated as:

$$\Delta\dot{\omega} = - \frac{\Delta P_{dis} - \sum_{i=1}^k \Delta P_{Lmt,i} + \sum_{i=k+1}^n J_i \dot{\omega}_{dz}}{\sum_{i=k+1}^{n+m} J_i} \quad (18)$$

$$J_{eq} = \frac{\Delta P_{dis}}{\Delta P_{dis} - \sum_{i=1}^k \Delta P_{Lmt,i} + \sum_{i=k+1}^n J_i \dot{\omega}_{dz}} \sum_{i=k+1}^{n+m} J_i \quad (19)$$

Comparing (19) with (11), we find that: the presence of dead zones reduces the equivalent inertia, given that $\dot{\omega}_{dz} > 0$; and the impact of the dead zone decreases with the increase of disturbance, since fewer GFL IBRs ($k+1 \sim n$) remain unsaturated.

A similar derivation can be carried out to analyze the effect of the dead zone on the equivalent PFR parameter, which is omitted here for brevity.

C. Influence of Control Delay

In the virtual inertia control of GFL IBRs, LPFs are commonly employed to suppress noise and mitigate small-signal instability [27]. These filters, along with other delays (e.g., in power control), can degrade the inertia response. Furthermore, since inertia and RoCoF are typically assessed over a short interval following a disturbance, damping can also contribute power and help reduce the RoCoF during this period, particularly in IBRs with strong damping. For these reasons, the virtual inertia parameters J^{GFL} and J^{GFM} defined in the control loops may not fully capture the actual effect of IBRs on mitigating RoCoF, even when nonlinear elements such as PRLs and dead zones are excluded.

Notably, damping, in a sense, can be interpreted as delayed inertia. The control delay is represented by an LPF, i.e., $T_f/(1+T_f s)$ with $T_f \rightarrow \infty$, where T_f is the time constant. Therefore, its influence is also categorized as a type of control delay and is treated in the same manner.

To address the aforementioned issue, the concept of effective inertia (denoted by J^e) is introduced, which represents the actual power response of IBRs during the short period after disturbance. Specifically, following the method in [22], we approximate ΔP^{GFL} or ΔP^{GFM} (the actual power output of GFL or GFM) with $-J^e s \Delta\omega$ over a short time interval. By integrating the two trajectories, the effective inertia can be calculated as:

$$J^e = \frac{\int_{t_0}^{t_0+T} \Delta P^{GFL}(t) dt}{\Delta\omega(t_0) - \dot{\omega}_{dz} - \Delta\omega(t_0+T)} \quad (20)$$

where t_0 is the disturbance onset time; and T is the selected short time interval. An excessively small T may fail to adequately suppress noise, whereas an excessively large T may

lead to the dominance of damping effects [28], [29]. Hence, T is typically chosen within the range of 200-500 ms [30]. In this paper, T is selected as 250 ms. Here, $\dot{\omega}_{dz}$ is subtracted to eliminate the dead zone effect. Meanwhile, the power response should not saturate during the selected interval.

There are several additional explanations regarding (20).

1) The power response in the numerator can be obtained using the model depicted in Fig. 2, or through simulations/practical measurements. The latter method effectively accounts for not only the LPF (T^{GFL}) but also other unmodeled dynamics, e.g., the delay introduced by the power control.

2) In multi-device systems, the power response from IBRs may result from multiple frequency response modes, including the system-wide frequency changes, i.e., the common-mode frequency [22] and inter-device oscillations. The latter method, however, is irrelevant for system-level inertia assessment and should be excluded. It can be achieved using techniques such as Prony analysis, or by carefully selecting T such that the integral of the oscillatory power components is minimized.

Regarding the assessment of PFR, this paper focuses on the steady-state response, where the control delay has no impact.

D. Assessment Process for Equivalent Inertia and PFR

Combining the above analysis, the procedure for assessing the equivalent inertia and PFR is summarized as follows.

First, collect the inertia, damping, and PFR parameters of all IBRs and SGs in the system. For equivalent inertia assessment, if the control delay (including the damping coupling effect) is non-negligible, calculate the effective inertia by (20).

Next, determine the PRLs. This can be achieved through the operational control system (OCS) or energy management system (EMS), which may provide power reserves directly, or offer necessary data for calculating (5) such as deloading parameters and current power output [31], [32].

Finally, the equivalent inertia and PFR, $J_{eq}(\Delta P_{dis})$ can be assessed using (11) or (19) (the latter takes dead zone into consideration). The PFR parameter $D_{eq}(\Delta P_{dis})$ can be obtained through similar method.

We note that assessment results, e.g., $J_{eq}(\Delta P_{dis})$, are functions of disturbance ΔP_{dis} . Representative ΔP_{dis} values such as the loss of a generator can be substituted to assess the equivalent inertia under typical disturbances. Furthermore, if specific RoCoF (or steady-state frequency deviation) threshold under certain disturbance is given, the proposed method can also assist in evaluating whether the requirements are satisfied.

We note that the proposed method can still be applied even certain information is unavailable. For instance, if the PRLs are unknown, they can be assumed to be infinite. If the dead zone is unknown, (11) can be used instead of (19). Similarly, if the effective inertia cannot be calculated, the inertia parameters in the control loop can be used instead. In fact, for small disturbances, the dead zone has the most significant impact on the assessment results, whereas for large

disturbances, the PRL dominates. It will be further discussed in Section V-C.

E. Comparison with Other Existing Assessment Methods

For a comprehensive understanding, the proposed method is compared with existing assessment methods, including aggregation-based [9], [10] and identification-based methods [13], [14], as summarized in Table II.

TABLE II
COMPARISON WITH EXISTING ASSESSMENT METHODS

Method	Description	Assessment result
Aggregation-based method [9], [10]	Aggregate equivalent inertia and PFR contributions of all devices and ignore PRLs, dead zones, and control delays	Yield optimistic results under large disturbances
Identification-based method [13], [14]	Assess equivalent inertia and PFR using data collected during large disturbances and neglect impact of disturbance magnitude on assessment	Lack accuracy when the system encounters and obtain disturbances of different magnitudes
Proposed method	Account for PRLs, dead zones, and control delays and require additional information for assessment	Provide more accurate results

1) Aggregation-based Method

This method aggregates the equivalent inertia and PFR of all generation devices in the system. The inertia or PFR parameters of individual devices can be obtained from pre-existing models or identification [33]. It should be noted that this individual identification differs from the identification-based method, where the equivalent inertia and PFR are directly identified.

A key drawback of the aggregation-based method is its inability to account for the PRLs of IBRs. As a result, the assessment may become overly optimistic when the system experiences large disturbances. Moreover, the aggregation-based method does not adequately address the effects of dead zones and control delays, e.g., by directly aggregating the inertia parameters in the control loop, which can lead to more inaccurate results.

2) Identification-based Method

This method assesses the equivalent inertia and PFR of the entire system based on measurement [13]. It typically relies on data collected during large disturbances, under which bus frequencies exhibit coherent behavior, thereby enabling system-wide assessment.

This method can accurately estimate the equivalent inertia and PFR for a specific disturbance, implicitly incorporating factors such as PRLs, dead zones, and control delays. However, the influence of these factors, especially PRLs, is disturbance-dependent. Consequently, the results may not be accurate for other disturbance scenarios.

3) Proposed Method

In contrast to the aforementioned methods, the proposed method explicitly incorporates PRLs, dead zones, and control delays into the assessment, resulting in more accurate results. A limitation of the proposed method is that it requires more detailed information such as PRLs. However, the meth-

od remains applicable when such information is unavailable, albeit with reduced accuracy.

IV. OPTIMAL PARAMETER-SETTING APPROACH FOR EQUIVALENT INERTIA AND PFR OF IBRS

In this section, an optimal parameter-setting approach for GFL IBRs is proposed to maximize their frequency support capability. Subsequently, it is shown that achieving this theoretical optimum in practice is challenging. As a result, the equivalent inertia may be considerably lower than expected.

A. Proposed Optimal Parameter-setting Approach

For simplicity, we neglect dead zones and control delays, and focus solely on PRLs in the subsequent analysis, as PRLs are the dominant factor during large disturbances (see Section V-C). Inertia parameter setting is used for demonstration.

According to (11), the equivalent inertia reaches its maximum when none of the IBRs are saturated, and it decreases as the number of saturated IBRs increases. Therefore, the key to parameter setting lies in minimizing saturation across all IBRs. The optimal condition is achieved when all GFL IBRs share the same ratio of $\Delta P_{Lmt,i}/J_i$, such that they reach their PRLs simultaneously as the disturbance increases. This condition can be interpreted as aligning the regulation parameters of the IBRs with their power reserves. Based on this principle, once J_{sum}^{GFL} is given, the optimal inertia J_i for each GFL IBR can be determined as:

$$J_i = \frac{\Delta P_{Lmt,i} J_{sum}^{GFL}}{\Delta P_{Lsum}^{GFL}} \quad i \in [1, n] \quad (21)$$

where ΔP_{Lsum}^{GFL} is the total power reserve of GFL IBRs.

In this optimal situation, the equivalent inertia can be calculated as:

$$J_{eq,max}(\Delta P_{dis}) = \begin{cases} J_{sum} & \Delta P_{dis} \leq \frac{J_{sum} \Delta P_{Lsum}^{GFL}}{J_{sum}^{GFL}} \\ \frac{\Delta P_{dis} (J_{sum}^{SG} + J_{sum}^{GFM})}{\Delta P_{dis} - \Delta P_{Lsum}^{GFL}} & \Delta P_{dis} > \frac{J_{sum} \Delta P_{Lsum}^{GFL}}{J_{sum}^{GFL}} \end{cases} \quad (22)$$

where $J_{eq,max}$ is the maximal equivalent inertia with the optimal parameter setting. The breakpoint corresponds to the condition where all GFL IBRs reach PRLs simultaneously.

The parameter J_{sum}^{GFL} can be determined by the requirement on RoCoF. For example, if it is required that the RoCoF should remain below a certain threshold $\dot{\omega}_{th}$ under disturbance ΔP_{dis} , the minimum value of J_{sum}^{GFL} is given by:

$$J_{sum}^{GFL} = \frac{\Delta P_{dis}}{\dot{\omega}_{th}} - (J_{sum}^{SG} + J_{sum}^{GFM}) \quad (23)$$

It is worth noting that the above requirement on RoCoF also imposes a lower bound on the total power reserve of GFL IBRs, which can be expressed as:

$$\Delta P_{Lsum}^{GFL} = \Delta P_{dis} - \dot{\omega}_{th} (J_{sum}^{SG} + J_{sum}^{GFM}) \quad (24)$$

B. Non-optimality of Parameter Settings in Practice

The above optimal parameter-setting approach represents a theoretical optimum. However, its practical implementation

is challenging because the inertia (or PFR) parameters of IBRs are typically fixed during operation, while their available power reserves can vary significantly due to the stochastic nature of weather conditions. In the following, we use WTGs as an example to model such operational uncertainties.

Considering that wind speed follows a Weibull distribution [34], the probability density function $f_w(v; c, k_{sh})$ is expressed as:

$$f_w(v; c, k_{sh}) = \frac{k_{sh}}{c} \left(\frac{v}{c} \right)^{k_{sh}-1} e^{-\left(\frac{v}{c} \right)^{k_{sh}}} \quad (25)$$

where c is the scale parameter; and k_{sh} is the shape parameter. For each sampled wind speed, the corresponding wind power and power reserve can be calculated using (3)-(5).

From (25), it is evident that wind speeds (and PRLs) vary and are not uniformly distributed among WTGs. However, the inertia parameters of WTGs are often uniformly set, e.g., to satisfy the minimum requirements specified by power system operators. As a result, the inertia parameter settings are far from optimal, and the equivalent inertia may be significantly lower than expected under large disturbance.

It is worth noting that this situation could be greatly improved if the inertia parameters of IBRs are adjustable during operation and if wind speeds are predictable. In such cases, the inertia parameter settings of WTGs could be coordinated with the forecasted PRLs in each prediction interval. Of course, forecast errors are inevitable. In this work, such errors are modeled using a normal distribution, as shown in (26) [35]. Nevertheless, even with such prediction errors, the equivalent inertia can be significantly enhanced compared with fixed settings, which will also be demonstrated in the case studies.

$$\frac{P_M - P'_M}{\sigma} \sim N(0, 1) \quad (26)$$

where P'_M is the forecasted wind power; σ is the standard deviation; and $N(0, 1)$ is a normal distribution with a mean of 0 and a variance of 1.

C. Comparison with Existing Parameter-setting Approaches

Several studies have also investigated inertia parameter setting while accounting for the PRLs, e.g., [21]. However, the involved approaches generally rely on complex optimization algorithms. By contrast, the proposed optimal parameter-setting approach is rooted in theoretical analysis, offering two main advantages.

1) Interpretability and generalizability: the proposed optimal parameter-setting approach reveals the principle underlying the optimal solution, i.e., matching the frequency regulation parameters of IBRs with their available PRLs. This physically intuitive concept can be readily extended to other systems, whereas optimization-based methods often produce case-specific and non-generalizable results.

2) Simplicity: the proposed optimal parameter-setting approach provides clear parameter-setting schemes, without the need for intricate model construction or computationally intensive optimization.

V. CASE STUDIES

A. Test System Description

Two test systems, including a modified IEEE 39-bus system and a 100-device system, are employed in this subsection. The modified IEEE 39-bus test system, as shown in Fig. 3, is used as the primary test system. It consists of four SGs and six WTGs as IBRs. All devices have a rated capacity of 600 MVA, which is also adopted as the system base power. The IBRs are modeled under either GFL or GFM control depending on the case configuration, and the corresponding models are illustrated in Supplementary Material A Fig. SA3. The dead zone, virtual inertia, and PRL control of GFL IBRs are shown in Supplementary Material A Fig. SA1. The overload mitigation control of GFM IBRs follows [26]; once activated, they behave as constant power sources. The main frequency-related parameters of devices are listed in Table III, and the network parameters are consistent with the IEEE standard system. Detailed model explanations are omitted here but can be found in [36]. In addition, the 100-device system is constructed as an idealized model to illustrate the impact of uncertainties on the proposed optimal parameter-setting approach.

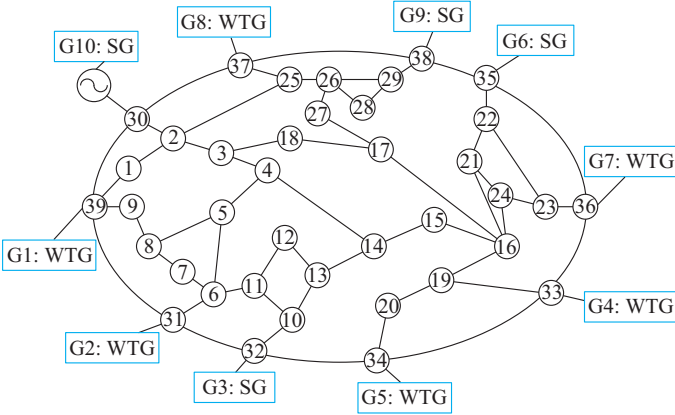


Fig. 3. Modified IEEE 39-bus test system.

TABLE III
FREQUENCY-RELATED PARAMETERS OF DEVICES

Parameter	Value	Parameter	Value
J^{SG}	10 p.u.	J^{GFM}	10 p.u.
K^{SG}	15 p.u.	D^{GFM}	10 p.u.
J^{GFL}	10 p.u.	K_{dl}	0.9
D^{GFL}	10 p.u.	$\dot{\omega}_{dz}$	0.05 Hz/s
T^{GFL}	0.05 s	ω_{dz}	0.05 Hz

A brief description of all cases is provided as follows. In the modified IEEE 39-bus test system, Cases 1-4 are designed to validate the proposed method, while Cases 5 and 6 are used to evaluate the proposed optimal parameter-setting approach.

1) Case 1: in this case, all IBRs are configured with GFL control and equipped with identical equivalent inertia and PFR parameters. The power reserves are detailed in Table IV, while SGs are assumed to have sufficient power reserves.

TABLE IV
POWER RESERVES IN CASES 1-6

Case	Output power before disturbance (p.u.)	Reserved power (p.u.)
1	$P_2^0=0.6, P_i^0=0.9 \ (i=1,4,5,7,8)$	$\Delta P_{Lmt,i}=0.1 \ (i=1,4,5,7,8)$
2, 3, 4	$P_2^0=0.8, P_i^0=0.54 \ (i=1,4,5,7,8)$	$\Delta P_{Lmt,i}=0.06 \ (i=1,4,5,7,8)$
5, 6	$P_2^0=0.6, P_i^0=0.9 \ (i=1,4,5),$ $P_i^0=0.45 \ (i=7,8)$	$\Delta P_{Lmt,i}=0.1 \ (i=1,4,5),$ $\Delta P_{Lmt,i}=0.05 \ (i=7,8)$

2) Case 2: it is almost consistent with Case 1, all IBRs adopt GFL control with the same equivalent inertia and PFR parameters. The key differences from Case 1 lie in the adoption of a larger disturbance magnitude and smaller power reserves for IBRs.

3) Case 3: derived from Case 2, this case retains the same disturbance magnitude and power reserve settings. The core modification is that two GFL IBRs are replaced with GFM IBRs, while the equivalent inertia and PFR parameters of all IBRs remain unchanged. Notably, the overload mitigation control function of the GFM IBRs is enabled in this case.

4) Case 4: it is developed on the basis of Case 3, sharing the same system topology, IBR control type configuration, and parameter settings. The only distinction is that the overload mitigation control function of the GFM IBRs is disabled in this case.

5) Case 5: all IBRs are configured with GFL control, applying the proposed optimal parameter-setting approach.

6) Case 6: it is designed as a comparative case for the evaluation of the proposed optimal parameter-setting approach. Like Case 5, all IBRs adopt GFL control, but the parameter setting employed here is a uniform parameter approach for the sake of comparison.

7) Case 7: this case employs a 100-device system. Its primary purpose is to further demonstrate the impact of system uncertainties on the equivalent inertia of the power system.

B. Verification of Proposed Method

Cases 1 and 2 are designed to validate the proposed method. In both cases, the disturbance is the trip of G2 at $t=1$ s. The two cases differ in initial power outputs and power reserves, as shown in Table IV: Case 2 has a larger disturbance and smaller reserves, leading to GFL IBR saturation, while GFL IBRs remain unsaturated in Case 1.

We first assess the equivalent inertia. Three methods are compared: aggregation-based method, identification-based method, and the proposed method.

Since all devices have the same inertia parameter (10 p.u.), the aggregated system inertia is 90 p.u. (after the trip of G2) in both Case 1 and Case 2. For the identification-based method, simulation results from Case 1 yield a system inertia of 84.11 p.u., which is subsequently applied to assess Case 2. As for the proposed method, the assessment process follows the steps outlined in Section III-D. First, the effective inertia of the GFL IBRs is calculated using historical data, which is 9.02 p.u.. Then, combining the PRLs and the disturbances, the equivalent inertia is then determined using (19), yielding 82.75 p.u. for Case 1 and 64.00 p.u. for

Case 2.

To verify the above analysis, the obtained inertia is used to calculate the RoCoFs, which are then compared with time-domain simulations (TDSs) conducted in MATLAB/Simulink, as shown in Fig. 4. P_3 is the actual active power of SG 3. P_7 is the measured active power of IBR unit 7. $P_{ref0,7}$ is the active power reference command for IBR unit 7 from its frequency regulator. The COI frequency from TDS serves as the benchmark, and a 250 ms time window is adopted for RoCoF calculation. The assessment results in Cases 1 and 2 are shown in Table V, where the percentages in parentheses denote the relative errors with respect to the TDS results.

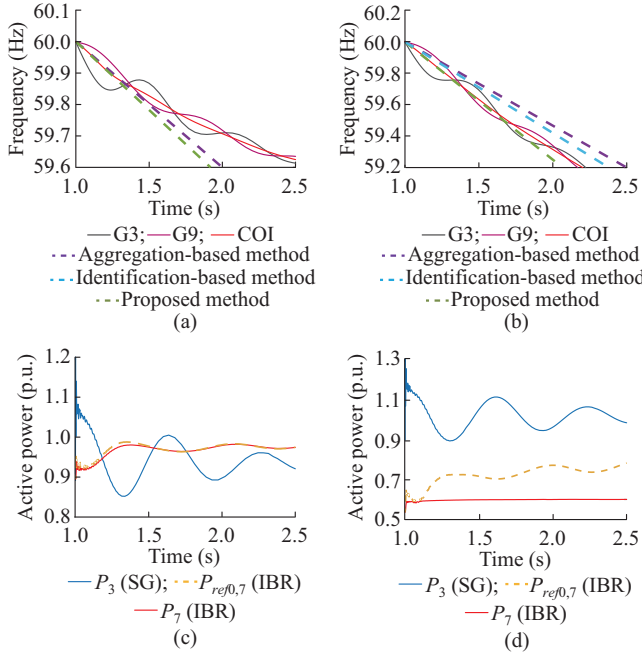


Fig. 4. Frequency and active power responses in Cases 1 and 2. (a) Frequency response in Case 1. (b) Frequency response in Case 2. (c) Active power response in Case 1. (d) Active power response in Case 2.

TABLE V
ASSESSMENT RESULTS IN CASES 1 AND 2

Case	Metric	Aggregation-based method	Identification-based method	Proposed method	TDS result
1	RoCoF (Hz/s)	-0.400 (-6.50%)		-0.435 (1.69%)	-0.428
	$\Delta\omega_{st}$ (Hz)	-0.327 (-6.25%)		-0.350 (0.25%)	-0.349
2	RoCoF (Hz/s)	-0.533 (-27.81%)	-0.571 (-22.76%)	-0.750 (1.51%)	-0.739
	$\Delta\omega_{st}$ (Hz)	-0.436 (-16.21%)	-0.465 (-10.62%)	-0.500 (-4.00%)	-0.521

From these results, we can find that:

- 1) The aggregation-based method is over-optimistic in both cases, particularly in Case 2. It results from neglecting dead zones and control delays in Case 1 and ignoring PRLs in Case 2.
- 2) The identification-based method considering the data of Case 1 overestimates inertia in Case 2, as it fails to account for the PRLs. Conversely, using Case 2 data for identifica-

tion would yield an overly conservative result when applied to Case 1.

3) The proposed method achieves higher accuracy in both cases with relative errors below 2%, showing its ability to account for the PRLs of IBRs.

Similarly, the PFR is assessed using the three methods. The frequency responses in Cases 1 and 2 are shown in Fig. 5. These results further verify the effectiveness of the proposed method.

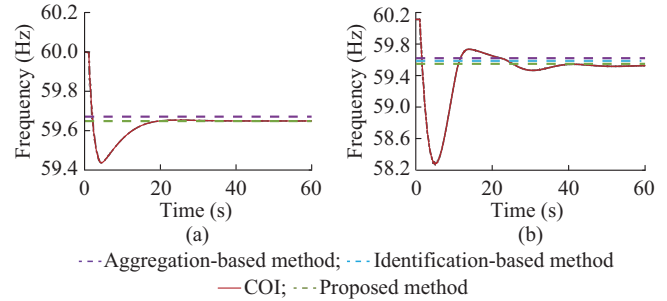


Fig. 5. Frequency responses in Cases 1 and 2. (a) Case 1. (b) Case 2.

It is also noteworthy that the frequency nadir in Case 2 is significantly lower than that in Case 1. However, the proposed method cannot deal with the transient behavior of PFR under saturation, and therefore cannot assess the frequency nadir. Addressing this limitation is a direction for our future work.

We further compare the effects of PRLs, dead zones, and control delays on J_{eq} based on Case 2. To this end, J_{eq} is assessed by considering each factor individually. Specifically, in (19), setting the dead zone to zero, and assuming infinite PRL, or using the nominal inertia (instead of the effective inertia) neglect the impact of each respective factor. These single-factor results are compared with that when all factors are included, as shown in Fig. 6. For reference, the aggregation-based method is also presented.

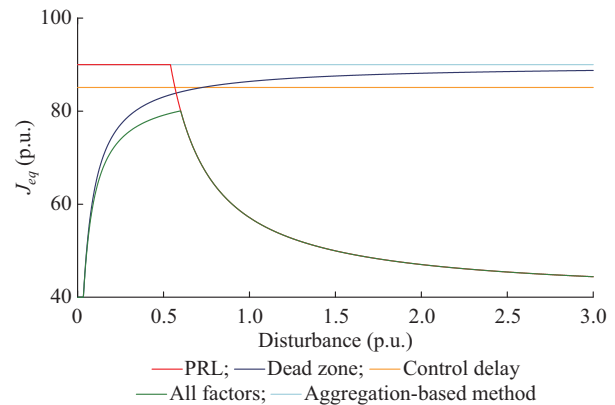


Fig. 6. Comparison of J_{eq} considering different factors and aggregation-based method.

From Fig. 6, it can be observed that for small disturbances, the dead zone has the greatest impact, as its result being closest to the accurate values. A slight discrepancy remains due to the effect of control delay. In contrast, for large disturbances, the PRLs dominate, considering that only the PRL

produces results that match the accurate values.

C. Different Characteristics Between GFL IBR and GFM IBR

To investigate the differences between GFL IBR and GFM IBR, G1 and G7 are replaced with GFM control based on Case 2, resulting in Cases 3 and 4. In Case 3, the GFM IBRs are assumed to have sufficient energy storage, allowing their inertia and damping responses to remain effective throughout the disturbance. In Case 4, it is assumed that the GFM IBRs can sustain overload for only 10 s, after which the overload mitigation control is activated [26]. Thereafter, the GFM IBRs operate at their maximum reserved power, behaving similarly to GFL IBRs. Consequently, when assessing the equivalent PFR, these GFM IBRs should be treated as GFL IBRs.

The frequency and active power responses are shown in Fig. 7. The proposed method accurately assesses the equivalent inertia and PFR with GFM converters. Besides, we find that: the effective inertia of the GFM IBRs (11.68 p.u.) is larger than that of GFL IBRs (9.02 p.u.), due to shorter control delay and the contribution of damping; and with overload capability of GFM IBRs, the equivalent inertia in Cases 3 and 4 (81.76 p.u.) is higher than that in Case 2 (64.00 p.u.).

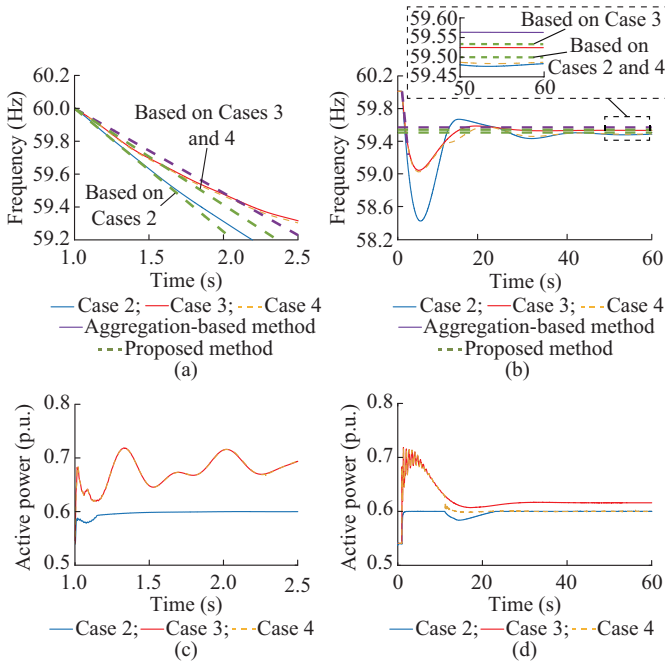


Fig. 7. Frequency and active power responses in Cases 2-4. (a) Frequency response from 1.0 s to 2.5 s. (b) Frequency response from 0 to 60 s. (c) Active power response of G7 from 1.0 s to 2.5 s. (d) Active power response of G7 from 0 to 60 s.

D. Verification of Proposed Optimal Parameter-setting Approach

In this subsection, the proposed optimal parameter-setting approach is validated using Cases 5 and 6. It is assumed that the system operator requires the RoCoF to remain below $\dot{\omega}_{th} = 0.35$ Hz/s under a disturbance of $\Delta P_{dis,th} = 0.6$ p.u.. Accordingly, the minimum required values of J_{sum}^{GFL} and ΔP_{Lsum}^{GFL}

are 62.85 p.u. and 0.367 p.u., respectively. The actual total power reserve of GFL IBRs is 0.4 p.u., which satisfies this requirement. The inertia parameters of individual GFL IBR are then determined by (21) (Case 5). For comparison, Case 6 sets identical inertia and damping parameters for all GFL IBRs while keeping their total power reserve unchanged. The inertia and damping settings for Case 5 are as follows: when $i = 1, 4, 5$, $J_i^{GFL} = 15.71$, $D_i^{GFL} = 12.5$; and when $i = 7, 8$, $J_i^{GFL} = 7.86$, $D_i^{GFL} = 6.25$. The inertia and damping parameter settings for Case 6 are as follows: when $i = 1, 4, 5, 7, 8$, $J_i^{GFL} = 12.57$, and $D_i^{GFL} = 10$.

J_{eq} in Cases 5 and 6 is shown in Fig. 8(a). Under a 0.6 p.u. disturbance, J_{eq} in Case 6 is smaller than that in Case 5, due to the saturation of certain IBRs. This observation is verified by the simulation in Fig. 8(b). A small deviation between the simulated RoCoF (0.366 Hz/s) and the target threshold (0.35 Hz/s) arises, mainly from the neglect of dead zone effects. Nevertheless, these results confirm the effectiveness of the proposed optimal parameter-setting approach.

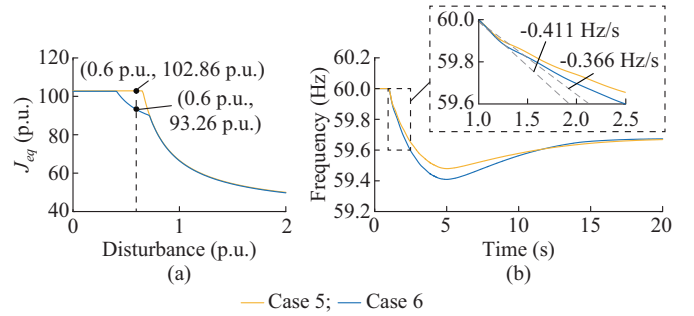


Fig. 8. J_{eq} and frequency response in Cases 5 and 6. (a) J_{eq} . (b) Frequency response.

E. Effects of Uncertainties on Equivalent Inertia

The inertia parameters of IBRs are typically preset and remain constant during operation, which is generally non-optimal due to the highly variable weather conditions. To illustrate the impact of such non-optimal parameter settings on the equivalent inertia, we consider a power system consisting of 80 GFL IBRs for WTGs and 20 SGs, each with identical capacity (Case 7).

In this case, wind speeds are generated according to the Weibull distribution with $c = 6.619$ and $k_{sh} = 2.088$ [37]. The corresponding wind power is calculated using (3), where $v_{in} = 5$ m/s, $v_0 = 13$ m/s, and $v_{out} = 25$ m/s. The probability distribution of wind power is shown in Fig. 9(a). For stable operation, WTGs with wind power below 0.2 p.u. (power reserve is less than 0.02 p.u., considering $K_{dl} = 0.9$) are assumed not to participate in frequency regulation, and their frequency control is disabled [38]. Therefore, in Fig. 9(b), WTGs with an initial power reserve below 0.02 p.u. are treated as having zero reserve.

The inertia parameters of all WTGs and SGs are 10 p.u., giving a nominal system inertia of 1000 p.u.. However, this nominal value is valid only under small disturbances, as shown in Fig. 10(a). Even a slight increase in disturbance size causes the equivalent inertia to drop sharply to around 400 p.u., far below the expected value. In contrast, if the in-

ertia of WTGs is set according to their power reserves, J_{eq} remains close to 1000 p.u. over a wider disturbance range, resulting in a much higher average value, as shown in Fig. 10(b). Note that in Figs. 10(a)-(c), each light-colored line represents a Monte Carlo simulation based on generated wind speed samples, while the central dark line indicates their average. In Fig. 10(d), the green line represents the equivalent inertia calculated by the proposed method, the orange line represents the result of the aggregation-based method, and the black line represents the identification-based method. We further assess the scenario in which the inertia parameters are adjusted based on predicted wind speeds, as described in Section IV-B, assuming a prediction standard deviation of $\sigma=0.1$. The result, as presented in Fig. 10(c), reveals a considerable improvement in J_{eq} . This demonstrates the importance of frequency regulation coordination among IBRs, especially when they serve as the primary sources to frequency support.

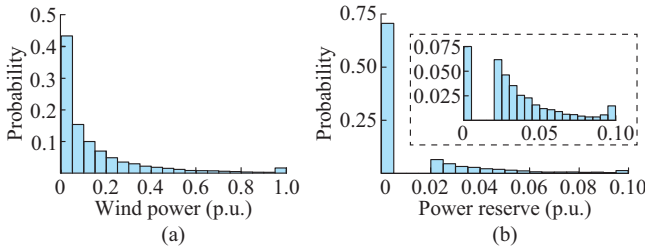


Fig. 9. Probability distributions of wind power and power reserve. (a) Wind power. (b) Power reserve.

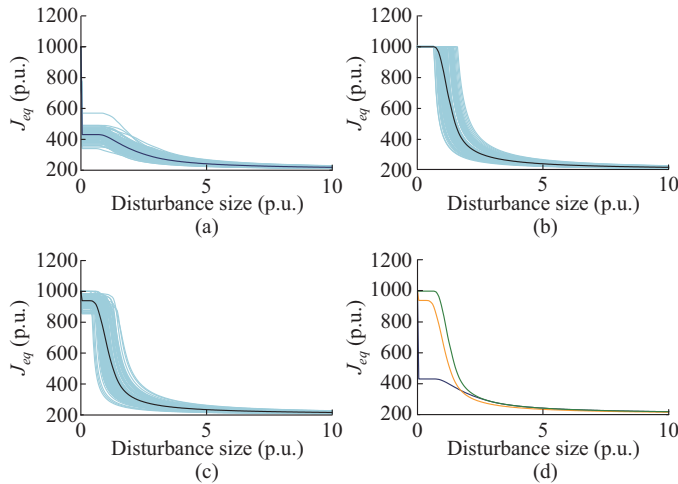


Fig. 10. Trajectories of J_{eq} under different disturbance sizes. (a) Uniform inertia parameter setting (all generation units are assigned an inertia of 10 p.u.). (b) Theoretical optimal setting. (c) Inertia parameter setting based on predicted wind speeds. (d) Comparison of average equivalent inertia values in three methods.

VI. CONCLUSION

This paper proposes an assessment method of equivalent inertia and PFR of power systems, considering the PRLs of IBRs. The relationship among the equivalent inertia and PFR, the PRLs of individual IBRs (especially GFL IBRs), and disturbances, is rigorously derived. Moreover, the effects of dead zones and control delays are included to further re-

fine the assessment results. Compared with existing aggregation- or identification-based methods, the proposed method significantly improves assessment accuracy when the PRLs of IBRs are non-negligible. Case studies demonstrate that the relative error in the assessed RoCoF can be reduced from up to about 30% to less than 2%.

Building on the insights from the assessment, an optimal parameter-setting approach is developed. Its core principle is to match the regulation parameters with the power reserves of IBRs. However, achieving this optimal configuration in practice is challenging, as these parameters of IBRs are generally fixed during operation. Consequently, the equivalent inertia may be significantly smaller than expected, especially in power systems with a high penetration of IBRs.

The proposed method focuses on equivalent inertia and PFR parameters, corresponding to RoCoF and steady-state frequency deviations, respectively. It cannot capture the complex PFR dynamics in transient, and therefore, cannot analyze frequency nadirs. Addressing this limitation is our future work direction.

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- Yu Shan** received the B.Eng. degree in electrical engineering from Southwest Jiaotong University, Chengdu, China, in 2020, and the Ph.D. degree in electrical engineering from Zhejiang University, Hangzhou, China, in 2025. In 2023, he was a Visiting Student with Imperial College London, London, UK. He is currently with Electric Power Research Institute of State Grid Fujian Electric Power Co., Ltd., Fujian, China. His research interests include power system frequency stability and control of inverter-based resources.
- Huisheng Gao** received the B.Eng. and Ph.D. degrees from Zhejiang University, Hangzhou, China, in 2017 and 2022, respectively. Currently, he is a Postdoctoral Researcher with the College of Electrical Engineering at Zhejiang University. His research interests include stability analysis and control of new power systems.
- Jialiang Wu** received the B.Eng. and Ph.D. degrees from the College of Electrical Engineering, Zhejiang University, Hangzhou, China, in 2018 and 2024, respectively. He is currently working at the Zhejiang Energy R&D Institute Co. Ltd., Hangzhou, China. His research interests include power system frequency stability analysis and control of renewable energy and voltage source converter-based high-voltage direct current (VSC-HVDC) transmission.
- Linbin Huang** received the B.Eng. and Ph.D. degrees from Zhejiang University, Hangzhou, China, in 2015 and 2020, respectively. Currently, he is a ZJU100 Young Professor at Zhejiang University. From 2020 to 2024, he was with ETH Zürich, Zürich, Switzerland, where he was a Postdoctoral Researcher and then promoted to a Senior Scientist. His research interests include power system stability, optimal control of power electronics, and data-driven control.
- Zhen Wang** received the B.Eng., M.Eng., and Ph.D. degrees from Xi'an Jiaotong University, Xi'an, China, Zhejiang University, Hangzhou, China, and The Hong Kong Polytechnic University, Hong Kong, China, in 1998, 2001, and 2009, respectively. He is currently a Full-time Professor and the Director of the Institute of Power and Energy Integration & Intelligence, Zhejiang University. His research interests include power system stability and control, renewable energy integration, and VSC-HVDC transmission.
- Huanhai Xin** received the Ph.D. degree in the College of Electrical Engineering, Zhejiang University, Hangzhou, China, in 2007. He was a Postdoctoral Researcher in the Department of Electrical Engineering and Computer Science, University of Central Florida, Orlando, USA, from June 2009 to July 2010. He is currently a Professor in the College of Electrical Engineering, Zhejiang University. His research interests include power system stability analysis and control of power systems with renewable energy.