Carbon-aware Multi-level Local Energy Market for Electricity-hydrogen Trading Based on Distributionally Robust Game Framework

Longyan Li, Abdulelah S. Alshehri, Maher M. Alrashed, and Chao Ning

***Abstract*—The proliferation of distribution-level green electricity and hydrogen resources entails an efficient local energy market (LEM). However, the existing LEM designed for electricity-hydrogen trading falls short of modeling multi-level mechanisms and accounting for the carbon intensity of hydrogen production. To bridge this gap, we propose a carbon-aware multi-level LEM for electricity-hydrogen trading based on a distributionally robust game framework, where hydrogen-based microgrids (HMGs) supply hydrogen to heterogeneous hydrogen users (HUs) including hydrogen refueling stations and industrial users. In this game framework, the coordination between HMGs and HUs is cast as a multi-leader multi-follower Stackelberg game. Specifically, HMGs determine an integrated hydrogen-carbon price, and carry out electricity trading through a non-cooperative game. Meanwhile, HUs act as followers, adjusting hydrogen purchasing strategies. Furthermore, the self-dispatching of HMGs and HUs is modeled as distributionally robust optimization problems considering source-load and hydrogen demand uncertainties, respectively. To hedge against these uncertainties, a novel Bayesian nonparametric hybrid ambiguity set is constructed based on local Wasserstein balls and moment information. Finally, the equilibrium of the proposed game framework is theoretically proved, and a distributed algorithm is developed to obtain this equilibrium. Comparative studies validate that the proposed game framework outperforms the existing ones, demonstrating a total income increasement of 12.3% and a carbon emission reduction of 11.6%.**

***Index Terms*—Local energy market (LEM), distributionally robust optimization (DRO), electricity-hydrogen trading, hydrogen-based microgrid (HMG), Stackelberg game.**

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L. Li and C. Ning (corresponding author) are with School of Automation and Intelligent Sensing, Shanghai Jiao Tong University, Shanghai 200240, China, and Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240, China (e-mail: lilongyan@sjtu. edu. cn; chao.ning@sjtu.edu.cn).

A. S. Alshehri and M. M. Alrashed are with Department of Chemical Engineering, College of Engineering, King Saud University, Riyadh 11421, Saudi Arabia (e-mail: aalshehri9@ksu.edu.sa; mabdulaziz@ksu.edu.sa)

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NOMENCLATURE

A. Sets and Indices

f, e	Superscripts indicating forecast value and forecast error
ι, I	Index and set of industrial users (IUs)
m, M	Index and set of hydrogen-based microgrids (HMGs)
$\theta, \mathcal{M}_+$	Index and set of positive Borel measures
n, N	Index and set of hydrogen refueling stations (HRSs)
$\mathcal{P}_e$	Ambiguity set constructed based on source-load uncertainty data
$\mathcal{P}_D$	Ambiguity set constructed based on hydrogen demand uncertainty data
t, T	Index and set of scheduling time slots
$\mathbb{W}^e$	Support set of uncertainties
z	Index of HRSs or IUs, $z \in N \bigcup I$

B. Parameters

$\alpha^{e,b/s}$	Unit price of purchasing/selling electricity from/to external grid
ε	Given risk level
ζ	Non-negative ramping rate
$\eta^{ch/dch}$	Charging/discharging efficiency of battery
η^{elz}	Electrolyzer efficiency
η	Optimal objective value of a second-order cone optimization problem
ρ_i	Wasserstein radius of basic ambiguity set
γ, γ_1	Given parameters
ψ	Step size
μ	Acceptable error
$\underline{\lambda}_m^{IHC}, \overline{\lambda}_m^{IHC}$	Lower and upper bounds of integrated hydrogen-carbon price
$c^{BT/H}$	Unit operation cost of battery/hydrogen storage
C^{elz}	Transformation factor of electrolyzer
c^G, c^{tax}	Carbon emission factor of external grid and carbon tax

$\boldsymbol{c}, \boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}, \hat{\boldsymbol{A}}, \hat{\boldsymbol{d}}$	Coefficient matrices or vectors
E, F	Parameters in support set
$\underline{E}_m^b$, $\bar{E}_m^b$	Lower and upper bounds for storage level of battery
k_1	Iteration index of electricity trading
k_2	Iteration index of solving non-cooperative electricity market clearing problem
$\underline{L}_{m/n}^{HS}$, $\bar{L}_{m/n}^{HS}$	Lower and upper bounds of hydrogen storage in HMG/HRS
$\bar{L}_{m,n/i}^{H_2}$	Upper bound of hydrogen purchased by HRS/IU from HMG
$\bar{P}_m^{ch/dch}$	The maximum charging/discharging power of battery
$\bar{P}_m^{elz}$	Capacity of electrolyzer
prc^H	Flat hydrogen retail price

C. Variables

η_m	Acceleration operator
$\lambda_m^{H_2/IHC}$	Hydrogen/integrated hydrogen-carbon price
$\lambda_{m,l}^P$	Unit price for electricity from HMG m to HMG l
$\pi_{m,l}$	Payment for electricity from HMG m to HMG l
$\delta_{\hat{\xi}_j}$	Dirac distribution concentrating unit mass at data $\hat{\xi}_j$
$\Pi\left(\xi_1, \xi_2\right)$	Joint distribution of ξ_1 and ξ_2 with marginal distributions $\mathbb{P}_i$ and $\hat{\mathbb{P}}_i$
$c_m^{b/HS,cum}$	Cumulated carbon emission of battery/hydrogen storage
$c_m^{b/HS}$	Carbon intensity of battery/hydrogen storage
C_m^{EMG}	Electricity trading cost with external grid
$C_m^{H_2}$	Hydrogen trading cost
$C_m^{op/sc}$	Operation/social cost
C_m^{non}	Cost without peer-to-peer electricity trading
c_m	Electricity carbon intensity
$D_m^{H_2}$	Amount of hydrogen sold by HMG
e_m^b	Storage level of battery
$k_{m,n}^{HRS}$, $k_{m,\iota}^{IU}$	Preference parameters of HRS and IU
$L_{n/\iota}^D$	Hydrogen demand of HRS/IU
$L_{m,n/\iota}^{H_2}$	Amount of hydrogen purchased by HRS/IU from HMG
$L_{m/n}^{HS}$	Hydrogen storage in HMG/HRS
N_i	Number of data points
$P_m^{PV/WT}$	Power generation of photovoltaic (PV)/wind turbine (WT)
$P_m^{ch/dch}$	Charging/discharging power of battery
P_m^{elz}	Power consumption of electrolyzer
$P_m^{HMG,b/s}$	Amount of electricity purchased from/sold to external grid by HMG
$P_{m,l}$	Amount of electricity from HMG m to HMG l
P_m^D	Power load of HMG
$\mathbb{P}_i$	True distribution of data points
$\hat{\mathbb{P}}_i$	Empirical distribution of data points
w_m^e, ΔL_n^D	Forecast errors of source-load and hydrogen demand
x	Decision variable of corresponding distributionally robust conditional value at risk (DR-CVaR) constraints
$\boldsymbol{x}$	Vector of continuous variables

I. Introduction

ALL major economies in the world have set green hydrogen development targets. For example, China has announced a roadmap to reach a green hydrogen production capacity of 7.7 million tons per year by 2030, resulting in an installed electrolyzer capacity of 100 GW [1]. As the scale of green hydrogen production, transportation, and consumption expands, the local energy market (LEM) for both electricity and hydrogen gradually takes shape. This LEM holds great promise to efficiently coordinate hydrogen-based microgrids (HMGs) with downstream hydrogen users (HUs) to address the growing hydrogen demands from transportation and industrial sectors [2]. Despite its potential, significant challenges exist in designing a multi-level LEM for the community of HMGs and HUs, especially when addressing comprehensive decarbonization and uncertainty management. Therefore, the main aim of this work is to design an LEM where both HMGs and HUs make cost-effective and low-carbon decisions through a privacy-protective decentralized market clearing paradigm.

Nowadays, the LEM for electricity trading has sparked a flurry of interest ever since the publication of [3]. The trading mechanism of this LEM can be categorized into two types according to whether there exists a centralized system operator. Traditional centralized LEM was proposed for the electricity trading of multi-microgrids, where the distribution system operator was able to maximize the total social utility in a centralized manner while sacrificing privacy [4], [5]. Thanks to the privacy-preserving attributes, distributed mechanisms such as the alternating direction method of multipliers (ADMM) [6], [7], the relaxed ADMM [8], and the bisection algorithms are emerging as more practical choices for the electricity trading [9].

Compared with the existing LEM for electricity trading, the LEM for electricity-hydrogen trading is far from mature. By fostering the synergy between electricity and hydrogen markets, the existing literature showcased positive impacts on system reliability, cost-effectiveness, and adaptability. Reference [10] studied the strategic bidding of a profitable HMG, which acted as a price taker. Meanwhile, an HMG can participate in the LEM as a price maker for more profits. Reference [11] proposed an auction-based pricing strategy with an auctioneer representing an HMG. The power system and hydrogen system operators belong to different stakeholders, so distributed market clearing methods were commonly implemented for the purpose of privacy protection [12]. An LEM for electricity-hydrogen trading with the single-level participants was considered in [13], and an iterative clearing method was developed using the merit-order principle. A single-level energy sharing model for hydrogen and electricity in integrated energy systems was proposed, and the market clearing problem was solved by a sub-gradient method [14]. An undiscriminating single-level peer-to-peer electricity-hydrogen trading was proposed for multi-HMGs [15], and the ADMM algorithm was utilized for problem solving.

Apart from the energy trading mechanism, research efforts have been devoted to addressing environmental issues in the LEM. Two categories for addressing environmental issues can be distinguished depending on the emission obligation allocation in the LEM. From the view of HMGs, the intermittent nature of renewable energy and limitations in current electrolysis technologies inevitably lead to indirect carbon emission from electrolytic hydrogen production, which relies on electricity from external grid. Therefore, the existing literature has focused on calculating the carbon intensity of the grid-connected electrolytic hydrogen in HMGs over a long time period [16]. The European Commission has proposed a standard for calculating annual average carbon intensity in electrolytic hydrogen production [17]. Carbon-aware consideration can also be incorporated into the HMG model by emission constraints [18], objective functions related to carbon emissions [19], and imposition of a carbon price [10]. From the view of HUs, the hydrogen price is an effective way to coordinate the hydrogen demand. The existing literature has studied the nodal gas price of a hydrogen-blended pipeline with a carbon emission price [20]. In [21], a dynamic hydrogen pricing mechanism was proposed, which correlates the hydrogen price with the proportion of renewable energy in the energy supply.

There has been a plethora of methods for addressing uncertainties stemming from renewable energy generation and load in the LEM, for which the distributionally robust optimization (DRO) is the state-of-the-art uncertainty-aware decision-making paradigm [22], [23]. According to the construction of ambiguity sets, DRO methods can be classified into two types, namely moment-based DRO and distance-based DRO [24]-[26]. Existing literature typically employs either of them in microgrid-related trading problems. Reference [27] applied the first- and second-order moment ambiguity sets in the local electricity market. The Wasserstein-based DRO was proposed for a multi-microgrid energy trading problem [28].

However, there are some shortcomings in the existing research work mentioned above. First, previous research has not adequately addressed the intricacies of a multi-level mechanism for electricity-hydrogen trading. In reality, various entities with conflict of interest coexist within a comprehensive LEM. Obviously, the rise in hydrogen price increases the income of HMGs, while also increasing the purchasing cost of HUs. Moreover, with a constant hydrogen demand, an increase in the hydrogen sales of one microgrid will inevitably decrease those of other microgrids, leading to conflict of interest among HMGs. Thus, it is essential to consider the multi-level interaction within the LEM and to achieve market clearing in a privacy-preserving distributed manner. Second, the existing literature fails to identify the carbon intensity of electrolytic hydrogen production and the carbon emission flow in the hydrogen market, and the environmental issues have not received appropriate attention in the hydrogen market design. In practice, HUs are the underlying driver of carbon emissions in the LEM. Therefore, the carbon emissions produced from the HMGs should be identified from the perspective of HU side. Third, the existing literature prefers using conventional DRO methods including moment-based DRO and Wasserstein-based DRO in hedging against uncertainties in energy trading. There is a lack of effective methods to model the global-local features of the uncertainty distribution in the market design and to prevent the trading results from being over-conservative. The unique features of this paper compared with prior research work are summarized in Table I.

TABLE I
UNIQUE FEATURES OF THIS PAPER COMPARED WITH PRIOR RESEARCH WORK

Reference	Market participant	Framework	Uncertainty consideration	Carbon-aware consideration
[29], [30]	Microgrids	Single-level	Yes (robust or stochastic optimization)	No
[11]	Hydrogen auctioneer and hydrogen refueling stations (HRSs)	Bilevel	No	No
[13]-[15]	Integrated energy systems	Single-level	No	No
[31]	Integrated energy systems	Single-level	No	Yes (objective function)
[32]	HMGs	Bilevel	Yes (Wasserstein-based DRO)	No
This paper	HMGs and HUs	Multi-level	Yes (Bayesian nonparametric DRO)	Yes (integrated hydrogen-carbon price)

To address the above research gaps, this paper proposes a novel carbon-aware multi-level LEM for electricity-hydrogen trading based on a distributionally robust game framework. The major contributions of this paper are summarized as follows.

1) The carbon-aware multi-level LEM is proposed for the electricity-hydrogen trading between HMGs and HUs. A distributionally robust game framework is established to capture the intricate interactions in the uncertain LEM, wherein electricity trading is formulated as a non-cooperative game and hydrogen trading is modeled as a multi-leader multi-follower (MLMF) Stackelberg game. The existence and uniqueness of the game equilibrium are rigorously proved.

2) A novel integrated hydrogen-carbon pricing strategy is

developed and incorporated into the LEM to achieve systematic decarbonization across energy value chains. This strategy integrates carbon signals into the hydrogen prices and charges HUs by tracing their responsible carbon emissions, thereby contributing to renewable integration and low-carbon hydrogen production.

2) Unlike conventional frameworks that oversimplify HMGs and HUs as lumped nodes or auctioneers, the proposed game framework enables simultaneous coordinative trading and dispatching. To address the uncertainties stemming from the renewable energy and demand, the decision processes of different stakeholders are modeled as data-driven DRO problems, where a novel Bayesian nonparametric hybrid ambiguity set is constructed. This ambiguity set effectively captures the local fine-grained feature of the uncertainty data based on both the Wasserstein balls and moment information, resulting in a less conservative and more trustworthy robust solution.

4) A privacy-preserved distributed algorithm is proposed to find the multi-level game equilibrium. The DRO problem is first reformulated into a mixed-integer linear program in light of the duality theory and distributionally robust conditional value at risk (DR-CVaR) definition. Further, a nested-loop iterative algorithm based on fast-ADMM algorithm is developed to protect the privacy of participants.

The remainder of this paper is organized as follows. Section II introduces the problem description. Section III describes the model formulation of market participants. The distributionally robust game framework for electricity-hydrogen trading and solution methodology are presented in Sections IV and V, respectively. Section VI provides case studies and corresponding computational results. The conclusion is given in Section VII.

II. Problem Description

The framework of the considered system is depicted in Fig. 1. The participants are HMGs and HUs, with HUs further categorized into HRSs and industrial users (IUs). Each HMG is equipped with renewable energy generation units such as wind turbines (WTs) and photovoltaics (PVs). An HMG can purchase electricity either from the external grid or through peer-to-peer transactions with other HMGs. Apart from satisfying the power load, the electricity is used for electrolytic hydrogen production. Then, the hydrogen is stored and sold to HUs. Each HRS has its own hydrogen storage system.

The topology in Fig. 1(b) forms a multi-level game framework for LEM. In the hydrogen trading, HMGs play the role of leaders and HUs are followers. Additionally, HMGs engage in peer-to-peer electricity trading to exchange surplus energy. Notably, the carbon footprint of LEM is induced by the electricity purchased from the external grid that utilizes fossil fuels for electricity generation. Consequently, the "virtual" carbon flow, coinciding with the electricity flow, accumulates in both battery and hydrogen storage systems. This carbon flow is ultimately transferred to downstream HUs through hydrogen sales.

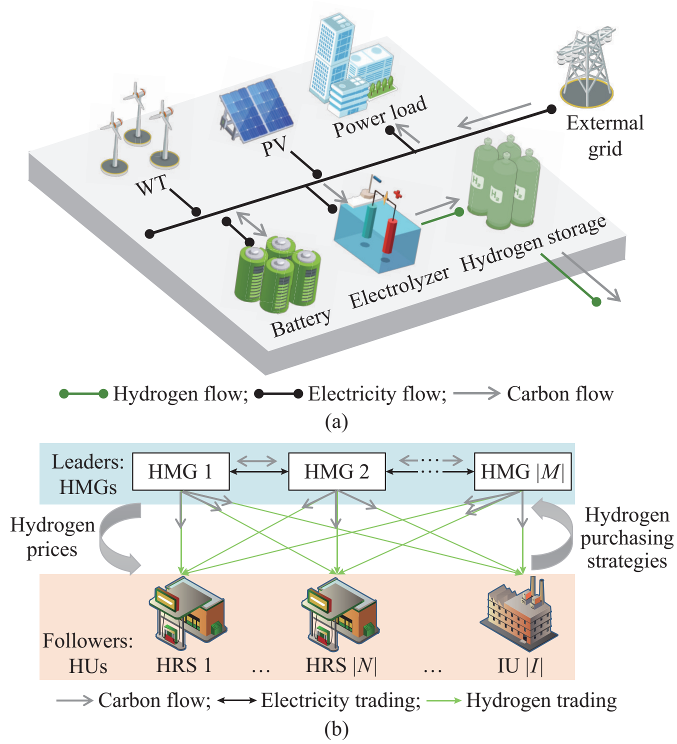

Fig. 1. Framework of considered system. (a) Structure of HMG. (b) Game framework for LEM.

Remark In this paper, we adopt the classical assumption of rational and self-interested participants to establish a tractable game-theoretic baseline, consistent with prior studies of electricity-hydrogen markets [11], [13].

III. Model Formulation of Market Participants

A. Bayesian Nonparametric Hybrid Ambiguity Set

The ambiguity sets can be constructed by the same method but based on different uncertainty datasets. In the following, we uniformly refer to ambiguity sets as $\mathcal{P}$ and the uncertainty as ξ. We first implement a Bayesian nonparametric method, i.e., the Dirichlet process mixture model (DPMM), to learn the local distributional information of the historical uncertainty data. For details of DPMM, please refer to Supplementary Material A.

By the variational inference algorithm, we can decompose the uncertainty distribution into K mixture components, where K is automatically learned from data without being presumed a priori. Specifically, the weight γ_i, mean μ_i, and covariance Σ_i of the i^{th} component are derived. By leveraging the local information extracted by the variational inference, we propose a novel ambiguity set $\mathcal{P}$, which incorporates both the local moment information and Wasserstein metric.

$$\mathcal{P}=\oplus_{i=1}^{K}\gamma_i\mathcal{P}_i\left(\mathbb{W}^e,\mu_i,\Sigma_i,\hat{\mathbb{P}}_i\right) \tag{1}$$

where $\mathbb{W}^e=\left\{\xi\,\middle|\,E\xi\le F\right\}$; $\hat{\mathbb{P}}_i=\dfrac{1}{N_i}\sum_{j=1}^{N_i}\delta_{\hat{\xi}_j}$; $\oplus$ denotes the Minkowski sum; and the basic ambiguity set $\mathcal{P}_i$ is defined as:

$$\mathcal{P}_i\left(\mathbb{W}^e,\mu_i,\Sigma_i,\hat{\mathbb{P}}_i\right)=\left\{\theta\in\mathcal{M}_+\left|\int_{\mathbb{W}^e}\theta\mathrm{d}\xi=1,\int_{\mathbb{W}^e}\xi\theta\mathrm{d}\xi=\mu_i,\right.\right.$$

$$\left.\int_{\mathbb{W}^e}\xi^2\theta\mathrm{d}\xi\leq\Sigma_i+\mu_i^2,W\left(\mathbb{P}_i,\hat{\mathbb{P}}_i\right)\leq\rho_i\right\} \tag{2}$$

where the Wasserstein metric $W\left(\mathbb{P}_i,\hat{\mathbb{P}}_i\right)$ is defined as:

$$W\left(\mathbb{P}_i,\hat{\mathbb{P}}_i\right)=\inf\int_{\mathbb{W}^e\times\mathbb{W}^e}\left\|\xi_1-\xi_2\right\|\mathrm{d}\Pi\left(\xi_1,\xi_2\right) \tag{3}$$

This novel ambiguity set is constructed in a nonparametric fashion, and the fine-grained local moment information is utilized rather than the global information. Thanks to the nonparametric property, the Bayesian nonparametric hybrid ambiguity set is well capable of dealing with uncertainty data, which (including the number of components) are unknown to us. This ambiguity set captures the global-local features of the uncertainty distribution via the DPMM, and it is constructed by considering both moment-based and Wasserstein-based information. Therefore, it mitigates conservatism in comparison to state-of-the-art DRO methods.

B. DR-CVaR Constraint and DRO Formulation

The DR-CVaR constraint is defined as (4). This constraint implies that the function $L(\xi)\leq 0$ is respected with a probability of at least $1-\varepsilon$.

$$\sup_{\mathbb{P}\in\mathcal{P}}\mathbb{P}-CVaR_\varepsilon\left(L(\xi)\right)\leq 0 \tag{4}$$

where the definition of $CVaR_\varepsilon(\cdot)$ refers to [22]. The DR-CVaR constraint has several merits. First, unlike chance constraints, it leads to tractable convex optimization problems. Second, the DR-CVaR constraint imposes a higher penalty on constraint violation, and restricts the constraint violation in both frequency and magnitude. Then, the general DRO problem is formulated as:

$$\begin{cases}\min \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}\\ \text{s.t. } \boldsymbol{A}^{\mathrm{T}}\boldsymbol{x}\leq\boldsymbol{b}\\ \quad\sup\limits_{\mathbb{P}\in\mathcal{P}}\mathbb{P}-CVaR_\varepsilon\left(L(\xi)\right)\leq 0\end{cases} \tag{5}$$

C. Model of HMGs

The HMG produces hydrogen via electrolyzer and sells hydrogen to HUs for profit. Formally, each HMG attempts to solve the following optimization problem.

1) Objective Function

The objective function of each HMG J_m is given in (6). Equations (7)-(9) define the operation cost, electricity trading cost with the external grid, and hydrogen trading cost, respectively. Equation (10) calculates the payment for electricity from HMG m to HMG l.

$$\min_{P_m^{ch},P_m^{dch},L_m^{HS},P_m^{HMG,b},e_m^b,P_m^{HMG,s},P_{m,l},P_m^{elz},D_m^{H_2}} J_m=$$

$$\sum_{t\in T}\left(C_m^{op}(t)+C_m^{EMG}(t)+\sum_{l\in M\backslash m}\pi_{m,l}(t)-C_m^{H_2}(t)\right) \tag{6}$$

$$C_m^{op}(t)=c^{BT}\left(\eta^{ch}P_m^{ch}(t)+P_m^{dch}(t)/\eta^{dch}\right)+c^H L_m^{HS}(t)\quad\forall t\in T \tag{7}$$

$$C_m^{EMG}(t)=P_m^{HMG,b}(t)\alpha^{e,b}(t)-P_m^{HMG,s}(t)\alpha^{e,s}(t)\quad\forall t\in T \tag{8}$$

$$C_m^{H_2}(t)=\lambda_m^{IHC}(t)D_m^{H_2}(t)\quad\forall t\in T \tag{9}$$

$$\pi_{m,l}(t)=\lambda_{m,l}^P(t)P_{m,l}(t)\quad\forall t\in T,l\in M\backslash m \tag{10}$$

2) Constraints

In HMGs with high renewable penetration, the power load may not always be equal to the uncertain renewable generation, and enforcing strict equality can result in high costs. Therefore, we introduce the following inequality:

$$P_m^{D,f}(t)+P_m^{ch}(t)+P_m^{elz}(t)+P_m^{HMG,s}(t)\leq P_m^{PV,f}(t)+P_m^{WT,f}(t)+ \\ P_m^{dch}(t)+P_m^{HMG,b}(t)+\sum_{l\in M\backslash m}P_{l,m}(t)+w_m^e(t)\quad\forall t\in T \tag{11}$$

Power generation is permitted to exceed the power load, with excess energy being curtailed at zero marginal cost. On this basis, the DR-CVaR constraint is expressed as (12) with the superscript f being omitted. The DR-CVaR constraint systematically coordinates the imbalance risks by allowing for temporary load shedding with probability not exceeding ε. The DR-CVaR constraint not only ensures probabilistic constraint satisfaction, but also penalizes severe constraint violations.

$$\sup_{\mathbb{P}\in\mathcal{P}_e}\mathbb{P}-CVaR_\varepsilon(P_m^D(t)+P_m^{ch}(t)+P_m^{elz}(t)+P_m^{HMG,s}(t)- \\ P_m^{PV}(t)-P_m^{WT}(t)-P_m^{dch}(t)-P_m^{HMG,b}(t)- \\ \sum_{l\in M\backslash m}P_{l,m}(t)-w_m^e(t))\leq 0\quad\forall t\in T \tag{12}$$

Constraints (13) and (14) are the dynamic state and operation range of battery, respectively. Constraint (15) defines the initial and final states of battery. Constraints (16) and (17) express the limits of charging and discharging power of battery, respectively.

$$e_m^b(t+1)=e_m^b(t)+\eta^{ch}P_m^{ch}(t)-P_m^{dch}(t)/\eta^{dch}\quad\forall t\in T \tag{13}$$

$$\underline{E}_m^b\leq e_m^b(t)\leq\bar{E}_m^b\quad\forall t\in T \tag{14}$$

$$e_m^b(|T|)=e_m^b(0) \tag{15}$$

$$0\leq P_m^{ch}(t)\leq\bar{P}_m^{ch}\quad\forall t\in T \tag{16}$$

$$0\leq P_m^{dch}(t)\leq\bar{P}_m^{dch}\quad\forall t\in T \tag{17}$$

Constraints (18) and (19) are the dynamic state and operation range of hydrogen storage, respectively. Constraint (20) defines the initial and final states of hydrogen storage. Constraint (21) depicts the operation range of electrolyzer. Constraint (22) is the maximum available hydrogen. Constraint (23) determines the amount of hydrogen sold by HMG.

$$L_m^{HS}(t+1)=L_m^{HS}(t)+\eta^{elz}P_m^{elz}(t)/C^{elz}-D_m^{H_2}(t)\quad\forall t\in T \tag{18}$$

$$\underline{L}_m^{HS}\leq L_m^{HS}(t)\leq\bar{L}_m^{HS}\quad\forall t\in T \tag{19}$$

$$L_m^{HS}(|T|)=L_m^{HS}(0) \tag{20}$$

$$0\leq P_m^{elz}(t)\leq\bar{P}_m^{elz}\quad\forall t\in T \tag{21}$$

$$\overline{L}_m^{HS}(t)=L_m^{HS}(t)-\underline{L}_m^{HS}+\eta^{elz}P_m^{elz}(t)/C^{elz}\quad\forall t\in T \tag{22}$$

$$D_m^{H_2}(t)=\min\left\{\overline{L}_m^{HS}(t),\sum_{z\in N\cup I}L_{m,z}^{H_2}(t)\right\} \tag{23}$$

Equation (24) calculates the electricity carbon intensity as the weighted average carbon emission with respect to the total power inflow. The cumulated carbon emissions of battery and hydrogen storage depend on the past charging and discharging actions, as presented in (25) and (26), respectively. The carbon intensities of battery and hydrogen storage are calculated by (27) and (28), respectively. The initial cumulated carbon emissions of battery and hydrogen storage are given in (29) and (30), respectively. Note that the carbon intensities are calculated after solving the self-dispatching problem of HMG.

$$c_m(t)=\frac{c^G(t)P_m^{HMG,b}(t)+c_m^b(t)P_m^{dch}(t)+\sum_{l\in M\setminus m}c_l(t)P_{l,m}(t)}{P_m^{PV}(t)+P_m^{WT}(t)+P_m^{dch}(t)+P_m^{HMG,b}(t)+\sum_{l\in M\setminus m}P_{l,m}(t)}\quad \forall t\in T \tag{24}$$

$$c_m^{b,cum}(t+1)=c_m^{b,cum}(t)-c_m^b(t)P_m^{dch}(t)+c_m(t)P_m^{ch}(t)\quad \forall t\in T \tag{25}$$

$$c_m^{HS,cum}(t+1)=c_m^{HS,cum}(t)-c_m^{HS}(t)D_m^{H_2}(t)+c_m(t)P_m^{elz}(t)\quad \forall t\in T \tag{26}$$

$$c_m^b(t)=c_m^{b,cum}(t)/\left(\eta^{dch}e_m^b(t)\right)\quad \forall t\in T \tag{27}$$

$$c_m^{HS}(t)=c_m^{b,cum}(t)/L_m^{HS}(t)\quad \forall t\in T \tag{28}$$

$$c_m^{b,cum}(0)=c_m^b(0)e_m^b(0) \tag{29}$$

$$c_m^{HS,cum}(0)=c_m^{HS}(0)L_m^{HS}(0) \tag{30}$$

D. Model of HUs

Hydrogen serves diverse applications, primarily finding utility in transportation and chemical industries. Therefore, HUs including HRSs and IUs are considered in this paper. As followers in the game, HUs decide the hydrogen purchasing and operation strategies to minimize their costs. In specific, the models of HRSs and IUs are given in the following.

1) HRSs

The follower-level problem of HRS n (P-HRS$_n$) is shown below. The first, second, and third terms of the right side in the objective function J_n in (31) are the hydrogen purchasing cost, logarithm utility function depicting the decreasing marginal utility [33], and hydrogen retail income, respectively. Note that the applicabilities of this model and the pricing scheme are not limited to the utility function in the logarithmic form. Instead, they are widely applicable as long as the utility function is strictly concave and non-decreasing. Constraint(32) denotes the dynamic state of hydrogen storage. Constraints (33) and (34) are the DR-CVaR constraints. Constraint (35) limits the amount of hydrogen that can be purchased.

P-HRS$_n$:

$$\min_{L_{m,n}^{H_2},L_n^{HS}} J_n=\sum_{t\in T}\sum_{m\in M}\Big(L_{m,n}^{H_2}(t)\lambda_m^{IHC}(t)+k_{m,n}^{HRS}\ln\left(1+L_{m,n}^{H_2}(t)\right)-L_n^D(t)\cdot prc^{H_2}\Big) \tag{31}$$

s.t.

$$L_n^{HS}(t)=L_n^{HS}(t-1)+\sum_{m=1}^{M}L_{m,n}^{H_2}(t)-L_n^D(t)\quad \forall t\in T \tag{32}$$

$$\sup_{\mathbb{P}\in\mathcal{P}_D}\mathbb{P}-CVaR_\varepsilon\left(L_n^{HS}(t)-\bar{L}_n^{HS}+\Delta L_n^D(t)\right)\le 0\quad \forall t\in T \tag{33}$$

$$\sup_{\mathbb{P}\in\mathcal{P}_D}\mathbb{P}-CVaR_\varepsilon\left(-L_n^{HS}(t)+\underline{L}_n^{HS}-\Delta L_n^D(t)\right)\le 0\quad \forall t\in T \tag{34}$$

$$0\le L_{m,n}^{H_2}(t)\le \bar{L}_{m,n}^{H_2}\quad \forall m\in M,\forall t\in T \tag{35}$$

2) IUs

The follower-level problem of IU i (P-IU$_i$) is given as follows. The objective function J_i of each IU in (36) is comprised of hydrogen purchasing cost and a logarithm utility function. The hydrogen demand of IUs is much higher than that of HRSs. It is obvious that a higher value of $k_{m,i}^{IU}$ indicates that a user is willing to consume more hydrogen to improve its utility. Therefore, the preference parameter of IUs is set higher than that of HRSs. Constraint (37) specifies the range of the planned hydrogen purchasing amount. Constraint (38) ensures that the purchased hydrogen equals to the hydrogen demand.

P-IU$_i$:

$$\min_{L_{m,\iota}^{H_2}} J_\iota=\sum_{t\in T}\sum_{m\in M}\left(L_{m,\iota}^{H_2}(t)\lambda_m^{IHC}(t)+k_{m,\iota}^{IU}\ln\left(1+L_{m,\iota}^{H_2}(t)\right)\right) \tag{36}$$

s.t.

$$0\le L_{m,\iota}^{H_2}(t)\le \bar{L}_{m,\iota}^{H_2}\quad \forall m\in M,\forall t\in T \tag{37}$$

$$\sum_{m\in M}L_{m,\iota}^{H_2}(t)=L_\iota^D(t)\quad \forall m\in M,\forall t\in T \tag{38}$$

IV. Distributionally Robust Game Framework for Electricity-hydrogen Trading

The distributionally robust game framework for electricity-hydrogen trading is mathematically formulated as two nested sub-models. The hydrogen trading is formulated as a Stackelberg game, and the electricity trading is formulated as an inner-loop peer-to-peer model nested in the upper layer of the hydrogen trading.

A. Hydrogen Trading

The hydrogen trading among $|M|$ HMGs, $|N|$ HRSs, and $|I|$ IUs is modeled as an MLMF Stackelberg game, where HMGs represent leaders and heterogeneous HUs playing the role of followers. HMGs optimize their self-dispatching strategies and engage in electricity trading to decide the hydrogen price, and then they propose the hydrogen price to all HUs. Each HU has a fixed hydrogen demand for a time horizon, and it aims to minimize its cost by determining the optimal hydrogen purchasing strategy and operation strategy based on the hydrogen prices announced by HMGs. The information asymmetry arises from the fact that the HUs have private knowledge of their actual hydrogen demand, which is inaccessible to HMGs. The MLMF Stackelberg game between the HMGs and HUs can be formulated as:

$$\Big\{M\cup N\cup I,\left\{\lambda_{m,z}^{H_2}\right\}_{m\in M,z\in N\cup I},\left\{L_{m,z}^{H_2}\right\}_{m\in M,z\in N\cup I},\left\{J_m\right\}_{m\in M},\left\{J_n\right\}_{n\in N},\left\{J_\iota\right\}_{\iota\in I}\Big\} \tag{39}$$

where the set of participants is $M \bigcup N \bigcup I$; the sets of trading strategies are $\left\{\lambda_{m,z}^{H_2}\right\}_{m \in M, z \in N \bigcup I}$ and $\left\{L_{m,z}^{H_2}\right\}_{m \in M, z \in N \bigcup I}$; and the sets of payoff functions are $\left\{J_m\right\}_{m \in M}$, $\left\{J_n\right\}_{n \in N}$, and $\left\{J_\iota\right\}_{\iota \in I}$.

B. Electricity Trading

In the upper layer of hydrogen trading, the non-cooperative electricity trading among HMGs is formulated as a two-step problem, as illustrated in Fig. 2. Firstly, the optimal energy sharing profiles for every HMG are derived to achieve a minimum operation cost, where the objective function (40) is the total social cost (P1); and (41) is the coupling constraint between HMGs.

P1:

$$\min \sum_{t \in T} \sum_{m=1}^{M} C_m^{sc}(t) = \sum_{t \in T} \sum_{m=1}^{M} \left(C_m^{op}(t) + C_m^{EMG}(t) - C_m^{H_2}(t)\right) \tag{40}$$

s.t.

$$(7)\text{-}(23)$$

$$P_{l,m}(t) = P_{m,l}(t) \quad \forall m \in M, l \in M \setminus m, t \in T \tag{41}$$

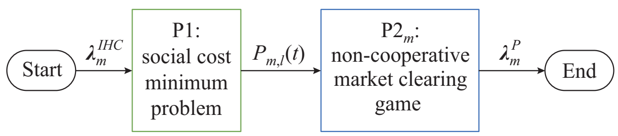

Fig. 2. Structure of non-cooperative electricity trading.

Then, the market clearing is developed as a non-cooperative market clearing game (P2$_m$), where each HMG $m \in M$ decides the price of purchasing/charging in the electricity trading price. Each HMG consistently aims to minimize its expenses for purchasing electricity from other HMGs. Consequently, the market clearing process, during which each HMG settles payments for others, essentially resembles a non-cooperative game characterized by interconnected constraints. The non-cooperative market clearing problem is given as follows. The objective function (42) is the electricity trading payment. Constraint (43) guarantees that the peer-to-peer electricity trading is more profitable than that with the external grid. Equation (44) is the payment coupling constraint between HMGs.

P2$_m$:

$$\min f_m\left(\boldsymbol{\lambda}_m^P, \boldsymbol{\lambda}_{-m}^P\right) = \sum_{t \in T} \sum_{l \in M \setminus m} P_{l,m}(t) \lambda_{m,l}^P(t) \quad \forall m \in M \tag{42}$$

s.t.

$$C_m^{non} \geq \sum_{t \in T} C_m^{sc}(t) + \sum_{l \in M \setminus m} P_{l,m}(t) \lambda_{m,l}^P(t) \tag{43}$$

$$\lambda_{m,l}^P(t) = \lambda_{l,m}^P(t) \quad \forall m \in M, \forall l \in M \setminus m, \forall t \in T \tag{44}$$

where $\boldsymbol{\lambda}_m^P = \left[\lambda_{m,l}^P\right] \geq \mathbf{0}$; and $\boldsymbol{\lambda}_{-m}^P = \left[\boldsymbol{\lambda}_l^P\right]$.

The electricity market clearing game is formulated as follows. In such a non-cooperative game, each HMG cannot improve its payoff function by changing its strategy $\boldsymbol{\lambda}_m^P$.

$$\left\{M, \boldsymbol{\lambda}_m^P \in \{(43),(44)\}, \left\{f_m\left(\boldsymbol{\lambda}_m^P, \boldsymbol{\lambda}_{-m}^P\right)\right\}_{m \in M}\right\} \tag{45}$$

where the set of participants is M; the sets of trading strategies are (43) and (44); and the set of payoff functions is $\left\{f_m\left(\boldsymbol{\lambda}_m^P, \boldsymbol{\lambda}_{-m}^P\right)\right\}_{m \in M}$.

C. Game Analysis

In this subsection, we prove the existence of the Nash equilibrium (NE), which is the optimal solution of the proposed game framework. Considering that the proposed LEM is a multi-level game model, a backward induction method is employed to prove that the LEM model has an equilibrium point [34]. Accordingly, we have Theorem 1.

Theorem 1 There exists a unique NE where each participant cannot improve its income.

Proof A unique SE exists if the following three conditions are satisfied [35]. ① Condition 1: the set of participants is finite. ② Condition 2: the sets of trading strategies are closed, bounded, and convex. ③ Condition 3: the payoffs of all participants are continuous and quasi-concave in the strategy space. We will verify that these three conditions are respected.

1) There are $|M|$ HMGs and $|N|$ off-site HRSs, so the set of participants $M \bigcup N$ is finite. Condition 1 is satisfied.

2) Combined with the reformulation of DR-CVaR constraints, the constraints of HRSs/IUs are all linear, which is convex and closed. The feasible sets are clearly nonempty. Furthermore, they are compact because, in practical applications, the variables associated with the hydrogen storage and hydrogen trading are all bounded.

For the feasible set of HMGs, the actual amount of hydrogen slod by HMG m at time t is given by:

$$D_m^{H_2}(t) = \begin{cases} L_m^{H_2,\max}(t) & \sum_{z \in N \bigcup I} L_{m,z}^{H_2}(t) > L_m^{H_2,\max}(t), \forall t \in T \\ \sum_{z \in N \bigcup I} L_{m,z}^{H_2}(t) & \sum_{z \in N \bigcup I} L_{m,z}^{H_2}(t) \leq L_m^{H_2,\max}(t), \forall t \in T \end{cases} \tag{46}$$

In either case, (46) is a linear. Combined with the reformulation of DR-CVaR constraints, the constraints of HMGs are all linear, convex, and bounded. The feasible sets are clearly nonempty. Furthermore, they are compact because the variables associated with the electrolyzer, battery, hydrogen storage, and energy transactions are all bounded. Condition 2 is satisfied accordingly.

3) The second-order derivatives of $\left\{J_z\right\}_{z \in N \bigcup I}$ can be obtained as:

$$\frac{\partial^2 J_z}{\partial\left(L_{m,z}^{IHC}(t)\right)^2} = -\frac{k_{m,z}^{HRS/IU}}{\left(1 + L_{m,z}^{H_2}(t)\right)^2} < 0 \quad \forall t \in T \tag{47}$$

Obviously, the objective functions of HUs are continuous and concave. Regardless of the value, when the energy prices of HMGs are given, there is a unique optimal hydrogen purchasing strategy of HUs. The second-order derivatives of the objective function J_m can be obtained as 0 by the backward induction. Therefore, J_m is also a continuous and concave function. Condition 3 is satisfied accordingly.

Therefore, there exists a unique NE for the non-cooperative clearing game.

V. SOLUTION METHODOLOGY

A. Equivalent Reformulation of DR-CVaR Constraints

The generic formulation of DR-CVaR constraints studied in this paper ((12), (33), and (34)) is shown as (48) with parameters H and h:

$$\sup_{\mathbb{P}\in\mathcal{P}} \mathbb{P}-CVaR_{\varepsilon}\left(Hx-h+\xi\right)\le 0 \tag{48}$$

We reformulate the DR-CVaR constraint (48) in Theorem 2.

Theorem 2 The DR-CVaR constraint is equivalent to the following deterministic constraint:

$$Hx-h+\eta\le 0 \tag{49}$$

Proof The proof is provided in Supplementary Material B.

By implementing the above reformulation, the original problem of HMGs and HUs (nonlinear programming with DR-CVaR constraints) can be recast into linear programs as follows, which can be directly solved by off-the-shelf solvers.

$$\begin{cases}\min \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}\\ \text{s.t. } \hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{x}\le\hat{\boldsymbol{d}}\end{cases} \tag{50}$$

B. Distributed Algorithm

To obtain the transactive energy in the LEM, a distributed algorithm with inner and outer iterations is proposed. The outer iteration involves interactions among HMGs and HUs, and the inner iteration is conducted to solve the electricity trading among HMGS.

The good performance of ADMM algorithm in solving practical problems makes it widely used in distributed peer-to-peer energy trading. However, the traditional ADMM algorithm converges slowly and is highly dependent on the initial penalty parameter selection. In this paper, we implement the fast-ADMM algorithm because it converges faster than the traditional ADMM algorithm. The fast-ADMM algorithm is widely used to divide the complex problems into smaller pieces for faster solution process [36]. Therefore, it is employed for solving the inner electricity trading problem, which involves iterations between two subproblems. By introducing auxiliary variables $\hat{P}_{m,l}(t)$, we can replace (41) with (51) and (52).

$$\hat{P}_{m,l}(t)=P_{m,l}(t)\quad \forall m\in M,\forall l\in M\setminus m,\forall t\in T \tag{51}$$

$$\hat{P}_{m,l}(t)+\hat{P}_{l,m}(t)=0\quad \forall m\in M,\forall l\in M\setminus m,\forall t\in T \tag{52}$$

By defining the dual variable of (51) as $\upsilon_{m,l}$, we can write the augmented Lagrangian function of P1:

$$\begin{cases}\min\left\{\sum\limits_{m\in M}C_m^{sc}+\right.\\ \left.\sum\limits_{l\in M\setminus m}\sum\limits_{t\in T}\left[\dfrac{\gamma_1}{2}\left(\hat{P}_{m,l}^{k_1}(t)-P_{m,l}(t)\right)^2+\upsilon_{m,l}(t)P_{m,l}(t)\right]\right\}\\ \text{s.t. (7)-(23)}\quad \forall m\in M\end{cases} \tag{53}$$

The fast-ADMM algorithm uses a predictor-corrector-type acceleration step in the update of dual variables, and the auxiliary variables, acceleration operator, and dual variables are updated by (54)-(57) [36].

$$\hat{P}_{m,l}^{k_1+1}(t)=\left[\gamma_1\left(P_{m,l}^{k_1}(t)-P_{l,m}^{k_1}(t)\right)-\left(\upsilon_{m,l}^{k_1}(t)-\upsilon_{l,m}^{k_1}(t)\right)\right]\Big/\left(2\gamma_1\right)$$
$$\forall m\in M,\forall l\in M\setminus m,\forall t\in T \tag{54}$$

$$\tilde{\upsilon}_{m,l}^{k_1+1}(t)=\upsilon_{m,l}^{k_1}(t)+\gamma_1\left(\hat{P}_{m,l}^{k_1+1}(t)-P_{l,m}^{k_1}(t)\right)$$
$$\forall m\in M,\forall l\in M\setminus m,\forall t\in T \tag{55}$$

$$\eta_m^{k_1+1}=1+\sqrt{1+4\left(\eta_m^{k_1}\right)^2}\,/2 \tag{56}$$

$$\upsilon_{m,l}^{k_1+1}(t)=\tilde{\upsilon}_{m,l}^{k_1}(t)+\left(\eta_m^{k_1}-1\right)\left(\upsilon_{m,l}^{k_1}(t)-\upsilon_{m,l}^{k_1-1}(t)\right)\Big/\eta_m^{k_1+1} \tag{57}$$

To obtain the NE in the non-cooperative electricity market clearing, we introduce a regularized Nikaido-Isoda-function $\Psi\left(\boldsymbol{\lambda}^P,\hat{\boldsymbol{\lambda}}^P\right)$ in (58). It indicates how much an HMG gains if it changes its trading strategy to a new one $\lambda_m^{IHC,k+1}(t)=\lambda_m^{H_2,k+1}(t)+c^{tax}c_m^{HS}(t)$, while all other HMGs hold their trading strategies [37].

$$\Psi\left(\boldsymbol{\lambda}^P,\hat{\boldsymbol{\lambda}}^P\right)=\sum_{m\in M}\left(f\left(\boldsymbol{\lambda}_m^P,\boldsymbol{\lambda}_{-m}^P\right)-f_m\left(\hat{\boldsymbol{\lambda}}_m^P,\boldsymbol{\lambda}_{-m}^P\right)-\frac{\gamma}{2}\left\|\hat{\boldsymbol{\lambda}}_m^P-\boldsymbol{\lambda}_m^P\right\|_2^2\right) \tag{58}$$

Then, we can rewrite $\mathrm{P2}_m$ as:

$$\begin{cases}\min\,(58)\\ \text{s.t. }(43)\end{cases} \tag{59}$$

Note that this problem can also be solved via the fast-ADMM algorithm, with trading strategy updated as:

$$\boldsymbol{\lambda}_m^{P,k_2+1}=k_2\boldsymbol{\lambda}_m^{P,k_2}\big/\left(k_2+1\right)+\hat{\boldsymbol{\lambda}}_m^{P,k_2}\big/\left(k_2+1\right) \tag{60}$$

The inner iteration based on the fast-ADMM algorithm is guaranteed to converge with a rate of order $O\left(1/\mu^2\right)$ [36]. This means that it requires $O\left(1/\mu^2\right)$ iterations to achieve a solution with an accuracy of μ.

After updating the electricity trading and receiving the hydrogen demand from HUs, the HMGs update the hydrogen price by (61) [38].

$$\lambda_m^{H_2,k+1}(t)=\lambda_m^{H_2,k}(t)+$$
$$\psi\left(\sum_{z\in N\cup I}P_{m,z}^{H_2}(t)-D_m^{H_2}(t)\right)\quad \forall m\in M,\forall t\in T \tag{61}$$

The integrated hydrogen-carbon price is updated as:

$$\lambda_m^{IHC,k+1}(t)=\lambda_m^{H_2,k+1}(t)+c^{tax}c_m^{HS}(t)\quad \forall m\in M,\forall t\in T \tag{62}$$

To resolve this problem and improve the convergence, a step-length control method is implemented in (63). The main idea is to limit the ramping rate of hydrogen price in the iterative process [38].

$$\max\left(\lambda_m^{IHC,k}(t)-\Delta,\underline{\lambda}_m^{IHC}\right)\le\lambda_m^{IHC,k+1}(t)\le$$
$$\min\left(\bar{\lambda}_m^{IHC,k+1}(t)+\Delta,\lambda_m^{IHC}\right)\quad \Delta=\zeta\left|\lambda_m^{IHC,k}(t)\right|,\forall m\in M,\forall t\in T \tag{63}$$

With the updated hydrogen prices, P-HRS_n ($n=1,2,\ldots,|N|$) and P-IU_ι ($\iota=1,2,\ldots,|I|$) are solved to rearrange the hydrogen purchasing plan. This process is iteratively performed until no adjustment on the hydrogen prices is found.

The distributed algorithm for solving the multi-level market problem is shown in Algorithm 1.

Algorithm 1: distributed algorithm for solving the multi-level market problem

Initialize error tolerances $\sigma=\sigma_1=\sigma_2=10^{-3}$ and step size ψ. Randomly initialize hydrogen price. Initialize iteration index $k=0$
Repeat (outer iteration)
  for each HU **do**
    Solve P-HRS$_n$/P-IC$_i$ with λ_m^{IHC}
    Update the hydrogen demand
  end for
  Initialize dual variables $v^0=0$. Initialize $k_1=0$ and $k_2=0$
  Repeat (inner iteration)
    for each HMG **do**
      Solve (53) based on the updated hydrogen demand
    end for
    Update auxiliary variables, acceleration operator, and dual variables by (54)-(57)
    $k_1=k_1+1$
  Until $\sum_{m\in M}\left\|\hat{P}_{m,l}^{k_1}-P_{m,l}^{k_1}\right\|\le\sigma_1$
  Repeat (inner iteration)
    Solve (59)
    Update λ_m^{P,k_2+1} by (60)
    $k_2=k_2+1$
  Until $\sum_{m\in M}\left\|\hat{\lambda}_m^{P,k_2}-\lambda_m^{P,k_2}\right\|\le\sigma_2$
  Update hydrogen and integrated hydrogen-carbon prices by (61) and (62), respectively
  $k=k+1$
Until $\sum_{m\in M}\sum_{t\in T}\left\|\lambda_m^{H_2,k+1}(t)-\lambda_{m,k}^{H_2,k+1}(t)\right\|\le\sigma$

VI. Case Studies

In this section, we start with case studies on an LEM with three HMGs, two HRSs, and one IU to demonstrate the effectiveness of the proposed game framework. Then, the scalability of the distributed algorithm is discussed. The experiments are implemented on a personal computer with an Intel Core i7 4.70 GHz CUP and 16 GB memory. The program is coded in MATLAB R2022a with YALMIP (Version R20210331). GUROBI (Version 10.0.2) and MOSEK (Version 10.0.20) are employed to solve the optimization problems. Following [39], the Wasserstein radius is directly obtained based on historical data. The historical data gathered from [40] are served as the daily profiles of WT and PV generation. The WT and PV capacities in HMGs are set to be 1.5 MW and 2 MW, respectively. The time-varying electricity prices and carbon emission factors of external grid are illustrated in Fig. 3. The hydrogen demands of HRSs and IUs are derived from [11] and [41], respectively. The specific parameters in the experiment setup are provided in Supplementary Material A Table SAI. The parameters in the utility function of HRSs are randomly generated uniformly within the range of 1 to 5, while those of IUs are randomly generated uniformly within the range of 20 to 50. The base distribution of DPMM is often chosen as the Normal-Wishart. The initial number of clusters is set to be 1. Note that it can be freely initialized and automatically optimized during the learning process [42].

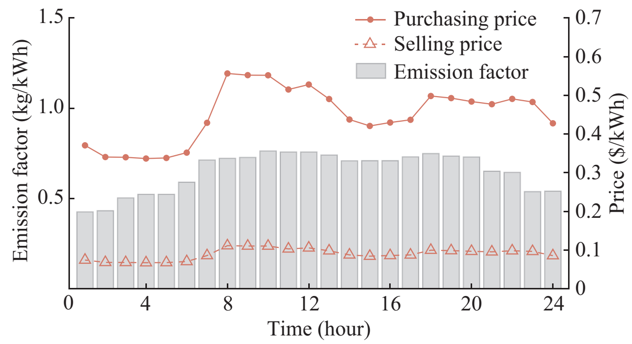

Fig. 3. Time-varying electricity prices and carbon emission factors of external grid.

A. Multi-level Electricity-hydrogen Trading Results

To evaluate the validity of the proposed LEM, this subsection analyzes the following five cases with different trading strategies. In these cases, the carbon tax is set to be 100 ¥/t.

Case 1: the proposed LEM, i. e., with multi-level game framework.

Case 2: only bi-level hydrogen trading. Without peer-to-peer electricity trading in multi-microgrids, the HMGs can only trade electricity with the external grid.

Case 3: only single-level peer-to-peer electricity trading in multi-microgrids. The hydrogen is traded at a flat price of 22.5 ¥/kg.

Case 4: the LEM without integrated hydrogen-carbon price. The carbon price is embedded in the objective function of HMGs.

Case 5: the LEM without carbon emission obligation.

Cases 1-3 are designed for comparison to show the effect of the proposed LEM. Cases 1, 4, and 5 are designed for the comparative analysis of the integrated hydrogen-carbon pricing strategy.

To validate the adaptability of the proposed game framework, all cases are examined across various energy and demand levels during three seasons. The diverse energy/demand levels are presented in Supplementary Material A Table SAII, and the computational results are given in Supplementary Material A Table SAIII. It can be seen that Case 1 shows the best economic performance during all three seasons. The total income in Case 1 increases on average by 3.4%, 8.9%, 3.5%, and 1.9% compared with that in Cases 2-5, respectively. In terms of carbon emissions, Case 1 is proven to be more effective in carbon emission abatement than Cases 2-5, achieving average total carbon emission reductions of 14.3%, 8.5%, 9.2%, and 13.3%, respectively. Compared with Case 5, although the conventional carbon emission obligation in Case 4 can decrease carbon emission by reducing energy consumption, not a small amount of income is sacrificed simultaneously. By contrast, Case 1 enjoys significant improvements in both income and carbon emission abatement. The results in Supplementary Material A Table SAIII demonstrate that the proposed LEM performs the best in both total income and carbon emission abatement.

Figure 4 illustrates the hydrogen trading results of different HMGs in Case 1, including the hydrogen price λ^{H_2}, integrated hydrogen-carbon price λ^{IHC}, and amount of hydrogen sold by HMGs to HRSs and IUs, i.e., D^{H_2}. The integrated hydrogen-carbon price rises in all periods while the price increment changes over time according to the carbon intensity of stored hydrogen. The settled hydrogen demand tends to be high when the integrated hydrogen-carbon price is low.

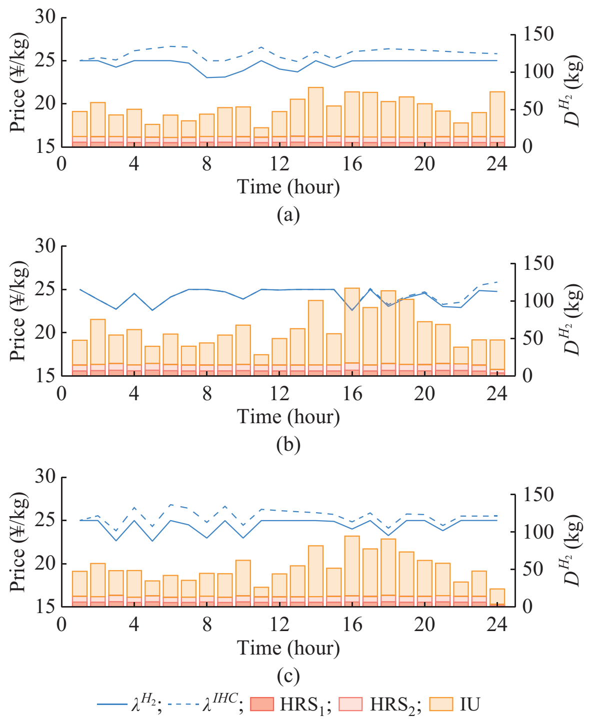

Fig. 4. Hydrogen trading results of different HMGs in Case 1. (a) HMG 1. (b) HMG 2. (c) HMG 3.

Figure 5 illustrates the carbon intensity of HMGs in Case 1. As shown in Fig. 5(a), the electricity carbon intensities in HMG 1 and HMG 3 are high during hours 0-7, during which we tend to purchase electricity from the external grid because of the relatively low electricity price. Similarly, the carbon intensities and cumulated carbon emissions of hydrogen storage and battery in HMG 1 and HMG 3 increase during hours 0-7. Note that in Fig. 5(b) and (c), the sizes of the squares correspond to the values of $c_m^{HS,cum}$ $c_m^{b,cum}$, ranging from 0-9 tCO_2 and 0-200 $kgCO_2$, respectively. From Fig. 5(b), we observe that the carbon intensity of hydrogen storage in HMG 2 increases since hour 7, coinciding with the increase of carbon intensity in HMG 2, as it has to purchase electricity from the external grid.

To present the change in the behaviors of HMGs and HUs by considering the integrated carbon-hydrogen price more intuitively, the specific hydrogen trading results of different HMGs and total hydrogen that flows from HMGs to HUs in Case 1 and Case 5 are demonstrated in Fig. 6. In Fig. 6(d) and (e), the numbers on the left side denote the average carbon intensity in the produced hydrogen, and the numbers on the right side are the total carbon emission in the hydrogen purchased.

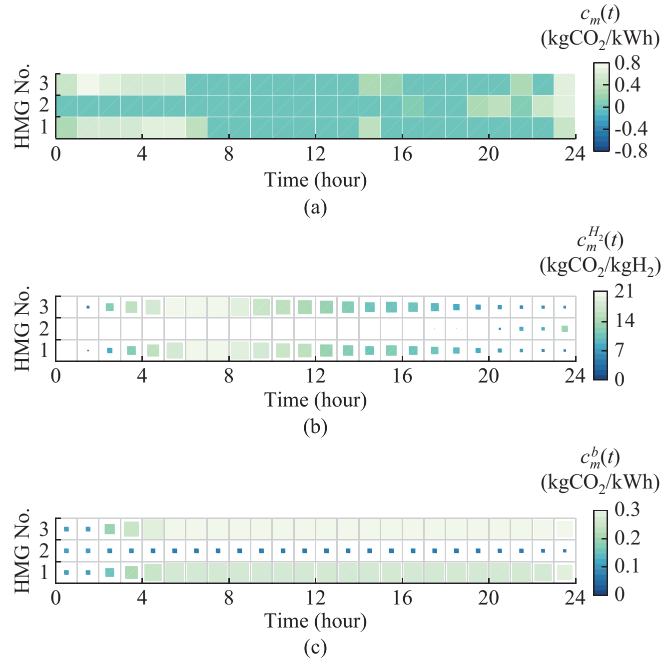

Fig. 5. Carbon intensity of HMGs in Case 1. (a) Electricity. (b) Hydrogen storage. (c) Battery system.

From Fig. 5 and Fig. 6(b), it is obvious that HMG 2 produces hydrogen with the lowest carbon intensity and cumulated carbon emission. This is directly reflected in the integrated hydrogen-carbon price, thus stimulating HUs to purchase significantly more hydrogen from HMG 2.

From Fig. 6(a)-(c), we observe that the integrated hydrogen-carbon pricing strategy pulls up the hydrogen prices of HMG 1 and HMG 3 in Case 5. Consequently, HUs significantly reduce the amount of hydrogen purchased from HMG 1 and HMG 3 and instead purchase more from HMG 2. The overall changes in hydrogen purchasing amount are shown in Fig. 6(d) and (e), illustrating that the integrated hydrogen-carbon pricing strategy leads HUs to purchase hydrogen primarily from HMG 2, which has the lowest average carbon intensity in the hydrogen production. This shift results in carbon emission reductions of 0.08-4.32 t.

B. Sensitivity Analysis

To evaluate the impact of the integrated hydrogen-carbon pricing strategy on the hydrogen trading, we perform a sensitivity test with the carbon tax increasing from 0 to 450 ¥/tCO_2 with a step size of 50 ¥/tCO_2. Figure 7 presents the experiment results under different carbon tax settings. From Fig. 7, it is evident that the average hydrogen price λ^{H_2} experiences an upward trend with the increasing carbon tax. This is because the high carbon tax drives up the average integrated hydrogen-carbon price λ^{IHC}. The effects of the carbon tax on the three HMGs are distinct. As the carbon tax increases, there has been a significant increase in the income of HMG 2, while there is no obvious increase in that of HMG 1 and HMG 3. The integration of carbon price serves as an incentive for HUs to opt for HMG 2, as its price is relatively low owing to its low carbon intensity in hydrogen production.

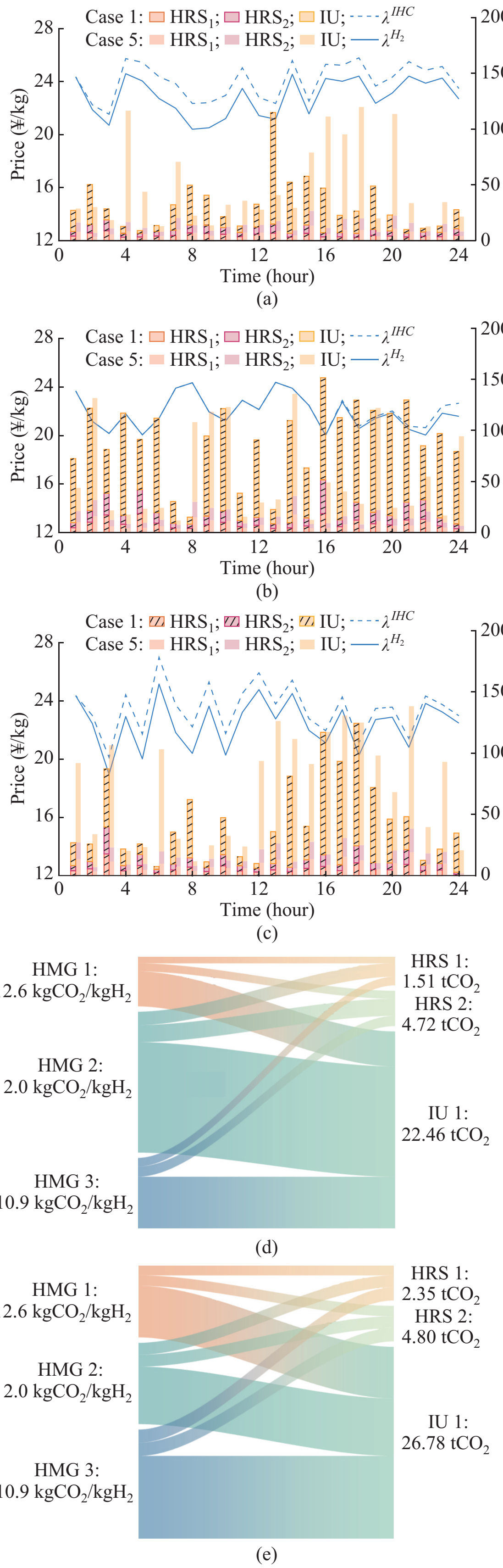

Fig. 6. Hydrogen trading results of different HMGs and total hydrogen flows from HMGs to HUs in Case 1 and Case 5. (a) Hydrogen trading results of HMG 1. (b) Hydrogen trading results of HMG 2. (c) Hydrogen trading results of HMG 3. (d) Total hydrogen flows from HMGs to HUs in Case 1. (e) Total hydrogen flows from HMGs to HUs in Case 5.

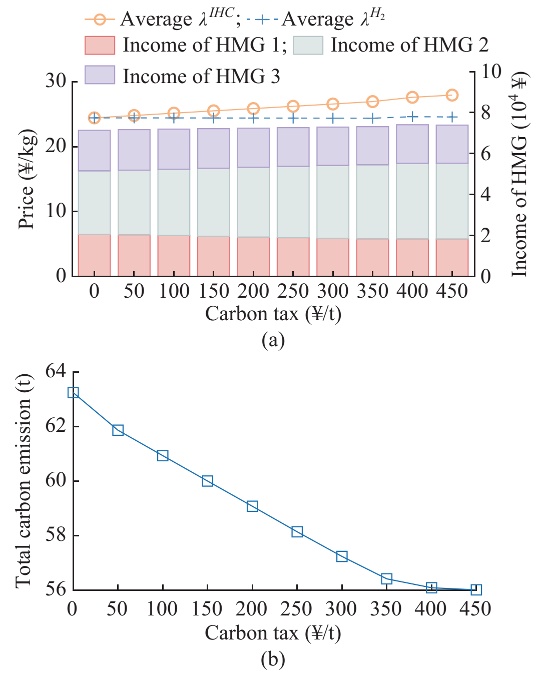

Fig. 7. Experiment results under different carbon tax settings. (a) Hydrogen price, integrated hydrogen-carbon price, and income of HMG. (b) Total carbon emission.

Hence, the integrated hydrogen-carbon pricing strategy is more capable of incentivizing low-carbon hydrogen production and trading.

From Fig. 7, we can also analyze the impact of the carbon tax on the total carbon emission. Initially, the total carbon emission decreases at a linear rate as the carbon tax increases. The HMGs have an incentive to produce hydrogen with a low carbon intensity. However, when the carbon tax reaches 350 ¥/t, a further increase in the carbon tax has a slight impact on the total carbon emission. This is because the carbon emissions associated with electricity purchased from the external grid are near the minimum level. In comparison to carbon emissions solely influenced by hydrogen prices, the proposed integrated hydrogen-carbon pricing strategy achieves an impressive emission reduction up to 12.9%. This result confirms the superiority of the proposed integrated hydrogen-carbon pricing strategy in carbon emission abatement.

C. Comparative Analysis

In this subsection, we compare the performance of the Bayesian nonparametric-based DRO (M1) in this paper with five benchmark methods, i.e., moment-based DRO (M2), Wasserstein-based DRO (M3), Wasserstein- and moment-based DRO (M4), unimodality- and moment-based DRO (M5) [43], [44], and the deterministic method (M6).

From Supplementary Material A Table SAIV, we can see that M1 outperforms M2 and M3 in terms of the total income of HMGs by ¥1205.2 and ¥91.8, respectively. Meanwhile, compared with M2 and M3, M1 increases the total income of HUs by ¥101.6 and ¥22.7, respectively. The advanced DRO methods M4 and M5 generate less conservative decisions compared with traditional DRO methods M2 and M3. By contrast, M1 obtains the highest income for both sides of HMGs and HUs. In terms of system reliability, M6 is the least reliable. In summary, M1 leverages the global-local moment information and Wasserstein balls of uncertainties, proving to be the least conservative and the most cost-efficient among all DRO methods.

We further present the solution robustness tests for different DRO methods, and the results are presented by the box-plot in Fig. 8, where M6 is not included because its reliability is far below the reliability threshold of 90%. For each test, we use 100 independent source-load uncertainty samples and hydrogen demand uncertainty samples. All DRO methods satisfy the reliability requirement of 90%. Among them, M1 is the least conservative among all DRO methods.

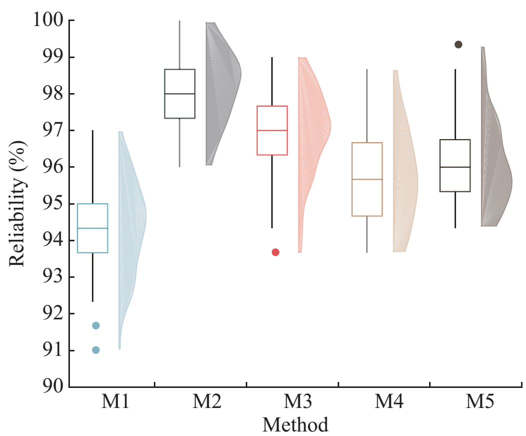

Fig. 8. Box-plot for solution robustness tests of different DRO methods.

D. Scalability

To further verify the scalability of the proposed game framework and distributed algorithm, we carry out case studies on different LEM scales. The considered LEM scales and their corresponding computational results are presented in Supplementary Material A Table SAV. Notably, the number of outer iterations experiences a modest increase with the augmentation of participants. Nevertheless, the computational time grows linearly with the LEM scale, rather than exhibiting exponential complexity. The computational time increases as the LEM size grows, but it is acceptable for the day-ahead market design problem.

VII. Conclusion

This paper proposes a novel multi-level LEM for electricity-hydrogen trading incorporating HMGs and heterogeneous HUs. An integrated hydrogen and carbon pricing strategy is proposed to incentivize low-carbon hydrogen trading. Further, a novel Bayesian nonparametric hybrid ambiguity set with the mixture of moment- and Wasserstein-based DRO is developed for distributionally robust self-dispatching. Case studies verify that both HMGs and HUs achieve cost-effective, sustainable, and reliable operation results under the proposed game framework, which also help boost the integration of renewable penetration and green hydrogen utilization. Although the proposed game framework exhibits satisfying results, it still has limitations.

The hydrogen transportation is neglected as we assume that HMGs and HUs are close to each other. Moreover, the situation where hydrogen demand from HUs drastically exceeds the production capacity of HMGs is not studied. In future work, we will consider large-scale trading, for which transportation constraints and external hydrogen sources are crucial. Future research will integrate a non-perfectly rational perspective based on prospect-theoretic behavioral models to accommodate non-rational participants [24], [45]. Additionally, competitive behaviors and information asymmetry can be further extended by reinforcement learning [46].

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Longyan Li received the B. Eng. degree from the School of Automation, Southeast University, Nanjing, China, in 2021. She is currently working toward the Ph.D. degree with the School of Electronic Information and Electrical Engineering, Shanghai Jiao Tong University, Shanghai, China. Her research interests include modeling and optimization of integrated energy system, smart hydrogen energy, and data-driven optimization under uncertainty.

Abdulelah S. Alshehri received the Bachelor of Chemical Engineering from Cleveland State University, Cleveland, USA, in 2016, the M.S. degree in chemical engineering from Carnegie Mellon University, Pittsburgh, USA, in 2017, and the Ph.D. degree in chemical engineering from Cornell University, Ithaca, USA, in 2024. He was a Lecturer with King Saud University (KSU), Riyadh, Saudi Arabia, from 2018 to 2024, and a KSU Doctoral Fellow and AI4Science Researcher at Cornell University, Ithaca, USA, from 2019 to 2024. Since August 2024, he has been an Assistant Professor with the Department of Chemical Engineering, KSU. His research interest includes artificial intelligence for science and engineering

Maher M. Alrashed received the B. Sc. degree in chemical engineering from King Saud University, Riyadh, Saudi Arabia, and M.Sc. and Ph.D. degrees in polymer engineering from The University of Akron, Akron, USA. He is currently an Associate Professor and Chairman of the Chemical Engineering Department at King Saud University. His research interests include sustainable polymer system, aerospace-grade hybrid coating, and smart material for energy, defense, and environmental applications.

Chao Ning received the B. Eng. degree in automation from the University of Electronic Science and Technology of China, Chengdu, China, in 2012, the M.S. degree in control science and engineering from Tsinghua University, Beijing, China, in 2015, and the Ph. D. degree from Cornell University, Ithaca, USA, in 2020. He is currently an Associate Professor with the Department of Automation, Shanghai Jiao Tong University, Shanghai, China. He is honored with National High-level Excellent Young Scholar. His research interests include data-driven optimization under uncertainty, learning-based model predictive control, artificial intelligence, and green electricity-hydrogen-chemical system.