

Optimal Coalition Structure Model for Multi-energy Retailers in Electricity and Gas Markets

Shuai He, Ming Yang, Nian Liu, Xiaohe Yan, and Jianpei Han

Abstract—Multi-energy retailer (MER) is a new type of retailer participating in both the electricity and gas markets. In a co-operative and competitive market environment, how to find the optimal coalition structure (OCS) model for MERs is an unsolved problem. Thus, this paper proposes an OCS model for MERs in electricity and gas markets. First, an MER optimization model is built to maximize profit, and the market-clearing constraints are considered. A coalition game model for MERs including coalition value and OCS model is introduced, which describes the behavior of forming a coalition among MERs considering coalition cost and scale limitation. Moreover, a hybrid algorithm including the diagonalization algorithm (DGA) and dynamic programming algorithm (DPA) is designed to solve the OCS model. Finally, the case studies are performed on a system with IEEE 14-bus electric network and 7-bus gas network. The numerical results show that the proposed model can effectively obtain OCS.

Index Terms—Multi-energy retailer, coalition game, optimal coalition structure, coalition value, coalition cost, electricity market, gas market.

NOMENCLATURE

A. Indices and Sets

b_e, B_e	Index and set of electric demand response segments
b_g, B_g	Index and set of gas demand response segments
i, N_d	Index and set of multi-energy retailers (MERs)
j, N_{gu}	Index and set of gas-fired unit
l_e, L_e	Index and set of electric network lines
l_g, L_g	Index and set of gas network lines

L_{g,n_g}	Set of gas network lines connected to the n_g^{th} node
$n_{e,j}, n_{g,j}, i_j$	Indexes of electric nodes, gas nodes, and MER ownership of the j^{th} gas-fired unit
n_e, N_e	Index and set of electric network nodes
n_g, N_g	Index and set of gas network nodes
t, T	Index and set of dispatch periods

B. Parameters

α_{gl}	Adjustment coefficient of power constraint of gas pipeline
η_G	Gas-fired unit efficiency
ϕ_{gl}	Coefficient of gas pipeline transmission
$\pi_{\min}, \pi_{\max}$	Lower and upper limits of gas pressure at start and end nodes of a gas network line, with sizes of $N_g \times 1$
ϵ_{gl}	Vector of violation of trigger criteria of Weymouth constraint
ϵ_{gcheck}	Criteria for satisfying Weymouth constraint
A_{es}, A_{gs}	Quadratic coefficient matrices of day-ahead (DA) electricity and gas supply costs, with sizes of $N_e \times N_e$ and $N_g \times N_g$
B_{es}, B_{gs}	Linear coefficient matrices of DA electricity and gas supply costs, with sizes of $N_e \times 1$ and $N_g \times 1$
c_{co}	Coalition cost factor
c_G	Cost factor vector of gas-fired unit, with size of $N_g \times 1$
G_e	Transfer matrix of electric network, with size of $N_e \times L_e$
k_{unit}	Unit conversion factor
$P_{\text{esup}}^{\min}, P_{\text{esup}}^{\max}$	Power vectors of lower and upper limits of electricity supply, with sizes of $N_e \times 1$
$P_{\text{el}}^{\min}, P_{\text{el}}^{\max}$	Power vectors of lower and upper limits of electric transmission line, with sizes of $L_e \times 1$
$P_{\text{gs}}^{\min}, P_{\text{gs}}^{\max}$	Power vectors of lower and upper limits of gas supply, with sizes of $N_g \times 1$
$P_{\text{gl}}^{\min}, P_{\text{gl}}^{\max}$	Power vectors of lower and upper limits of gas transmission line, with sizes of $L_g \times 1$
$P_j^{\text{GUE}, \min}, P_j^{\text{GUE}, \max}$	Lower and upper power limits of the j^{th} gas-fired units

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$P_{i,t,b_e}^{\text{ed,max}}, P_{i,t,b_g}^{\text{gd,max}}$ Demand response load vectors of upper limits for electric and gas networks in the b^{th} segment of the i^{th} MER, with sizes of $N_e \times 1$ and $N_g \times 1$

$q_{i,t,b_e}^{\text{ed}}, q_{i,t,b_g}^{\text{gd}}$ Load price vectors for electric and gas networks in the b^{th} segment of the i^{th} MER, with sizes of $N_e \times 1$ and $N_g \times 1$

S_{max} Scale limitation of coalition

C. Variables

μ_t^e, μ_t^g Dual variable vectors of DA electric and gas power balance constraints

$\mu_{n_g,t}^g$ Dual variable of DA gas power balance constraint in the n_g^{th} node

$\lambda_t^{\text{es,min}}, \lambda_t^{\text{es,max}}$ Dual variable vectors of lower and upper limits of DA electricity supply constraints, with sizes of $N_e \times 1$

$\lambda_t^{\text{el,min}}, \lambda_t^{\text{el,max}}$ Dual variable vectors of lower and upper limits of DA electric network line power constraints, with sizes of $L_e \times 1$

$\lambda_t^{\text{gs,min}}, \lambda_t^{\text{gs,max}}$ Dual variable vectors of lower and upper limits of DA gas supply constraints, with sizes of $N_g \times 1$

$\lambda_t^{\text{gl,min}}, \lambda_t^{\text{gl,max}}$ Dual variable vectors of lower and upper limits of DA gas network line power constraints, with sizes of $L_g \times 1$

$\pi_t^{\text{gl,start}}, \pi_t^{\text{gl,end}}$ Gas pressure vectors at start and end nodes of a gas network line, with sizes of $L_g \times 1$

ΔP_t^{wey} Vector of violation of Weymouth constraint, with size of $L_g \times 1$

c_i^{GU} Operation cost of gas-fired unit of the i^{th} MER

c_i^{DA} Cost of purchasing electricity and gas in DA market

D_{OCS} Number of coalitions within OCS

$E_t^{\text{gl,linepack}}$ Linepack energy vector of gas network line, with size of $L_g \times 1$

$E^{\text{gl,linepack,min}}$ Linepack energy vector of lower limit of gas network line, with size of $L_g \times 1$

$E^{\text{gl,linepack,max}}$ Linepack energy vector of upper limit of gas network line, with size of $L_g \times 1$

$f_{\text{eda}}, f_{\text{gda}}$ Market-clearing objectives of DA electricity and gas markets

f_i Profit of the i^{th} MER

$f_S, f_{0,S}$ Profits of coalition S with and without considering coalition costs

f_{gcheck} Indicator for evaluating deviation of Weymouth equation

$f_{e,\text{DR}}$ Mapping relationship between electricity retail price and user electric load

$f_{g,\text{DR}}$ Mapping relationship between gas retail price and user gas load

$P_{n_g,t}^{\text{gsnode}}$ Supply power of the n_g^{th} node in gas network

$P_{n_g,i,t}^{\text{gd}}$ Power of DA purchased gas for the n_g^{th} node of the i^{th} MER

$P_{i,n_e,t}^{\text{eload}}, P_{i,n_g,t}^{\text{gload}}$ Electric and gas node loads for the n_e^{th} and n_g^{th} nodes of the i^{th} MER

$P_t^{\text{esup}}, P_t^{\text{gsup}}$ Vectors of DA electricity and gas supplies, with sizes of $N_e \times 1$ and $N_g \times 1$

$P_{i,t,b_e}^{\text{ed}}, P_{i,t,b_g}^{\text{gd}}$ Vectors of electric and gas node loads of users in the b_e^{th} and b_g^{th} segments of the i^{th} MER, with sizes of $N_e \times 1$ and $N_g \times 1$

$P_{i,t}^{\text{ed}}, P_{i,t}^{\text{gd}}$ Power vectors of DA electric and gas node loads of the i^{th} MER, with sizes of $N_e \times 1$ and $N_g \times 1$

$P_{i,t}^{\text{GUe}}, P_{i,t}^{\text{GUg}}$ Power vectors of electric and gas node loads of gas-fired unit of the i^{th} MER, with sizes of $N_e \times 1$ and $N_g \times 1$

$P_{j,n_e,t}^{\text{GUe}}, P_{j,n_g,t}^{\text{GUg}}$ Power of the n_e^{th} electric node and the n_g^{th} gas node of the j^{th} gas-fired unit

$P_{j,n_e,t}^{\text{GUe}}, P_{j,n_g,t}^{\text{GUg}}$ Power of the $n_{e,j}^{\text{th}}$ electric node and the $n_{g,j}^{\text{th}}$ gas node of the j^{th} gas-fired unit of the i^{th} MER

$P_t^{\text{gl,ave}}$ Vector of average transmission power of gas network line, with size of $L_g \times 1$

$P_{l_g,t}^{\text{gl,start}}, P_{l_g,t}^{\text{gl,end}}$ Transmission power at start and end of the l_g^{th} gas network line

$P_t^{\text{gl,start}}, P_t^{\text{gl,end}}$ Vectors of transmission power at start and end of gas network line, with sizes of $L_g \times 1$

q_t^e, q_t^g Vectors of DA electric and gas node prices, with sizes of $N_e \times 1$ and $N_g \times 1$

$q_{i,n_e,t}^{\text{e,retail}}, q_{i,n_g,t}^{\text{g,retail}}$ Electricity and gas retail prices of the n_e^{th} electric node and the n_g^{th} gas node of the i^{th} MER

ΔT Duration of dispatch periods

u_i^{profit} Electricity and gas sale profit of the i^{th} MER

$V_S, V_{0,S}$ Coalition values of coalition S with and without considering coalition costs

V_S^{SC} Coalition value of coalition S after super-additive coverage

V_M^{SC} Coalition value of sub-coalition M after super-additive coverage

I. INTRODUCTION

WITH the increase in the proportion of multi-energy equipment such as gas-fired units (GUs) and power-to-gas (P2G) units, multiple energy systems and energy markets are strongly coupled, and this trend is continuing [1]-[5]. The traditional electricity and gas retailers have gradually become multi-energy retailers (MERs) [6]-[8]. MER submits bids to market operators to purchase energy in the wholesale market. In contrast, MER signs energy contracts with users to determine the retail price in the retail market, which lasts for months to years. In addition to purchasing the energy on behalf of users in the wholesale market, some MERs also have self-owned energy equipment such as GU and P2G unit [6].

As a relatively new field in energy retailer research, the

existing studies of MERs are mainly focused on the decision-making of a single MER in the wholesale market. It is assumed that MER signs a contract with the user and agrees on the retail price. The decision-making problem of a single MER can be considered as an energy scheduling problem. The decision variables include the clearing power of wholesale market [6] - [9], the self-owned equipment power [6], and the load of demand response [7], etc. Generally, due to the complexity of the wholesale market model, it is common to simplify the wholesale price as a constant or random variable [6], [7], [9] or to define an equivalent function for the wholesale price and clearing power [8]. The self-owned equipment such as GU and P2G unit can be set as a cost model of operation. During energy scheduling, stochastic optimization, robust optimization [6], [9], and information gap decision [7] are used to deal with the uncertainty of the user load, renewable energy, and wholesale energy price. Moreover, the constraints of distribution network are considered within the MER scheduling [8]. However, studies that consider the interaction among multiple MERs are still very limited. Generally, there are two types of interactions among MERs: competition and cooperation. For the competition among multiple MERs, a bi-level model based on game theory for energy producers and MERs is proposed in [10]. As for the cooperation among multiple MERs, there is no relevant research so far. In addition, this paper mainly studies the coalition behavior of MER. Whether to consider uncertainty has little impact on the modeling and analysis of MER coalition behavior. Therefore, to highlight the research focus, uncertainty is not considered.

It is stressed that recent studies have shown that coalition game theory is very effective and suitable for formulating cooperation among traditional electricity retailers. The formation of alliances among market participants can achieve complementary advantages, effectively enhancing their individual profits [11]-[14]. Coalition game theory analyzes the formulation, joint actions, and collective payoffs of coalitions. Generally, the profit of all electricity retailers in the formed coalition is defined as the coalition value [11]-[14], which is the evaluation index of the coalition. By maximizing coalition value, all retailers can form a unified coalition called a grand coalition [11], [12]. Then, a reasonable operation strategy is formulated to maximize the profits of the grand coalition. In contrast, the coalition cost is also considered in [13] and [14], which is defined as the communication and coordination cost required by retailers to form a coalition. In this regard, if there are many retailers in the grand coalition, the cost would be huge, which would reduce the coalition profit. Therefore, it is possible to form multiple coalitions in some scenarios, and the combined form of multiple coalitions is defined as a coalition structure (CS) [13], [14]. Moreover, the optimal coalition structure (OCS) can be formed to maximize the profits of MERs. Finally, it is important to determine the profit allocation of retailers in the coalition, which includes the Core method [11], Shapley method [12], [14], multi-index allocation method [13], etc.

In general, OCS is a complex structure composed of multiple coalitions. There is a cooperation among members of the coalition and a competition among different coalitions [15].

Currently, the studies of OCS for retailers are very limited. The OCS model of the electricity retailer is established in [13], and the deviation power of the retailers within the coalition can be complemented and balanced, thereby reducing the cost and risk of energy deviation. In [14], in the environment of the electricity market with uncertainty, electricity retailers reduce their risk by forming OCS. To obtain OCS, the enumeration method [13] and the heuristic method [14] have been adopted. However, the coalition profits are not affected by other coalition strategies in [13]. That is, competition is not considered between different coalitions. In [14], the profit of the retailer coalition is only related to the market price and its own strategy. As the market price is assumed to be a random variable in the modeling, the competition between different coalitions is ignored.

Therefore, addressing the problem of multiple CSs for MERs requires bridging three key gaps: the absence of MER optimization model participating in integrated electricity and gas markets; the absence of an approach to quantifying coalition value under an OCS that captures internal cooperation and external competition; and the absence of an algorithm capable of solving OCS while maintaining equilibrium among multiple coalitions.

To this end, an OCS model for MERs in electricity and gas markets is proposed. The main contributions are as follows.

- 1) An MER optimization model participating in both electricity and gas markets is established. Decision variables for MER include energy trading power in electricity and gas markets and the self-owned GU power. The MER optimization model is built to describe the competition among MER coalitions.

- 2) Considering the competition among MER coalitions, a representation method of MER coalition value based on the min-max theory is given. The implication is the maximum profit that can be achieved in the context of the worst decisions of all other coalitions. Based on the MER coalition value, an OCS model of MER considering coalition cost and scale limitation is proposed.

- 3) A hybrid algorithm including a diagonalization algorithm (DGA) and a dynamic programming algorithm (DPA) is designed to solve the OCS model of MERs. The DGA is used to solve the equilibrium among coalitions to obtain the coalition value. Based on the coalition value, the DPA is applied to find OCS.

II. OPERATION MODEL OF ELECTRICITY AND GAS MARKETS

A. Structure of Electricity and Gas Markets

The electricity and gas markets are composed of power generation companies (Genco), gas companies (GCO), MERs, and users, and the structure is shown in Fig. 1.

Under general operation rules, energy suppliers and MERs participate in the wholesale market and submit bids containing quantity and price information to market operators, and then market operators clear the market according to the principle of social welfare maximization. In the retail market, MER signs energy contracts with users, which lasts for months to years. It is assumed that each user selects an

MER to sign a contract, and the replacement of MER is not considered in this paper.

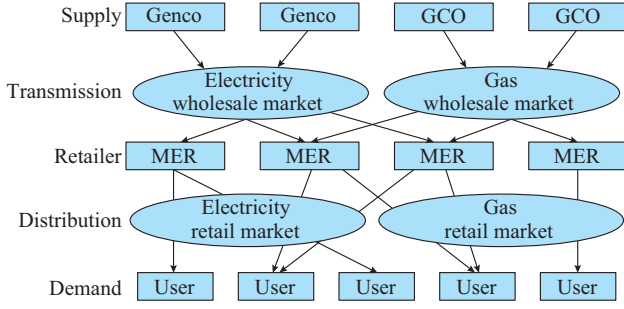


Fig. 1. Structure of electricity and gas markets.

MER can be composed of GUs, electricity, and gas, and the internal users of MER can be connected to multiple buses in the electric and gas networks. The focus of this paper is MER, so it is considered that suppliers are non-strategic stakeholders and bid on marginal cost. This paper assumes that MERs are mutually aware of each other's complete information, implying a scenario of a perfect information game. In reality, interactions among MERs often constitute an imperfect information game. However, the number of MERs in the market is relatively small, and they interact with each other over a long period. It can be assumed that MERs have a relatively high accuracy degree in the information about other MERs. Furthermore, perfect information games serve as a foundation for imperfect information games, offering insights for decision-making by MERs and laying the groundwork for equilibrium analysis and social welfare assessment in the market.

The bidding formats generally include block quantity price bidding, only quantity bidding, and linear quantity price bidding, as shown in Fig. 2, where P_0 and Q_0 represent the market-clearing price and quantity, respectively.

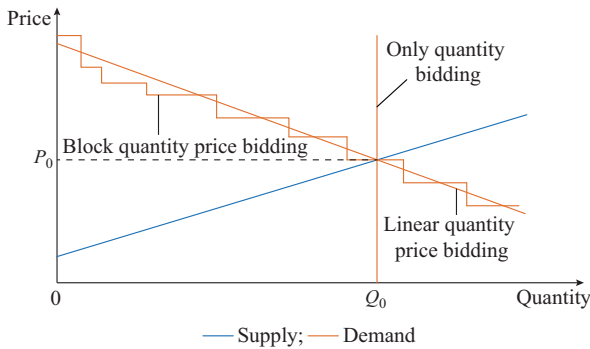


Fig. 2. Comparison of different bidding formats.

MER is on the demand side, hence it is assumed that the supply curve is a marginal cost function. When the supply curve remains unchanged, the clearing result can be understood as a point on the supply curve. Therefore, the actual decision of MER is the quantity to be cleared, and the bidding decision variables of different bidding formats are only intermediate variables. Provided that the supply side has quantity and price information, the three bidding formats of MER can achieve the same clearing result.

To simplify the calculation, the bidding format of MER in this paper only provides the quantity. Under some electricity market rules and related references, the bidding format of electricity retailers only provides quantity, too [16], [17]. In terms of network constraints, the electric network uses the DC power flow model, and the gas network uses the Weymouth equation model. Meanwhile, the influence of linepack in the gas network is considered. This paper mainly models the power distribution of the gas network. Therefore, it is more appropriate to use the energy corresponding to linepack as a variable for modeling.

In this paper, it is assumed that the electricity and gas markets clear the day-ahead (DA) market simultaneously on the same timescale [18]–[21]. With the deep coupling of the electricity and gas markets, this is also one of the future development trends.

B. Operation Model of DA Electricity Market

The clearing process of DA electricity market is formulated as follows, which is equivalent to DC optimal power flow.

$$\min f_{\text{eda}} = \sum_{t \in T} ((P_t^{\text{esup}})^T A_{\text{es}} P_t^{\text{esup}} + (P_t^{\text{esup}})^T B_{\text{es}}) \Delta T \quad (1a)$$

s.t.

$$\mathbf{1}^T P_t^{\text{esup}} = \mathbf{1}^T \sum_{i \in N_d} P_{i,t}^{\text{ed}} \cdot \mu_t^e \quad (1b)$$

$$P_{\text{esup}}^{\min} \leq P_t^{\text{esup}} \leq P_{\text{esup}}^{\max} \cdot \lambda_t^{\text{es}, \min}, \lambda_t^{\text{es}, \max} \quad (1c)$$

$$P_{\text{el}}^{\min} \leq G_e \left(P_t^{\text{esup}} - \sum_{i \in N_d} P_{i,t}^{\text{ed}} \right) \leq P_{\text{el}}^{\max} \cdot \lambda_t^{\text{el}, \min}, \lambda_t^{\text{el}, \max} \quad (1d)$$

The objective function (1a) optimizes the energy in the DA electricity market by minimizing the total cost of energy offered by all energy suppliers. Constraint (1b) enforces the DA electric power balance. Constraint (1c) represents the upper and lower power limits of the energy supplier. Constraint (1d) imposes the transmission capacity limit of each power line.

C. Operation Model of DA Gas Market

The clearing process of DA gas market is formulated as follows, aiming to minimize costs while meeting the operation constraints of the gas network.

$$\min f_{\text{gda}} = \sum_{t \in T} ((P_t^{\text{gsup}})^T A_{\text{gs}} P_t^{\text{gsup}} + (P_t^{\text{gsup}})^T B_{\text{gs}}) \Delta T \quad (2a)$$

s.t.

$$P_{n_g, t}^{\text{gsnode}} - \sum_{i \in N_d} P_{n_g, i, t}^{\text{gd}} + \sum_{l_g \in L_{g, n_g}} (P_{l_g, t}^{\text{gl, end}} - P_{l_g, t}^{\text{gl, start}}) = 0; \mu_{n_g, t}^g \quad (2b)$$

$$P_{\text{gs}}^{\min} \leq P_t^{\text{gsup}} \leq P_{\text{gs}}^{\max} \cdot \lambda_t^{\text{gs}, \min}, \lambda_t^{\text{gs}, \max} \quad (2c)$$

$$P_{\text{gl}}^{\min} \leq P_t^{\text{gl, ave}} \leq P_{\text{gl}}^{\max} \cdot \lambda_t^{\text{gl}, \min}, \lambda_t^{\text{gl}, \max} \quad (2d)$$

$$P_t^{\text{gl, ave}} = \frac{P_t^{\text{gl, start}} + P_t^{\text{gl, end}}}{2} \quad (2e)$$

$$E_t^{\text{gl, linepack}} = E_{t-1}^{\text{gl, linepack}} + (P_t^{\text{gl, start}} - P_t^{\text{gl, end}}) \Delta T \quad (2f)$$

$$E_t^{\text{gl, linepack, min}} \leq E_t^{\text{gl, linepack}} \leq E_t^{\text{gl, linepack, max}} \quad (2g)$$

$$P_t^{\text{gl, ave}} = \phi_{\text{gl}} \text{sgn}(\pi_t^{\text{gl, start}} - \pi_t^{\text{gl, end}}) \sqrt{(\pi_t^{\text{gl, start}})^2 - (\pi_t^{\text{gl, end}})^2} \quad (2h)$$

$$\begin{cases} \pi_{\min} \leq \pi_t^{\text{gl, start}} \leq \pi_{\max} \\ \pi_{\min} \leq \pi_t^{\text{gl, end}} \leq \pi_{\max} \end{cases} \quad (2i)$$

The objective function (2a) optimizes the energy in the DA gas market by minimizing the total cost of energy offered by all energy suppliers. Constraint (2b) enforces the DA gas power balance. Constraint (2c) represents the upper and lower power limits of the energy supplier. Constraint (2d) imposes the transmission capacity limit of each power line. Constraint (2e) represents the relationship between the average transmission power and the transmission power at the start and end of the gas network line. Constraint (2f) represents the relationship between the linepack energy of transmission line and the transmission power at the start and end of the gas network line. Constraint (2g) represents the upper and lower limits of the linepack energy. Constraint (2h) represents the relationship between the power of the gas network line and the node gas pressure, with $\text{sgn}(\cdot)$ as the sign function. Constraint (2i) represents the upper and lower limits of the node gas pressure. Constraint (2h) is highly nonlinear and nonconvex. Generally, it is solved after relaxation. However, the relaxation method has the following problems. ① The equation after relaxation is nonlinear, which increases the difficulty in solving the model after the Karush-Kuhn-Tucker (KKT) transformation. ② Relaxation will lead to a loss of model accuracy. Therefore, the relaxation method is not suitable for this paper. The relaxation method is temporarily removed during the solution process. After the MER optimization is completed, the removed relaxation method is tested, and the MER optimization model is adjusted based on the verification results.

III. COALITION GAME MODEL FOR MERS

Since there are competition and cooperation among different MERs in the market, the coalition game theory is adopted to analyze the operation strategy of MER. A coalition game is one where players can communicate with each other and enforce cooperative behavior by forming coalitions. Hence, competition appears at the level of coalition, rather than between individual players. As the basis of the coalition game model for MERS, the MER optimization model is established first. Secondly, a representation method of MER coalition value based on min-max theory is given, assessing the priority level of coalitions. Building upon the MER coalition value, the OCS model of MER is formulated, representing CS with the highest sum of MER coalition values. Finally, the profit allocation model of MER in the coalition is proposed.

A. MER Optimization Model

The electricity and gas market-clearing models can be transformed into KKT conditions, which is used as a constraint of MER optimization model [22], [23]. The MER optimization model can be described as follows. Θ_i is the strategy set of the i^{th} MER, and its decision variable is $\Xi_i = \{\mathbf{P}_{i,t,b_e}^{\text{ed}}, \mathbf{P}_{i,t,b_g}^{\text{gd}}, \mathbf{P}_{i,t}^{\text{ed}}, \mathbf{P}_{i,t}^{\text{gd}}, \mathbf{P}_{i,t}^{\text{GUe}}, \mathbf{P}_{i,t}^{\text{GUg}}\}$. MER aims to subtract the operation costs of GU and the purchasing costs in the DA wholesale market from the user revenue by selling electricity

and gas. The objective function is expressed as:

$$\max_{\Xi_i} f_i = u_i^{\text{profit}} - c_i^{\text{GU}} - c_i^{\text{DA}} \quad (3)$$

Among these, the revenue from electricity sales is correlated with the user load and retail prices. MER has signed a retail contract with the user, establishing the retail price. Building upon this foundation, MER can enhance its revenue by implementing demand response, i.e., altering the user power by reducing the retail price. The relationship between the user power and the retail price is illustrated as:

$$\begin{cases} P_{i,n_e,t}^{\text{eload}} = f_{e,DR}(q_{i,n_e,t}^{\text{e,retail}}) \\ P_{i,n_g,t}^{\text{gload}} = f_{g,DR}(q_{i,n_g,t}^{\text{g,retail}}) \end{cases} \quad (4)$$

The relationship between u_i^{profit} and user load is expressed as:

$$u_i^{\text{profit}} = \sum_{t \in T} \left(\sum_{n_e \in N_e} q_{i,n_e,t}^{\text{e,retail}} P_{i,n_e,t}^{\text{eload}} + \sum_{n_g \in N_g} q_{i,n_g,t}^{\text{g,retail}} P_{i,n_g,t}^{\text{gload}} \right) = \sum_{t \in T} \left(\sum_{n_e \in N_e} f_{e,DR}^{-1}(P_{i,n_e,t}^{\text{eload}}) P_{i,n_e,t}^{\text{eload}} + \sum_{n_g \in N_g} f_{g,DR}^{-1}(P_{i,n_g,t}^{\text{gload}}) P_{i,n_g,t}^{\text{gload}} \right) \quad (5)$$

Theorem 1 u_i^{profit} is a concave function of the user load.

Proof See (S1a)-(S1c) in Supplementary Material A for details.

Because u_i^{profit} is a concave function, its linearized model can omit the SOS-2 (an ordered set of non-negative variables, of which at most two can be non-zero, and if two are non-zero, these two must be consecutive in their ordering) constraints or 0-1 variables, thereby reducing the difficulty in solving the model. u_i^{profit} can be linearized through the following differential form, reducing computation costs.

$$u_i^{\text{profit}} = \sum_{t \in T} \left(\sum_{b_e \in B_e} (q_{i,t,b_e}^{\text{ed}})^T \mathbf{P}_{i,t,b_e}^{\text{ed}} + \sum_{b_g \in B_g} (q_{i,t,b_g}^{\text{gd}})^T \mathbf{P}_{i,t,b_g}^{\text{gd}} \right) \Delta T \quad (6)$$

$$\mathbf{P}_{i,t,1}^{\text{ed}} = \mathbf{P}_{i,t,1}^{\text{ed,max}} \quad 0 \leq \mathbf{P}_{i,t,b_e}^{\text{ed}} \leq \mathbf{P}_{i,t,b_e}^{\text{ed,max}}, b_e \geq 2 \quad (7)$$

$$\mathbf{P}_{i,t,1}^{\text{gd}} = \mathbf{P}_{i,t,1}^{\text{gd,max}} \quad 0 \leq \mathbf{P}_{i,t,b_g}^{\text{gd}} \leq \mathbf{P}_{i,t,b_g}^{\text{gd,max}}, b_g \geq 2 \quad (8)$$

Regarding the electric power load, q_{i,t,b_e}^{ed} for each segment is the average differential of u_i^{profit} within that load segment. Since u_i^{profit} is concave, q_{i,t,b_e}^{ed} gradually decreases with an increase in the number of segments. $q_{i,t,1}^{\text{ed}}$ is the contracted electricity price offered by the i^{th} MER to users, and $\mathbf{P}_{i,t,1}^{\text{ed,max}}$ is the load under the contracted electricity price. The same logic applies to gas load. As the number of segments increases, the error of u_i^{profit} gradually decreases.

The operation costs of GU and the purchasing costs in the DA wholesale market are shown in (9) and (10), respectively.

$$c_i^{\text{GU}} = \sum_{t \in T} c_G^T \mathbf{P}_{i,t}^{\text{GUg}} \Delta t \quad (9)$$

$$c_i^{\text{DA}} = \sum_{t \in T} ((q_{i,t}^{\text{e}})^T \mathbf{P}_{i,t}^{\text{ed}} + (q_{i,t}^{\text{g}})^T \mathbf{P}_{i,t}^{\text{gd}}) \Delta t \quad (10)$$

$$\mathbf{P}_{i,t}^{\text{ed}} + \mathbf{P}_{i,t}^{\text{GUe}} - \sum_{b_e \in B_e} \mathbf{P}_{i,t,b_e}^{\text{ed}} = 0 \quad (11)$$

$$\mathbf{P}_{i,t}^{\text{gd}} - \mathbf{P}_{i,t}^{\text{GUg}} - \sum_{b_g \in B_g} \mathbf{P}_{i,t,b_g}^{\text{gd}} = 0 \quad (12)$$

$$\begin{cases} P_{i,t}^{\text{ed}} \geq 0 \\ P_{i,t}^{\text{gd}} \geq 0 \end{cases} \quad (13)$$

$$\begin{cases} P_j^{\text{GUe}, \min} \leq P_{j, n_{e,j}, i_j, t}^{\text{GUe}} \leq P_j^{\text{GUe}, \max} \\ P_{j, n_{e,j}, i_j, t}^{\text{GUe}} = k_{\text{unit}} \eta_G P_{j, n_{g,j}, i_j, t}^{\text{GUg}} \end{cases} \quad (14)$$

$$\begin{cases} P_{j, n_{e,j}, t}^{\text{GUe}} = \sum_{j \in N_{\text{gu}}, n_{e,j} = n_{e,j}, i = i_j} P_{j, n_{e,j}, i_j, t}^{\text{GUe}} \\ P_{j, n_{g,j}, t}^{\text{GUg}} = \sum_{j \in N_{\text{gu}}, n_{g,j} = n_{g,j}, i = i_j} P_{j, n_{g,j}, i_j, t}^{\text{GUg}} \\ \mathbf{P}_{i,t}^{\text{GUe}} = [P_{i,1,t}^{\text{GUe}}, P_{i,2,t}^{\text{GUe}}, \dots, P_{i,|N_{e,i}|,t}^{\text{GUe}}]^T \\ \mathbf{P}_{i,t}^{\text{GUg}} = [P_{i,1,t}^{\text{GUg}}, P_{i,2,t}^{\text{GUg}}, \dots, P_{i,|N_{g,i}|,t}^{\text{GUg}}]^T \end{cases} \quad (15)$$

$$kkt(\text{con}_{\text{eda}} \times_{t \in T} \mathbf{P}_t^{\text{esup}}), kkt(\text{con}_{\text{gda}} \times_{t \in T} \mathbf{P}_t^{\text{esup}}) \quad (16)$$

$$\mathbf{q}_t^e = \boldsymbol{\mu}_t^e + \mathbf{G}_e^T \boldsymbol{\lambda}_t^{\text{el}, \min} - \mathbf{G}_e^T \boldsymbol{\lambda}_t^{\text{el}, \max} \quad (17)$$

$$\mathbf{q}_t^g = \boldsymbol{\mu}_t^g + \mathbf{G}_g^T \boldsymbol{\lambda}_t^{\text{gl}, \min} - \mathbf{G}_g^T \boldsymbol{\lambda}_t^{\text{gl}, \max} \quad (18)$$

Equations (11) and (12) present the node power balance constraints of DA electricity and gas and real-time electricity and gas. Constraint (13) indicates that MER only allows the purchase of electricity and gas in the DA market. Constraints (14) and (15) present the electric and gas power constraints of GUs. Constraint (16) presents the KKT conditions of the DA electricity and gas market-clearing model, where con_{eda} denotes the electricity market-clearing constraint, con_{gda} denotes the gas market-clearing constraint, and $\times$ denotes the Cartesian product operation. Equations (17) and (18) present the relationship between node price and dual variables.

The MER profit is affected by the market-clearing price and energy, and the decision variable of MER is the bid. Therefore, it is necessary to describe the relationship between the bid and the market-clearing price. Under the KKT condition, the market-clearing model is transformed into a constraint, which is conducive to solving the MER optimization model. The first input of function $kkt(\cdot)$ is the constraint of the MER optimization model, the second input is the objective function, and the third input is the optimization variable.

It is worth noting that when performing the KKT transformation on the gas market-clearing model, constraints (2h) and (2i) are not included in the model. This is because they bring nonlinear constraints to the model, increasing the difficulty in solving the model. In order to ensure that the solution meets constraints (2h) and (2i), a post-verification process is added after the above model is solved. The gas flow obtained by solving the above model is input as a parameter into the verification model. In order to speed up the model solution, the square of the node gas pressure is used as a variable.

$$\min f_{\text{gcheck}} = \sum_{t \in T} |\Delta \mathbf{P}_t^{\text{wey}}| \Delta T \quad (19)$$

s.t.

$$(\boldsymbol{\pi}_t^{\text{gl}, \text{start}})^2 - (\boldsymbol{\pi}_t^{\text{gl}, \text{end}})^2 = \text{sgn}(\mathbf{P}_t^{\text{gl}, \text{ave}}) \left(\frac{\mathbf{P}_t^{\text{gl}, \text{ave}}}{\phi_{\text{gl}}} \right)^2 + \Delta \mathbf{P}_t^{\text{wey}} \quad (20)$$

$$\boldsymbol{\pi}_{\min}^2 \leq (\boldsymbol{\pi}_t^{\text{gl}, \text{start}})^2 \leq \boldsymbol{\pi}_{\max}^2 \quad (21)$$

$$\boldsymbol{\pi}_{\min}^2 \leq (\boldsymbol{\pi}_t^{\text{gl}, \text{end}})^2 \leq \boldsymbol{\pi}_{\max}^2 \quad (22)$$

The objective function (19) is to minimize the violation of the Weymouth constraint. Constraint (20) is the Weymouth equation constraint with bias relaxation, and constraints (21) and (22) are the upper and lower limit constraints of the node pressure.

After the gas market-clearing model is solved, the difference between the transmission power of gas transmission line and the power determined by the Weymouth equation can be calculated by:

$$\Delta \mathbf{P}_t^{\text{gl}, \text{ave}} = \phi_{\text{gl}} \sqrt{(\boldsymbol{\pi}_t^{\text{gl}, \text{start}})^2 - (\boldsymbol{\pi}_t^{\text{gl}, \text{end}})^2} - |\mathbf{P}_t^{\text{gl}, \text{ave}}| \quad (23)$$

The gas transmission line constraints can be modified in the MER optimization model according to the difference. The specific modification plan is expressed as follows.

If $\Delta \mathbf{P}_t^{\text{gl}, \text{ave}} \geq \varepsilon_{\text{gl}}$ and $\mathbf{P}_t^{\text{gl}, \text{ave}} \geq 0$,

$$|\mathbf{P}_t^{\text{gl}, \text{ave}}| + \alpha_{\text{gl}} \Delta \mathbf{P}_t^{\text{gl}, \text{ave}} \leq \mathbf{P}_t^{\text{gl}, \text{ave}} \leq \mathbf{P}_{\text{gl}}^{\max} \quad (24)$$

If $\Delta \mathbf{P}_t^{\text{gl}, \text{ave}} \geq \varepsilon_{\text{gl}}$ and $\mathbf{P}_t^{\text{gl}, \text{ave}} < 0$,

$$\mathbf{P}_{\text{gl}}^{\min} \leq \mathbf{P}_t^{\text{gl}, \text{ave}} \leq -|\mathbf{P}_t^{\text{gl}, \text{ave}}| - \alpha_{\text{gl}} \Delta \mathbf{P}_t^{\text{gl}, \text{ave}} \quad (25)$$

If $\Delta \mathbf{P}_t^{\text{gl}, \text{ave}} \leq -\varepsilon_{\text{gl}}$ and $\mathbf{P}_t^{\text{gl}, \text{ave}} \geq 0$,

$$\mathbf{P}_{\text{gl}}^{\min} \leq \mathbf{P}_t^{\text{gl}, \text{ave}} \leq |\mathbf{P}_t^{\text{gl}, \text{ave}}| + \alpha_{\text{gl}} \Delta \mathbf{P}_t^{\text{gl}, \text{ave}} \quad (26)$$

If $\Delta \mathbf{P}_t^{\text{gl}, \text{ave}} \leq -\varepsilon_{\text{gl}}$ and $\mathbf{P}_t^{\text{gl}, \text{ave}} < 0$,

$$-|\mathbf{P}_t^{\text{gl}, \text{ave}}| - \alpha_{\text{gl}} \Delta \mathbf{P}_t^{\text{gl}, \text{ave}} \leq \mathbf{P}_t^{\text{gl}, \text{ave}} \leq \mathbf{P}_{\text{gl}}^{\max} \quad (27)$$

After the constraints are modified, the MER optimization is re-calculated until f_{gcheck} is less than $\varepsilon_{\text{gcheck}}$, which means that the Weymouth equation constraints are satisfied.

Taking the operation model of DA electricity market as an example of KKT function, the original MER optimization model is (1a)-(1d). In the conversion process, $\mathbf{P}_t^{\text{esup}}$ is defined as a variable and $\mathbf{P}_{i,t}^{\text{ed}}, \mathbf{A}_{\text{es}}, \mathbf{B}_{\text{es}}, \mathbf{P}_{\text{esup}}^{\min}, \mathbf{P}_{\text{esup}}^{\max}, \mathbf{G}_e, \mathbf{P}_{\text{el}}^{\min}, \mathbf{P}_{\text{el}}^{\max}$ are defined as parameters. The converted KKT conditions are given as:

$$2\mathbf{A}_{\text{es}} \mathbf{P}_t^{\text{esup}} + \mathbf{B}_{\text{es}} - \boldsymbol{\mu}_t^e - \boldsymbol{\lambda}_t^{\text{es}, \min} - \boldsymbol{\lambda}_t^{\text{es}, \max} - \mathbf{G}_e^T \boldsymbol{\lambda}_t^{\text{el}, \min} + \mathbf{G}_e^T \boldsymbol{\lambda}_t^{\text{el}, \max} = \mathbf{0} \quad (28)$$

$$\mathbf{1}^T \mathbf{P}_t^{\text{esup}} = \mathbf{1}^T \sum_{i \in N_d} \mathbf{P}_{i,t}^{\text{ed}} \quad (29)$$

$$\mathbf{0} \leq (\mathbf{P}_t^{\text{esup}} - \mathbf{P}_{\text{es}}^{\min}) \perp \boldsymbol{\lambda}_t^{\text{es}, \min} \geq \mathbf{0} \quad (30)$$

$$\mathbf{0} \leq (\mathbf{P}_{\text{es}}^{\max} - \mathbf{P}_t^{\text{esup}}) \perp \boldsymbol{\lambda}_t^{\text{es}, \max} \geq \mathbf{0} \quad (31)$$

$$\mathbf{0} \leq [\mathbf{G}_e (\mathbf{P}_t^{\text{esup}} - \mathbf{P}_t^{\text{ed}}) - \mathbf{P}_{\text{el}}^{\min}] \perp \boldsymbol{\lambda}_t^{\text{el}, \min} \geq \mathbf{0} \quad (32)$$

$$\mathbf{0} \leq [\mathbf{P}_{\text{el}}^{\max} - \mathbf{G}_e (\mathbf{P}_t^{\text{esup}} - \mathbf{P}_t^{\text{ed}})] \perp \boldsymbol{\lambda}_t^{\text{el}, \max} \geq \mathbf{0} \quad (33)$$

Equation (28) represents the stationarity constraint, (29) represents the primal equality constraint, and (30)-(33) represent the complementary slackness conditions. From (29), it can be observed that in the market-clearing constraints, there are variables related to other MERs. The revenue of the i^{th} MER is influenced not only by its own decision but also by the decisions of other MERs. Therefore, the MER optimization problem falls into a multi-agent game problem. The game among MERs can be expressed as $\Gamma_1 = \langle N_d, \Theta_i, f_i \rangle$.

B. Coalition Value Model of MER

1) Min-max Representation

As the primary objective of the MER is monetary gain, its strategic interaction can be modeled as a coalition game with transferable utility. There are three different representation methods for its coalition value: min-max representation, defensive-equilibrium representation, and rational-threat representation [15]. The coalition value is super-additive when the min-max representation is used and the coalition cost is not considered, so it is easier to produce a profit allocation located in the core. Therefore, this paper selects the min-max representation to calculate the coalition value.

$$\begin{cases} V_{0,S} = \min_{\mathcal{E}_{N_d-S}} \max_{\mathcal{E}_S} \sum_{i \in S} f_i(\mathcal{E}_S, \mathcal{E}_{N_d-S}) \\ \text{s.t. (7), (8), (11)-(18)} \quad \forall i \in N_d \end{cases} \quad (34)$$

In (34), $\mathcal{E}_S = \{\mathcal{E}_i, i \in S\}$, and coalition S is a subset of N_d , where the MERs cooperate. $N_d - S$ represents the complementary coalition of coalition S . Equation (34) represents the value of coalition S without coalition cost, and an empty coalition is a null set.

2) Coalition Cost and Scale Limitation

Besides coalition surplus, the coalition will also increase the communication and coordination costs among MERs. Therefore, it is necessary to establish a coalition value function considering coalition cost. The more members of MERs the coalition has, the more complex the coalition relationship network needs to be built. The total number of lines of the coalition relationship network with $|S|$ members is $0.5|S|(|S| - 1)$, which indicates that the coalition cost of coalition S is proportional to the quadratic square of $|S|$ [12].

$$C_S^{\text{co}} = c_{\text{co}} |S|^2 \quad (35)$$

Considering the coalition cost, we can obtain:

$$V_S = V_{0,S} - C_S^{\text{co}} \quad (36)$$

The basic attitude towards coalitions in this paper is to allow coalitions within a certain scale. By forming a coalition, multiple MERs can collaborate to improve operation efficiency. However, a coalition that is too large may form a monopoly and have too much market power, thereby raising market prices, which will be regulated by the government. Consequently, the coalition scale should have a limitation. When the coalition scale exceeds the limitation, the coalition value is considered to be negatively infinite.

$$V_S = -\infty \quad \forall S > S_{\max} \quad (37)$$

C. OCS Model

Based on the coalition value, the coalition game among MERs can be expressed in the standard form, i.e., $\Gamma_2 = \langle N_d, V_S \rangle$. A partition of N_d into disjoint and exhaustive coalitions is called a CS.

Taking ten MERs as an example, a schematic case of CS is shown in Fig. 3. The coalition in Fig. 3 does not represent OCS in the following cases. Note that there are four coalitions in Fig. 3, and MERs in the same color belong to the same coalition. The internal motivation for the coalition formation is the pursuit of higher profits by MER. For a certain number of entities, multiple different coalitions can be

formed. When forming a coalition, MER faces the choice of whom to form a coalition with, and its selection criteria are based on the coalition value, i.e., the ability of the coalition to obtain benefits. After multiple MERs select coalition objects based on the coalition value, the scenario of multiple coalitions in Fig. 3 is finally formed.

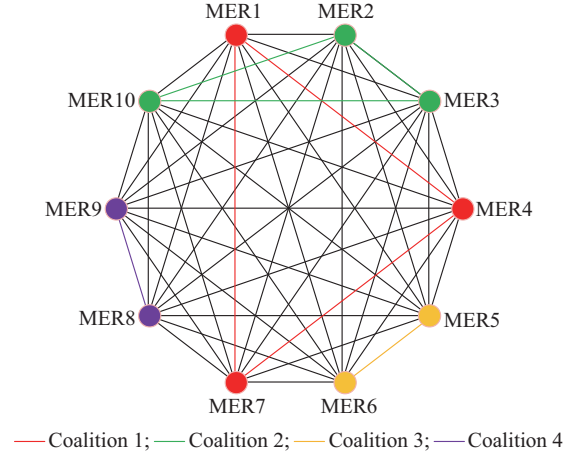


Fig. 3. Schematic case of CS.

The CS value is defined as the sum of each coalition value.

$$V(\text{CS}) = \sum_{S \in \text{CS}} V_S \quad (38)$$

The OCS model seeks to find a coalition structure CS^* that satisfies:

$$V(\text{CS}^*) \geq V(\text{CS}) \quad \forall \text{CS} \quad (39)$$

OCS can be found from a single objective optimization problem, which is a group formation that can be accepted by all MERs. Under OCS, the utility of each MER is the highest [13]. All MERs have no intention to change CS at this time. Therefore, OCS is achieved.

D. Coalition Profit in OCS

In OCS, each coalition aims to maximize its profit, i.e., there is a non-coalition game among coalitions in OCS. The profit of each coalition in OCS can be obtained by solving the equilibrium of the non-coalition game as:

$$\begin{cases} f_S = f_{0,S} - C_S^{\text{co}} = \sum_{i \in S} f_i(\mathcal{E}_S^{\text{NC}}, \mathcal{E}_{N_d-S}^{\text{NC}}) - C_S^{\text{co}} \quad \forall S \in \text{CS}^* \\ \text{s.t. (7), (8), (11)-(18)} \quad \forall i \in N_d \end{cases} \quad (40)$$

$$f_{0,S} \geq \sum_{i \in S} f_i(\mathcal{E}_S, \mathcal{E}_{N_d-S}^{\text{NC}}) \quad \forall \mathcal{E}_S, \forall S \in \text{CS}^* \quad (41)$$

Equation (40) represents the profit of coalition S , and (41) indicates that a non-coalition game equilibrium has been reached. Further, the relationship between coalition profit and coalition value can be obtained.

Theorem 2 The coalition profit in OCS is greater than or equal to the coalition value.

Proof See (SA2a)-(SA2d) in Supplementary Material A for details.

Note that coalition value and coalition profit have different meanings. Coalition value is the coalition benefit under the min-max model, i.e., the coalition benefit in the worst scenario. Coalition profit is the coalition benefit under the

game equilibrium when each coalition aims to maximize its benefit. The goal of the coalition is to maximize the coalition value. Since it is impossible to know the coalition strategies of other entities, it is more appropriate to adopt the min-max model, which can more robustly reflect the ability of the coalition to obtain benefits.

E. Profit Allocation Within Coalition

In a transferable utility game [24], the goal of the coalition is to maximize the total profit of the coalition and then distribute this amount among the members. A challenging problem in a coalition game is the profit allocation. It is desired that the profit allocation benefits each player who cooperates in a game and satisfies individual and global rationality conditions.

1) Core Theory

The core theory is an essential part of the coalition game, which gives the criterion of whether the profit allocation is stable and whether it is accepted by all participants [15]. A profit allocation is $f^d = (f_i^d)_{i \in N_d}$, where each component f_i^d is the distribution profit of the i^{th} MER. The requirements for f_i^d in the core theory are expressed as:

$$\sum_{i \in S} f_i^d \geq V_S \quad \forall S \subseteq N_d \quad (42)$$

$$\sum_{i \in S} f_i^d = f_S \quad \forall S \in CS^* \quad (43)$$

Among the requirements, (42) indicates that the profit of all participants in coalition S under f_i^d is greater than the utility of coalition S , and (43) indicates that the profit of all participants in coalition S under f_i^d is equal to the actual profit of coalition S under OCS.

2) Shapley Method and Super-additive Coverage

The Shapley method is unique and can well reflect the contribution of each MER [25]–[27]. Therefore, the Shapley method has been adopted to determine the profit allocation of each MER in coalition S .

When the coalition value is super-additive, the profit allocation based on Shapley value will be in the core, i.e., the profit allocation is stable. When the coalition cost is not considered, the coalition value represented by min-max model is super-additive. However, due to the existence of coalition costs, the coalition value is not necessarily super-additive. Within coalition S , by adopting the concept of super-additive coverage, the value of the sub-coalition within the coalition S is modified so that the value of the sub-coalition in the coalition S is super-additive.

A partition of a coalition S is a family of non-empty sets $\{S_1, S_2, \dots, S_n\}$ such that:

$$S_1 \cup S_2 \cup \dots \cup S_n = S \quad \forall j, l \neq j, S_j \cup S_l = \emptyset \quad (44)$$

Let $F_{\text{seg}}(S)$ denote the set of all partitions of S . Then the definition of the coalition value of S under super-additive coverage is expressed as:

$$V_S^{\text{SC}} = \max \left\{ \sum_{j=1}^n V_{S_j} \mid \{S_1, S_2, \dots, S_n\} \in F_{\text{seg}}(S) \right\} \quad (45)$$

In OCS, since the objective is to maximize the sum of co-

alition value, the coalition value of coalition S , $S \in CS^*$ is the same as the one after the super-additive coverage correction, and the value of its sub-coalition is less than or equal to the one after the super-additive coverage correction.

The Shapley value φ_i of the i^{th} MER in coalition S is calculated as the average of its marginal contribution to the coalition S . $M \subseteq S - i$ represents any possible sub-coalition (from empty to the entire group) formed by the members of coalition S excluding player i .

$$\varphi_i = \sum_{M \subseteq S-i} \frac{|M|!(|S| - |M| - 1)!}{|S|!} (V_{M+i}^{\text{SC}} - V_M^{\text{SC}}) \quad (46)$$

Considering that the actual coalition profit is greater than the coalition value, the allocation profit of the i^{th} MER is expressed as:

$$f_i^d = \frac{\varphi_i f_S}{V_S} \quad (47)$$

Since the coalition profit is greater than the coalition value, the profit allocation plan is at the core. Therefore, the profit allocation of multiple MERs is stable.

IV. SOLUTION METHODOLOGY

The coalition value model of MER and the coalition profit in the OCS model belong to the equilibrium problem with equilibrium constraint (EPEC), which can be solved by DGA [28]. The DPA is used to obtain OCS [29], and then the Shapley method is used to determine the profit allocation. The solution flow chart is shown in Supplementary Material B Fig. SB1. The specific details are introduced as follows.

A. Solving Coalition Value

1) Model Transformation

Owing to non-convexities from a bilinear term and a complementary constraint, the original MER optimization model is computationally challenging. Thus, reformulating the model is essential to facilitate computation.

The bilinear term in the objective function is linearized by recasting the objective and introducing a mixed-integer quadratic constraint. Concurrently, the complementary relaxation constraint from the KKT conditions is handled via the big- M method, which reformulates it as a mixed-integer linear constraint [30], [31]. Through the above conversion, the original model can be converted into mixed-integer quadratic constraint programming, and then it can be solved by the spatial branch method [32] and is readily solvable by using Yalmip [33] and Gurobi 9.0 or above.

2) DGA

The coalition value model can be regarded as a non-coalition game model, hence the coalition value can be obtained by solving the game equilibrium. The coalition to be sought is one subject, and the remaining coalitions are the other subject. At present, the commonly used solution algorithm is the DGA, as shown in Supplementary Material C Algorithm SC1, which can obtain the equilibrium iteratively.

The decision for coalition S in an odd iteration is obtained by solving the modified MER optimization model, employ-

ing the values of the decision variables for coalition $N_d - S$ derived from the last iteration. In the even iteration, the decision of the coalition S is constant, and the decision of the coalition $N_d - S$ is variable. The execution of DGA ends when the preset stopping criterion is met, or the preset maximum number of iterations is reached, i.e., the decisions of the two entities no longer change significantly. At this time, it can be considered that the game equilibrium has been reached, and the coalition benefit corresponds to the coalition value.

Integer variables and min-max models increase the solution difficulty. However, integer variables only exist in complementary constraints, which can be accelerated by removing complementary constraints, post-testing, and supplementing constraints. Except for the complementary constraints and the bilinear terms of the objective function, the rest of constraints and terms are linear. Therefore, the solution of the min-max model can be completed within the acceptance time.

B. Finding OCS

Based on the coalition value calculated above, OCS can be obtained by using the traditional DPA. Meanwhile, the constraint of scale limitation has been added. The DPA for OCS, as detailed in Supplementary Material C Algorithm SC2, aims to start with a smaller coalition, establish a split strategy for each coalition with the maximum coalition value, and then gradually expand to obtain the best split strategy for each coalition. The optimal split strategy of the grand coalition is the optimal alliance structure. The time complexity of DPA is $O(3^n)$ [29].

C. Obtaining Coalition Profit in OCS

The model of coalition profit in OCS is also a non-coalition game model, hence it can be solved by DGA. Each coalition can be regarded as an entity, and the goal of each coalition is to maximize its own profit. Specifically, the number of participants is generally more than two. When the strategies of each coalition no longer change significantly, it can be considered that the game equilibrium has been reached. The profit of each coalition in this scenario is deemed as its final profit.

After obtaining the coalition profit, the profit allocation within each coalition can be determined. The Shapley value of each MER can be solved by (46), and the profit of each MER can be obtained by (47). The details of DGA for obtaining coalition profit in OCS is shown in Supplementary Material C Algorithm SC3.

V. CASE STUDIES

The IEEE 14-bus electric network (with 5 generators) and the 7-bus gas network (with 7 gas sources) that obtain 10 MERs, as shown in Fig. 4, are used to validate the proposed OCS model, where mmBtu is the million British thermal unit. In Fig. 4, the MER load represents the peak load within the day under contract prices. The details of parameter setting can be found in Supplementary Material D.

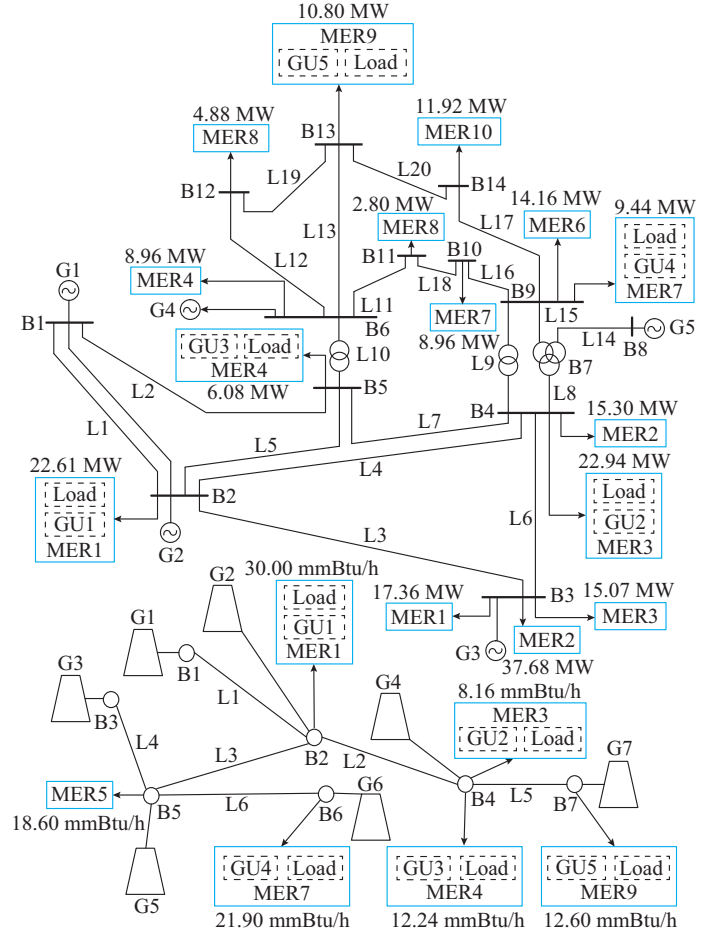


Fig. 4. Diagram of IEEE 14-bus electric network and 7-bus gas network.

A. Results and Analysis

1) Coalition Game Results

There are 10 MERs in the case, and the scale limitation is 3. Hence, the number of coalitions is $C_{10}^1 + C_{10}^2 + C_{10}^3 = 175$. The sorting order of serial numbers of coalition is 1, 2, ..., 10, 1 + 2, 1 + 3, ..., 9 + 10, ..., 1 + 2 + 3, 1 + 2 + 4, ..., 8 + 9 + 10. Based on Supplementary Material A Algorithm SA1, the coalition values can be obtained, as shown in Fig. 5. For a clearer presentation, the MER composition and coalition value of some coalitions in Fig. 5 are shown in Supplementary Material E Table SE1.

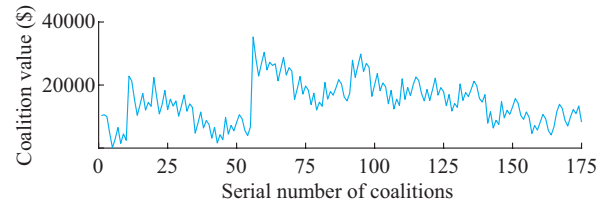


Fig. 5. Coalition value.

Ten MERs can form multiple coalitions, as well as different CSs. The composition of some CSs and corresponding coalition values are shown in Table I. It can be observed that different CSs have distinct coalition compositions and coalition values, aiming to obtain OCS, that is, to maximize the coalition value of CS.

TABLE I
COMPOSITION OF SOME CSS AND CORRESPONDING COALITION VALUES

Number	Composition of some CSSs	Coalition value (\$)
1	{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}	53620.04
2	{1, 2}, {3, 4}, {5, 6}, {7, 8}, {9, 10}	55813.35
3	{1, 2, 3}, {4}, {5, 6}, {7, 9, 10}, {8}	57987.46
4	{1, 2, 3}, {4, 8}, {5}, {6, 7, 10}, {9}	58821.80

Based on Supplementary Material A Algorithm SA2, OCS can be obtained. Based on Supplementary Material A Algorithm SA3, the coalition profit can be obtained. The data of OCS are shown in Table II.

TABLE II
DATA OF OCS

Coalition	OCS	Coalition value (\$)	Coalition profit (\$)
Coalition 1	{1, 2, 3}	35242.93	41321.31
Coalition 2	{4, 8}	6363.41	9252.77
Coalition 3	{5}	101.85	186.56
Coalition 4	{6, 7, 10}	12619.49	17809.41
Coalition 5	{9}	4494.12	4795.62

OCS contains five coalitions, and the profit of each coalition is greater than its coalition value, which is consistent with the theoretical analysis. For each MER, the more internal users, the greater the load, and the greater its market power. In the process of forming a coalition, one MER tends to form a coalition with another MER with larger market power so that the coalition can obtain more profit. For example, MER1, MER2, and MER3 have a large user load, and they are located in coalition 1.

Due to costs involved, when the benefits of forming coalitions are less than the coalition costs, MER chooses to form a small coalition. For example, MER4 and MER8 have relatively small market power and low coalition benefits, hence they form a small coalition of two MERs. The MER profit based on the Shapley method is shown in Table III.

TABLE III
MER PROFIT BASED ON SHAPLEY METHOD

MER	MER profit (\$)	MER	MER profit (\$)
1	13659.97	6	4412.08
2	14454.68	7	9641.32
3	13206.67	8	2244.11
4	7008.66	9	4795.62
5	186.56	10	3756.01

The profit distribution to an MER is positively correlated with its market power. Since there is no congestion, the price of each node is the same. The DA location marginal electricity price (DA-LMEP) and DA location marginal gas price (DA-LMGP) are shown in Fig. 6.

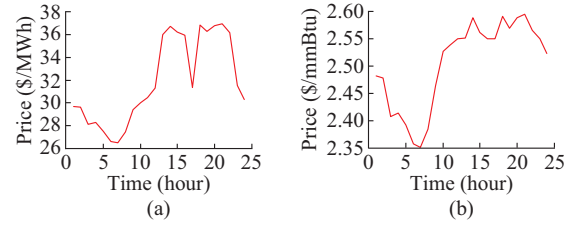


Fig. 6. DA-LMEP and DA-LMGP. (a) DA-LMEP. (b) DA-LMGP.

Due to the formation of coalitions among different MERs, market bidding is conducted from the perspective of coalitions. Therefore, the result analysis of bidding strategy is approached from the standpoint of coalition. The electric power of each coalition purchased by DA market in OCS is shown in Fig. 7. The gas power of each coalition in OCS is shown in Fig. 8.

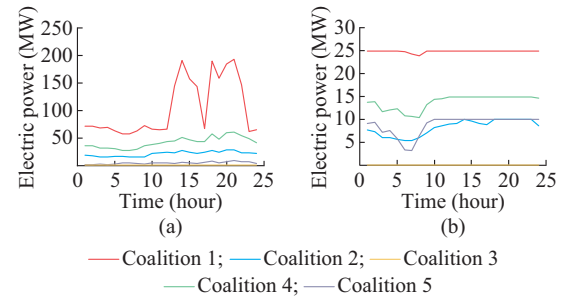


Fig. 7. Electric power of each coalition purchased by DA market in OCS. (a) Electric power. (b) Electric power of GU.

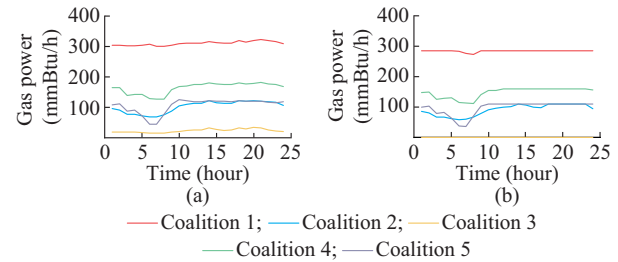


Fig. 8. Gas power of each coalition in OCS. (a) DA purchased gas power. (b) Gas power of GU.

During demand peaks, the increased power procurement of the coalition drives up electricity prices. The GU operates with high output at a relatively low cost. Conversely, during demand valleys, the reduced coalition purchases electricity at lower prices, making GU generation less economical. Consequently, the GU reduces its electric power output, leading to a drop in gas demand and thereby lowering gas prices. By adopting the post-verification method, it can be ensured that the transmission power, gas pressure, and linepack energy of the gas network are within the constraint range. The details are shown in Supplementary Material F.

2) With Considering Coalition Versus Without Considering Coalition

Without considering the coalition, all MERs constitute a non-cooperative game, and their profits can be obtained by solving the game equilibrium. The comparison of MER profit with and without considering coalition is shown in Fig. 9.

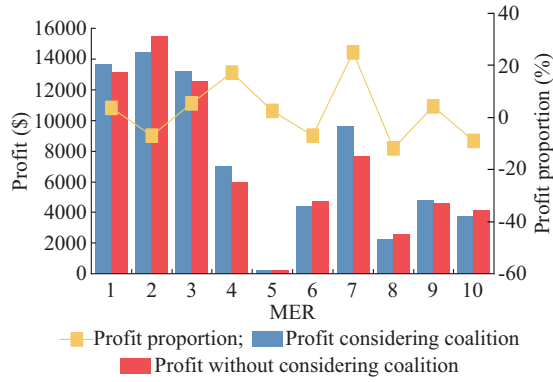


Fig. 9. Comparison of MER profit with and without considering coalition.

When the coalition is considered, the total profit of all MERs is \$73365.68, and when the coalition is not considered, the total profit of all MERs is \$71110.77. For the profit of a single MER, when considering the coalition, the profit of most MERs has increased. This is because the coalition has brought greater market power and the overall profit of the coalition increases. Therefore, the profit of the MER within the coalition also increases. However, the profit of some MERs such as MER2 has decreased because other MERs have greater power when considering the coalition, squeezing out their original profit space.

Taking MER2 as an example, we examine how its participation affects the profits of other MERs when they remain in the coalition. In this scenario, OCS changes to {1,3}, {2}, {4,8}, {5}, {6,7,10}, {9}. It can be observed that after MER2 stops participating in the coalition, it will also affect the coalition strategies of other MERs. The comparison of MER profit when MER2 participates in or leaves coalition is shown in Fig. 10.

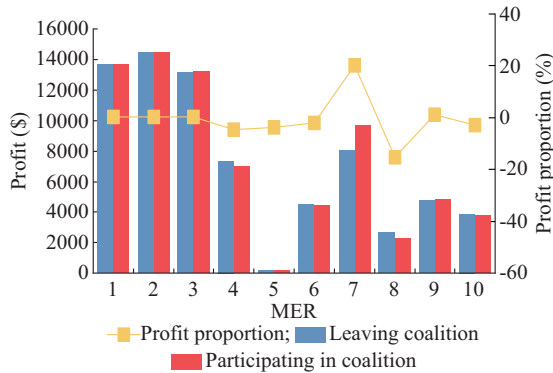


Fig. 10. Comparison of MER profit when MER2 participates in or leaves coalition.

When MER2 participates in the coalition, its own profits will be increased by about 0.2%. Due to the competition between MERs, the profits of most MERs decrease. For a single MER, it has the motivation to increase its own profit by participating in the coalition. However, the profit of a single MER depends not only on its own decision, but also on the decisions of other MERs. If other MERs choose to form a coalition, the profit of a single MER will decrease. There-

fore, when all MERs choose to participate in the coalition, the profit of some MERs will increase, while the profit of other MERs will decrease.

3) Comparison of Different Coalition Cost Coefficients and Scale Limitations

The results of the cases under different coalition cost coefficients and scale limitations are shown in Supplementary Material G Figs. SG1-SG3. As the coalition cost coefficient increases, the number of coalitions in OCS gradually increases, and the maximum scale of the coalition gradually decreases, i.e., MER gradually leaves the coalition. This is because the increase in coalition value is less than the coalition cost. Meanwhile, due to the dissolution of the coalition, the coalition value is gradually reduced.

As the scale limitation increases, the number of coalitions in OCS gradually decreases, and the maximum scale of coalition gradually increases, which shows that forming a coalition is beneficial to improve the MER profit. However, due to the existence of coalition cost, the maximum scale of the coalition in OCS is not necessarily equal to the scale limitation. As the scale limitation increases, the feasible region of OCS gradually increases. Therefore, the overall coalition value under OCS also gradually increases.

4) Comparison of Different Bidding Formats

When the bidding formats of MER are different, the solving speed and accuracy are also different. Although the three bidding formats are theoretically equivalent in their clearing outcomes, the computation constraints of the solution algorithm lead to observable differences in practice. By taking the grand coalition as an example, the optimization goal is to maximize the overall coalition profit. The comparison of three bidding formats is shown in Table IV.

TABLE IV
COMPARISON OF THREE BIDDING FORMATS

Bidding format	Coalition profit (\$)	Time (s)
Only quantity	81650.33	380.64
Block quantity price	81651.64	405.36
Linear quantity price	79780.51	4146.48

The three bidding formats can be ranked in descending order of solution speed as: only quantity bidding, block quantity price bidding, and linear quantity price bidding. The reason is that the only quantity bidding has the fewest optimization variables. For the linear quantity price bidding, the stationarity constraint under the KKT condition will contain the non-convex term of the product of the slope and the clearing power, which greatly increases the computation complexity. Therefore, this format not only has a slower solution speed but also a lower solution accuracy than the first two formats. Thus, using only quantity bidding can improve the solution speed of OCS model.

5) Comparison of Computation Time at Different Scales

The computation burden of OCS model grows significantly with scale, primarily due to two parts: coalition value calculation ($O(n^3)$) and OCS solution via DPA ($O(3^n)$), and the detailed numerical simulation can be found in Supplementa-

ry Material H.

Since there are a large number of parallel processes in the computation process, parallel computation can be used for acceleration. The acceleration effect of parallel computation is roughly proportional to the number of cores. The solution of coalition value can be completely parallel, while the solution of OCS needs to be serial from small coalitions to large coalitions and the same scale computation can be parallel. Therefore, in theory, if there are enough cores, the solution of coalition value can reach $O(1)$ at the lowest, and the solution of OCS can reach $O(n)$ at the lowest.

If the computational burden remains prohibitive, heuristic, artificially intelligent, and other algorithms can be used to obtain a faster solution speed by sacrificing a certain degree of accuracy.

VI. CONCLUSION

In this paper, we propose the OCS model for MERs in electricity and gas markets. The coalition cost and scale limitation are studied, and the OCS, profit allocation, and operation strategy of MER in electricity and gas markets can be determined. The following conclusions can be obtained.

1) The existence of coalition cost and scale limitation makes OCS change from a grand coalition to multiple coalitions. Compared with the grand coalition, the total profit of multiple coalitions increases. Thus, the grand coalition is unstable.

2) Most MERs can get higher profits by forming coalitions, since the market competitiveness can be strengthened through coalitions. However, the coalition profit of some MERs decreases. This is because, although their market competitiveness has increased, the market competitiveness of other MERs has increased even more.

3) The different bidding formats of MER will not affect the market-clearing results. The only quantity bidding format offers the advantages of fast solution speed and high solution accuracy. The proposed hybrid algorithm including DGA and DPA can effectively solve OCS of MERs

This study addresses the decision-making problems of CS, including operation strategy and profit allocation of MERs. The findings provide a theoretical foundation for MER decision-making in market environments and demonstrate the potential to significantly improve MER profitability. Future research should explore methods for formulating MER coalition strategies under incomplete information and uncertain conditions.

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