

Reduced-order Bus Frequency Response Model for Bulk Power Systems

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Abstract—Bulk power systems show increasingly significant frequency spatial distribution characteristics (FSDCs), leading to a huge difference in the frequency response between regions. Existing uniform-frequency models based on analytical methods are no longer applicable. This paper develops a reduced-order bus frequency response (BFR) model to preserve the FSDC and describe the frequency response of all buses. Its mathematical equation is proved to be isomorphic to the forced vibration of a mass-spring-damper system, and the closed-form solution (CFS) of the BFR model is derived by the modal analysis method and forced decoupling method in vibration mechanics. The correlation between its mathematical equation and the state equation for small-signal stability analysis is discussed, and related parameters in the CFS are defined by the eigen-analysis method without any additional devices or tools. Case studies show that the proposed reduced-order BFR model and its CFS can improve the solution accuracy while keeping the solution speed within milliseconds, which can preserve the significant FSDC of bulk power systems and represent a normalized mathematical description of distinct-frequency models.

Index Terms—Bulk power system, frequency response, frequency spatial distribution characteristic, closed-form solution, small-signal stability.

NOMENCLATURE

A. Mathematical Symbols

Δ	Deviation from rated value
A_i	Corresponding value in A of the i^{th} generator bus
$\tilde{A}$	Corresponding value in A of center of inertia (COI)
$\dot{A}$	First-order differential to time
$\ddot{A}$	Second-order differential to time
A	Column vector
$\bar{A}$	Matrix with A_i as the i^{th} diagonal element

$\bar{I}$ Identity diagonal matrix

S Laplacian operator

B. Constant Parameters

ω_0, f_0	Nominal angular speed and nominal frequency
$\bar{B}$	Bus admittance matrix
$\bar{B}_d$	Initial moment distribution matrix
$\bar{B}_{gg}, \bar{B}_{gl}, \bar{B}_{ll}, \bar{B}_{ll}$	Four submatrices of $\bar{B}$ corresponding to different generator and load buses
$\bar{B}_s$	Inter-generator oscillation matrix
g, l	Generator bus and load bus
N	Number of generators in system

C. Variable Parameters

$\delta_g, f_g, \Delta P_g$	Rotor angle, frequency, and unbalanced electromagnetic power vectors of generator bus
$\theta_l, f_l, \Delta P_l$	Phase angle, frequency, and unbalanced electromagnetic power vectors of load bus
ΔP_m	Unbalanced mechanical power vector
ΔP_f	Unbalanced mechanical power vector generated by the first-order inertia feedback
ΔP_{coi}	Distribution power vector of COI at initial moment
ΔP_{osc}	Inter-generator oscillation power vector
Δf_{coi}	Frequency response (FR) vector of system COI
Δf_{osc}	FR matrix of inter-generator oscillation
$\bar{F}_h, \bar{T}_r, \bar{R}$	Matrices of high-pressure turbine power fraction, reheat time constant, and governor regulation coefficient
$\bar{H}, \bar{D}$	Matrices of inertia time constant and damping coefficient
S	Capacity of generator

D. Parameter Definitions in Vibration Mechanics

ω, ϕ, λ	Eigenvalue, eigenmode, and eigenroot
$\bar{M}, \bar{C}, \bar{K}, F$	Matrices of mass, damping, stiffness, and vector of external excitation
$\bar{M}_p, \bar{C}_p, \bar{K}_p, F_p$	Matrices of mass, damping, stiffness, and vector of external excitation in principal coordinates
x, x_p	Coordinate and principal coordinate

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E. Parameter Definitions in Bus Frequency Response (BFR) with Closed-form Solution (CFS)

α, β	Proportional coefficient and constant coefficient
ω_n, ω_r	Natural frequency and damped natural frequency
ζ, φ	Damping ratio and angular coefficient

F. Frequency Security Indicators

$\Delta f_{\max}$	The maximum frequency deviation
$\dot{\Delta f}_{\text{initial}}$	Initial rate of change of frequency
Δf_{qss}	Quasi-steady-state frequency deviation
t_{nadir}	Time to the maximum frequency deviation
T_{osc}	Frequency oscillation period

1. INTRODUCTION

THE frequency response (FR) focuses on the arresting period, frequency nadir, and initial parts of the recovery period within the first tens of seconds under large disturbances. These aspects are crucial for mitigating frequency deviations and preventing system instability [1]. The high proportion of renewable generation connected to the grid displaces the output capacity of synchronous generators, resulting in a continuous decline in the available FR resources to withstand disturbances. This has led to an increasingly critical frequency stability scenario [2]. Therefore, more accurate and efficient FR modeling and solution are essential to provide foundational tools for the planning, operation, protection, and control of bulk power systems [3].

Based on its physical principles, the FR modeling can be achieved using a set of differential-algebraic equations [4]. Depending on the model complexity, the FR solution methods can be broadly categorized into two methods: simulation methods based on full-order models and analytical methods based on reduced-order models. Simulation methods describe all components and their topological relationships within the system by establishing full-order nonlinear differential algebraic equations, and employ numerical integration techniques to iteratively solve for high-precision FR curves [5]. Therefore, simulation methods are widely applied in studies requiring high precision such as security check, operational planning, and protection scheme development, making them the mainstream FR solution methods [6], [7]. However, due to the computational complexity of iterative integration processes, simulation methods are relatively inefficient and unsuitable for the applications requiring high speed, such as online calculations and large-scale computations [8].

To enhance the computational efficiency, analytical methods derive the closed-form solution (CFS) [9] between frequency and power deficit, which can directly solve the FR and frequency security indicators such as frequency nadir without iterations. Compared with simulation methods, analytical methods have an absolute advantage in speed but relatively lower precision. The reason is that analytical methods require ignoring the factors with weaker influences on the

FR and developing reduced-order models [10]. The representatives are the reduced-order system frequency response (SFR) model and its several improved models [11]–[14]. These uniform-frequency models ignore the network topology and aggregate the dynamic behavior of all generators into a single equivalent generator, which can solve the system center-of-inertia (COI) of the FR [15]. Therefore, uniform-frequency models have been well applied in traditional power systems with insignificant frequency spatial distribution characteristics (FSDCs) [16]–[18].

However, with the continuous increase of renewable generation penetration and the expansion of the system scale, the FR resources in bulk power systems nowadays are becoming more limited and unevenly distributed, leading to increasingly pronounced FSDCs [19]. As a result, different regions may face varying frequency stability situations, exhibiting significant discrepancies in FR and frequency security indicators [20]. At this point, continuing to ignore the FSDC and relying solely on the COI results derived from uniform-frequency models for frequency stability analysis and control may lead to the following issues [21]: ① using a uniform frequency as the analysis basis may result in the frequency being out-of-limit or even misjudgment at certain unknown locations; ② using a uniform frequency as the control basis may result in over-/under-tuning or even misoperation of some generators. Both scenarios can introduce significant safety risks. Therefore, it is crucial for different regions to formulate specific control strategies based on local frequencies [22]. The foundation for achieving this lies in distributed-frequency modeling that accurately captures the significant FSDC.

Compared with uniform-frequency models, distributed-frequency models can consider the FSDC by preserving the network structure of the system [23]. However, network modeling introduces the rotor angle coupling between generators, resulting in an exponential increase in model order and presenting significant challenges for the analytical solution of FR [24]. Reference [25] proposes a two-area SFR model and provides its infinite Taylor series. However, the Taylor series requires expansion into at least a 91st-order polynomial to achieve acceptable accuracy, with the polynomial order increasing exponentially as the number of generators grows. References [26] and [27] propose two-/three-machine equivalent SFR models and derive their CFSs to solve the FRs of two and three regions. However, in bulk power systems with significant FSDC, generators are typically clustered into multiple coherent groups. Using two-/three-machine models in such cases may introduce substantial computational errors, failing to meet the precision requirements for frequency stability analysis and control. Reference [28] deduces a post-fault CFS of the multi-region system by several theoretical techniques, which can support frequency stability conditions for unit commitment [29]. However, since the mathematical structure of the derived CFS is based on engineering experience and empirical observations rather than rigorous mathematical derivation, it has two key limitations: ① the CFS does not cover the entire state space, restricting its applicability; and ② the explicit expressions for some parameters

in the CFS are not clear and still require estimation through numerical regression techniques. As a result, it cannot be considered a purely analytical method in the strict sense. Meanwhile, the aforementioned studies focus exclusively on generator buses, leaving a gap in effective analytical methods for solving the FR of load buses.

To accurately preserve the significant FSDCs in bulk power systems through analytical methods, this paper proposes a reduced-order bus frequency response (BFR) model and derives its CFS. The novel contributions are twofold.

1) The proposed reduced-order BFR model and its CFS can solve the FR of any bus within the system, thus completely preserving the FSDC and providing a standardized mathematical description for distributed-frequency models.

2) The CFS of the BFR model is derived through strict mathematical derivation, and the relationship between its parameters and the eigenroots already obtained in small-signal stability analysis are revealed, thus achieving a purely analytical solution without relying on any simulation software or hardware investments.

The remainder of this paper is organized as follows. The proposed reduced-order BFR model is developed in Section II. The analytical solution of the BFR model is derived in Section III. Some relevant parameters in the CFS of BFR model are defined in Section IV. Case studies and simulation results are presented in Section V. Discussions are given in Section VI. Finally, Section VII concludes this paper.

II. PROPOSED REDUCED-ORDER BFR MODEL

During system transients, operating parameters such as frequency and voltage are mutually coupled. These interactions result in full-order FR models that can only be resolved through simulation methods. To preserve the FSDCs and achieve analytical solution, this section proposes a reduced-order BFR model that integrates both network dynamics and FR resources.

A. Network Modeling

The voltage characteristics of the load result in a correlation between its active power dynamics and voltage variations, leading to a coupling relationship between system frequency dynamics and voltage dynamics. This paper focuses on bulk power systems with significant FSDCs, which exhibit the following features [23]: ① the resistance of transmission lines is much smaller than the reactance; ② the bus voltages are typically near their rated values, with amplitude deviations rarely exceeding 10%; and ③ the voltage phase angle difference at both sides of transmission lines rarely exceeds 20°.

Based on the above features, when bulk power systems experience an active power disturbance such as load shedding and disconnection of tie lines, if there is no system angle or voltage instability, the active power-frequency dynamics and reactive power-voltage dynamics can be decoupled. Following the voltage constant assumption, the DC power flow network modeling can compute the active power flow of the initial operating condition with much lower computational burden and acceptable precision [24]. This assumption is com-

monly found in studies related to frequency stability analysis and control [30]-[32].

According to the system network topology, the bus admittance matrix $\bar{\mathbf{B}}$ can be formed, which includes generator buses (subscripted as g) and load buses (subscripted as l). If a load bus connects to both constant power and other loads, the constant power loads can be treated as a slack bus connected via a small impedance. This allows for the elimination of internal buses, constant impedance, and constant current load buses through Kron reduction [33]. In this paper, “load buses” refer to buses with only constant power loads after reduction.

The relationship between the phase angles at each bus and the active power can be described by the power flow equation:

$$\begin{bmatrix} \Delta \mathbf{P}_g \\ \Delta \mathbf{P}_l \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{B}}_{gg} & \bar{\mathbf{B}}_{gl} \\ \bar{\mathbf{B}}_{lg} & \bar{\mathbf{B}}_{ll} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta}_g \\ \Delta \boldsymbol{\theta}_l \end{bmatrix} \quad (1)$$

For constant power load buses, by eliminating $\Delta \boldsymbol{\theta}_l$ in (1), $\Delta \mathbf{P}_g$ can be obtained as:

$$\Delta \mathbf{P}_g = \bar{\mathbf{B}}_d \Delta \mathbf{P}_l + \bar{\mathbf{B}}_s \Delta \boldsymbol{\delta}_g \quad (2)$$

$$\begin{cases} \bar{\mathbf{B}}_d = \bar{\mathbf{B}}_{gl} \bar{\mathbf{B}}_{ll}^{-1} \\ \bar{\mathbf{B}}_s = \bar{\mathbf{B}}_{gg} - \bar{\mathbf{B}}_{gl} \bar{\mathbf{B}}_{ll}^{-1} \bar{\mathbf{B}}_{lg} \end{cases} \quad (3)$$

At the initial moment of disturbance, the imbalance power $\Delta \mathbf{P}_l$ flows from the disturbance location to each generator based on the initial moment distribution matrix $\bar{\mathbf{B}}_d$. Each generator in the system will bear a portion of $\Delta \mathbf{P}_{coi}$. Subsequently, the differences in the FR characteristics of each generator will cause inter-generator oscillation power $\Delta \mathbf{P}_{osc}$ based on the inter-generator oscillation matrix $\bar{\mathbf{B}}_s$, driving the frequencies of all generators towards synchronization. Therefore, $\Delta \mathbf{P}_g$ is the superposition of $\Delta \mathbf{P}_{coi}$ and $\Delta \mathbf{P}_{osc}$.

$$\begin{cases} \Delta \mathbf{P}_{coi} = \bar{\mathbf{B}}_d \Delta \mathbf{P}_l \\ \Delta \mathbf{P}_{osc} = \bar{\mathbf{B}}_s \Delta \boldsymbol{\delta}_g \end{cases} \quad (4)$$

Since $\bar{\mathbf{B}}_s$ represents the network Laplacian matrix (phase angle-active power Jacobian) after the elimination of load buses, it reflects the electrical connections between generator buses. Consequently, $\bar{\mathbf{B}}_s$ inherently possesses the following properties: it is a symmetric matrix, and the sum of each row equals zero.

$$\begin{cases} B_{s,ij} = B_{s,ji} & i, j = 1, 2, \dots, N \\ \sum_{j=1}^N B_{s,ij} = 0 & i = 1, 2, \dots, N \end{cases} \quad (5)$$

B. FR Resource Modeling

The electromechanical transient of bulk power systems persists for tens of seconds following a large disturbance. Power angle stability analysis focuses on whether the synchronous generators can maintain synchronous operation during the first and second swings (defined as the first stage). Frequency stability analysis focuses on whether the frequency can remain within or recover to an acceptable range within the first tens of seconds (defined as the second stage). Only when power angle stability is maintained in the

first stage can the system transition to the second stage to assess whether frequency stability can be achieved.

As shown in Fig. 1, FR focuses on the arresting period, frequency nadir, and initial parts of the recovery period in the second stage [1], and therefore, the premise of this paper is that the power system maintains power angle stability following large disturbances [3]. In the second stage, the inertia response and governor response provided by synchronous generators, along with the fast FR provided by other forms of controls such as renewable energy sources, energy storage, and loads, collectively act to help arrest the frequency excursions [5]. Meanwhile, the stabilizers and various other types of additional damping controllers are gradually weakened [34]. Therefore, only the inertia response, the governor response, and the fast FR resources are included in the modeling.

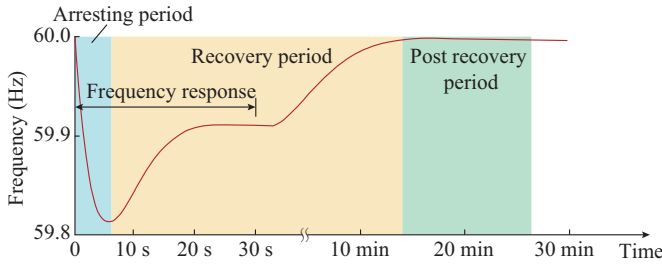


Fig. 1. Time frames of frequency transients.

1) Inertia Response

The swing equation is a second-order differential equation, and its solution provides information about the rotor angle dynamics and the response of generator to disturbances [4]. Therefore, the inertia response can be represented by the swing equation as:

$$\begin{cases} \Delta\delta_g s = \omega_0 \Delta f_g = 2\pi f_0 \Delta f_g \\ 2\bar{H}s\Delta f_g = \Delta P_m - \Delta P_g - \bar{D}\Delta f_g \end{cases} \quad (6)$$

2) Governor Response

The governor responds to changes in the electrical output and adjusts the mechanical power input to maintain system frequency within the desired range. Standard governor models such as IEEE G1 model have been recommended by institutions such as the IEEE Standard Committee [35]. These full-order governor models provide high precision in capturing the governor response but have complex structures that are not convenient for analytical solution. To address this issue, it is acceptable to preserve the essential elements and develop a reduced-order governor model. The most representative one consists of the governor regulation coefficient R , the reheat time constant T_r , and the high-pressure turbine power fraction F_h [11]. Its transfer function can be expressed as:

$$\Delta P_m = -(\bar{I} + \bar{F}_h \bar{T}_r s)(\bar{I} + \bar{T}_r s)^{-1} \bar{R}^{-1} \Delta f_g \quad (7)$$

3) Fast FR Resources

Existing resources including wind power, photovoltaics, battery energy storage systems, loads, and virtual power plants can participate in frequency regulation. This is achieved through the simulation of FR functions applied via

power electronic converter interfaces. According to different activation mechanisms, fast FR resources can be divided into the following two categories [1].

1) Proportional or derivative response based on the power-frequency characteristic curve. Examples include renewable energy sources such as wind power and photovoltaics. By incorporating feedback loops such as virtual inertia control and droop control, these resources can simulate the FR function and reduced-order model structure of synchronous generators. Under these conditions, these fast FR resources can be considered by adjusting the values of $\bar{H}$ and $\bar{D}$ in (6) without changing the model structure.

2) Step response based on a preset power reference value. Examples include battery energy storage systems, electrolytic aluminum, electrical fused magnesium, and other loads. Under these conditions, these fast FR resources can be considered by adjusting the value of ΔP_l in (2) without changing the model structure.

Kindly note that for these two categories of resources, we retain only the primary factors that accurately describe the relationship between frequency deviation and power, while ignoring the internal dynamics to capture their frequency regulation behaviors. This is also a common practice in analytical methods [16]–[18].

C. Reduced-order BFR Modeling and Simplification

Based on (2)–(7), the reduced-order BFR model is developed, as shown in Fig. 2 (taking the i^{th} generator bus as an example).

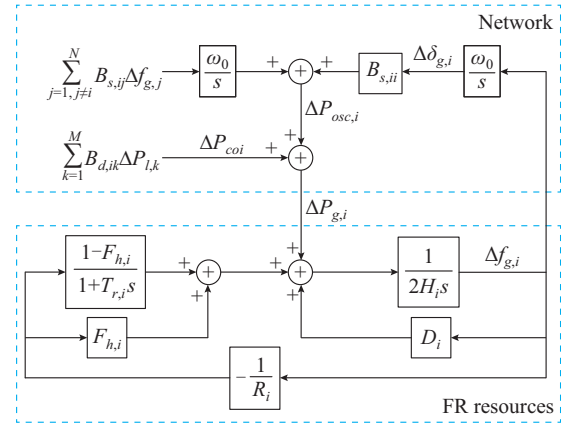


Fig. 2. Reduced-order BFR model.

The mathematical equation can be expressed as:

$$\bar{M}\Delta\delta_g s^2 + \bar{C}\Delta\delta_g s + \bar{K}\Delta\delta_g = \omega_0 (\Delta P_f - \Delta P_{coi}) \quad (8)$$

$$\begin{cases} \bar{M} = 2\bar{H} \\ \bar{C} = \bar{D} + \bar{F}_h \bar{R}^{-1} \\ \bar{K} = \omega_0 \bar{B}_s \\ \Delta P_f = -(\bar{I} - \bar{F}_h)(\bar{I} + \bar{T}_r s)^{-1} \bar{R}^{-1} \Delta f_g \end{cases} \quad (9)$$

As shown in (9), due to the first-order inertial correlation between ΔP_f and Δf_g in the reduced-order BFR model, (8) is a nonlinear formulation without an analytically solvable CFS. Therefore, reasonable simplification is required.

The simplification process of the reduced-order BFR model is shown in Fig. 3 and can be summarized in three steps.

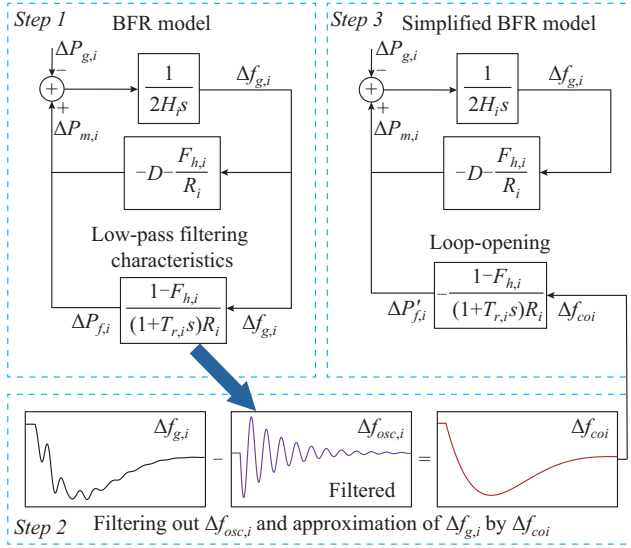


Fig. 3. Simplification process of reduced-order BFR model.

Step 1: the BFR model before simplification is shown in Fig. 3.

Step 2: compared with uniform-frequency models, the proposed reduced-order BFR model can not only solve the uniform FR of COI Δf_{coi} caused by ΔP_{coi} , but also the distributed inter-generator oscillation FR Δf_{osc} caused by ΔP_{osc} :

$$\Delta f_g = \Delta f_{coi} + \Delta f_{osc} = \begin{bmatrix} \Delta f_{coi} \\ \Delta f_{coi} \\ \vdots \\ \Delta f_{coi} \end{bmatrix} + \begin{bmatrix} \Delta f_{osc,1} \\ \Delta f_{osc,2} \\ \vdots \\ \Delta f_{osc,N} \end{bmatrix} \quad (10)$$

For an N -generator power system, the i^{th} generator is influenced by the remaining $N-1$ generators, and generates $N-1$ corresponding frequency oscillations [28]. Therefore, $\Delta f_{osc,i}$ is the superposition of $N-1$ components:

$$\Delta f_{osc,i} = \sum_{j=1, j \neq i}^N \Delta f_{osc,i \leftarrow j} \quad (11)$$

Step 3: as shown in Fig. 3, the low-pass filtering characteristics of the first-order inertia link between ΔP_f and Δf_g in (9) can filter out Δf_{osc} and approximate Δf_g by Δf_{coi} [36]. By opening the first-order inertial feedback loop of Δf_g and approximating it by Δf_{coi} based on the low-pass filtering characteristics, ΔP_f can be replaced by $\Delta P'_f$:

$$\Delta P'_f = -(\bar{I} - \bar{F}_h)(\bar{I} + \bar{T}_r s)^{-1} \bar{R}^{-1} \Delta f_{coi} \quad (12)$$

Since $\Delta P'_f$ is solvable and uncorrelated with Δf_g , (8) can be simplified to a quadratic nonhomogeneous differential equation:

$$\bar{M} \Delta \delta_g s^2 + \bar{C} \Delta \delta_g s + \bar{K} \Delta \delta_g = F \quad (13)$$

$$F = \omega_0 (\Delta P'_f - \Delta P_{coi}) \quad (14)$$

By solving (13), the CFS of the reduced-order BFR model can be derived. The validity of the above simplification is validated in Section V-A.

III. ANALYTICAL SOLUTION OF BFR MODEL

Many real-world physical problems including the BFR analytical solution of bulk power systems and the forced vibration of mass-spring-damper (MSD) systems can be described using differential-algebraic equations. The field of vibration mechanics has well-established theories for solving differential-algebraic equations related to forced vibrations [37]. By introducing the principles and theories from vibration mechanics, it is possible to draw analogies between the dynamic behavior of MSD systems and bulk power systems, thus providing valuable insights and mathematical tools for the BFR analytical solution.

A. Analogy Between BFR Solution and Forced Vibration

A vibration system with more than one degree of freedom and limited independent coordinates can be abstracted as an MSD system with multi-degrees of freedom, as shown in Fig. 4. Any practical engineering structure can be approximated as an MSD system with multi-degrees of freedom if it can be simplified to a finite number of centralized masses, springs, and dampers.

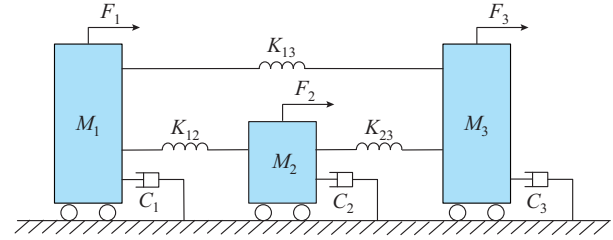


Fig. 4. MSD system with multi-degrees of freedom (three in this example).

An MSD system generates forced vibrations under the excitation of external controls. The motion equation in the time domain can be expressed as:

$$\bar{M} \ddot{x}(t) + \bar{C} \dot{x}(t) + \bar{K} x(t) = F(t) \quad (15)$$

It is worth noting that the generalized coordinate x can represent any variable of interest in the system. In bulk power systems, let $x = \Delta \delta_g$ and perform the Laplace transform on (15). The vibration equation of $\Delta \delta_g$ can be obtained as the same mathematical structure with (13), where $\bar{M}$ describes the mass distribution in the MSD system corresponding to the inertia distribution in the power system; $\bar{C}$ represents energy dissipation in the MSD system corresponding to damping in the power system; $\bar{K}$ represents elastic connections in the MSD system corresponding to electrical connections in the power system; and external excitation F in the MSD system corresponds to disturbances in the power system.

Therefore, the BFR analytical solution of bulk power systems is isomorphic to the mathematical equation for the forced vibration of MSD systems. It can be considered that these two systems follow the same physical principles. This isomorphism allows to draw analogy between the two problems and introduce well-established theories in vibration mechanics to solve the CFS of the reduced-order BFR model.

B. Derivation of CFS of BFR Model

In vibration mechanics, (13) is a forced vibration equation

of viscous damped MSD system with multi-degrees of freedom, which can be solved by the modal analysis method and forced decoupling method [37]. The specific steps of the derivation of CFS are as follows.

Step 1: the free vibration equation without damping corresponding to (13) can be expressed as:

$$\bar{M}\Delta\delta_g s^2 + \bar{K}\Delta\delta_g = \mathbf{0} \quad (16)$$

For an MSD system with N degrees of freedom, there are N eigenvalues $\{\omega_1, \omega_2, \dots, \omega_N\}$, each corresponding to an eigenmode ϕ_i . Each eigenmode $\phi_i = [\phi_{1i}, \phi_{2i}, \dots, \phi_{Ni}]^T$ is an N -dimensional column vector. Therefore, the eigenmode matrix Φ is an $N \times N$ matrix consisting of N eigenmodes $\{\phi_1, \phi_2, \dots, \phi_N\}$. The process of solving ω_i and ϕ_i in mathematics can be expressed as:

$$|\bar{K} - \bar{M}\omega_i^2| = \mathbf{0} \Rightarrow \omega = [\omega_1, \omega_2, \dots, \omega_N]^T \quad (17)$$

$$(\bar{K} - \bar{M}\omega_i^2)\phi_i = \mathbf{0} \Rightarrow \Phi = [\phi_1, \phi_2, \dots, \phi_N] = \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1N} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2N} \\ \vdots & \vdots & & \vdots \\ \phi_{N1} & \phi_{N2} & \dots & \phi_{NN} \end{bmatrix} \quad (18)$$

According to (5) and (9), $\bar{B}_s$ is non-full-rank, and its determinant is zero, indicating that it has a zero eigenvalue. The corresponding right eigenvector is $\mathbf{1}$ (a column vector with all elements equal to 1). Since $\bar{K}$ is proportional to $\bar{B}_s$, the determinant of $\bar{K}$ is also zero, signifying that the system is in neutral equilibrium:

$$\begin{cases} \omega_1 = 0 \\ \phi_1 = \mathbf{1} = [1, 1, \dots, 1]^T \end{cases} \quad (19)$$

Under this condition, the modal corresponding to (ω_1, ϕ_1) is the rigid body displacement generated by the overall MSD system, while the internal masses of the system remain relatively motionless, which represents the FR of COI of the power system. The remaining $N-1$ modals corresponding to $(\omega_2, \phi_2), \dots, (\omega_N, \phi_N)$ are the damped oscillation of each mass itself, which represents the frequency oscillation of each generator affected by the remaining $N-1$ generators.

Therefore, Δf_g and $\Delta\delta_g$ can be expressed by the eigenmode matrix Φ and the principal coordinate vector $\mathbf{x}_p$:

$$\Delta f_g = \frac{1}{\omega_0} \Delta\delta_g s = \frac{1}{\omega_0} \mathbf{x} s = \frac{1}{\omega_0} (\Phi \mathbf{x}_p) s \quad (20)$$

Step 2: substitute (20) into (13) and premultiply Φ^T on both sides of the equation, then perform the inverse Laplace transform. The motion equation in principal coordinates can be expressed as:

$$\bar{M}_p \ddot{\mathbf{x}}_p(t) + \bar{C}_p \dot{\mathbf{x}}_p(t) + \bar{K}_p \mathbf{x}_p(t) = \mathbf{F}_p(t) \quad (21)$$

where $\bar{M}_p$ and $\bar{K}_p$ are definitely diagonal based on modal orthogonality, while $\bar{C}_p$ is generally non-diagonal because of the energy coupling between different degrees of freedom in practical engineering applications:

$$\begin{cases} \bar{M}_p = \Phi^T \bar{M} \Phi \\ \bar{C}_p = \Phi^T \bar{C} \Phi \\ \bar{K}_p = \Phi^T \bar{K} \Phi \\ \mathbf{F}_p = \Phi^T \mathbf{F} \end{cases} \quad (22)$$

For bulk power systems with the significant FSDC, the non-diagonal elements of $\bar{C}_p$ represent the reactance of the transmission lines between generator buses [38]. Since these lines are typically high-voltage and long-distance (e.g., 500 kV in China), the reactance between generator buses is relatively small. As a result, the non-diagonal elements of $\bar{C}_p$ can be ignored by the forced decoupling method with minimal impact on accuracy. This allows (21) to be decoupled into N mutually independent principal coordinate equations, each representing an independent principal vibration:

$$M_{p,i} \ddot{x}_{p,i}(t) + C_{p,i} \dot{x}_{p,i}(t) + K_{p,i} x_{p,i}(t) = F_{p,i}(t) \quad (23)$$

$$\begin{cases} M_{p,i} = \sum_{j=1}^N \phi_{ij}^2 M_j \\ C_{p,i} = \sum_{j=1}^N \phi_{ij}^2 C_j \\ K_{p,i} = - \sum_{j=1}^{N-1} \sum_{k=j+1}^N (\phi_{ij} - \phi_{ik})^2 K_{jk} \\ F_{p,i} = \sum_{j=1}^N \phi_{ij} F_j \end{cases} \quad (24)$$

Step 3: when $i=1$, substituting (19) into (23), we can obtain:

$$\sum_{j=1}^N M_j \ddot{x}_{p,1}(t) + \sum_{j=1}^N C_j \dot{x}_{p,1}(t) = \omega_0 \sum_{j=1}^N (\Delta P_{f,j}(t) - \Delta P_{l,j}) \quad (25)$$

Equation (25) represents the mathematical equation of the aggregated SFR model [12], and the CFS of $\dot{x}_{p,1}(t)$ can be derived as:

$$\dot{x}_{p,1}(t) = \omega_0 \alpha_1 e^{-\zeta_1 \omega_{n,1} t} \sin(\omega_{r,1} t + \varphi) + \omega_0 \beta \quad (26)$$

where the expressions of α_1 and other parameters in (26) are given in Supplementary Material A.

Step 4: when $i=2, 3, \dots, N$, numerical results indicate that the magnitude of $\Delta P'_{f,i}$ is approximately several thousand times smaller than that of $\Delta P'_{f,1}$, rendering its impact on the FR minimal. By ignoring $\Delta P'_{f,i}$ (23) can be simplified from a nonlinear equation to a linear one:

$$\ddot{x}_{p,i}(t) + 2\zeta_i \omega_{n,i} \dot{x}_{p,i}(t) + \omega_{n,i}^2 x_{p,i}(t) = \omega_{r,i} a_i \quad i=2, 3, \dots, N \quad (27)$$

$$\begin{cases} \omega_{n,i} = \sqrt{K_{p,i}/M_{p,i}} \\ \zeta_i = (\omega_{n,i} C_{p,i}) / (2K_{p,i}) \\ \omega_{r,i} = \omega_{n,i} \sqrt{1 - \zeta_i^2} \quad i=2, 3, \dots, N \\ \alpha_i = \sum_{j=1}^N \frac{\phi_{ij} \Delta P_{coi,j}}{\omega_{r,i} M_{p,i}} \end{cases} \quad (28)$$

Therefore, the CFS of $\dot{x}_{p,i}(t)$ can be derived as:

$$\dot{x}_{p,i}(t) = \omega_0 \alpha_i e^{-\zeta_i \omega_{n,i} t} \sin \omega_{r,i} t \quad i=2, 3, \dots, N \quad (29)$$

Step 5: according to (20), (26), and (29), the CFS of Δf_g can be derived as:

$$\Delta \mathbf{f}_g(t) = \frac{1}{\omega_0} \Phi \dot{\mathbf{x}}_p(t) \quad (30)$$

To be specific, the FR of the i^{th} generator bus is:

$$\begin{cases} \Delta \mathbf{f}_{g,i}(t) = \Delta \mathbf{f}_{coi}(t) + \Delta \mathbf{f}_{osc,i}(t) \\ \Delta \mathbf{f}_{coi}(t) = \alpha_i e^{-\zeta_i \omega_{n,i} t} \sin(\omega_{r,i} t + \varphi) + \beta \\ \Delta \mathbf{f}_{osc,i}(t) = \sum_{j=2}^N \phi_{ij} \alpha_j e^{-\zeta_j \omega_{n,j} t} \sin \omega_{r,j} t \end{cases} \quad (31)$$

Step 6: according to (1), we can obtain:

$$\Delta \mathbf{P}_l = \bar{\mathbf{B}}_{lg} \Delta \boldsymbol{\delta}_g + \bar{\mathbf{B}}_{ll} \Delta \boldsymbol{\theta}_l \quad (32)$$

For constant power load buses, since $\Delta \mathbf{P}_l$ is a constant column vector, the derivation of (32) can be obtained as:

$$0 = \omega_0 \bar{\mathbf{B}}_{lg} \Delta \mathbf{f}_g + \omega_0 \bar{\mathbf{B}}_{ll} \Delta \mathbf{f}_l \quad (33)$$

Further, the CFS of $\Delta \mathbf{f}_l(t)$ can be derived as:

$$\Delta \mathbf{f}_l(t) = -\bar{\mathbf{B}}_{ll}^{-1} \bar{\mathbf{B}}_{lg} \Delta \mathbf{f}_g(t) \quad (34)$$

Equations (30) and (34) can solve the FR of generator buses and load buses, respectively. Therefore, the proposed reduced-order BFR model and its CFS can solve the FR of any bus within the system, thus completely preserving the FSDC and providing a standardized mathematical description for distributed-frequency models. The derivation of (32)-(34) is conceptually similar to the frequency divider formula [39]. Our derivation is based on the relationship between power and voltage phase angles, while the frequency divider formula is derived from the relationship between voltage and current. This offers an alternative perspective, mutually validating the rigor of both derivations.

However, the expressions of $(\omega_{n,i}, \zeta_i, \omega_{r,i}, \alpha_i)$ in (29) are related to (ω_i, ϕ_i) . Typically, numerical analysis methods (such as QR decomposition) are required in mathematics to iteratively obtain the numerical solutions of ω_i and ϕ_i . Therefore, the CFS cannot achieve a purely analytical solution and still needs to be combined with numerical analysis methods. This not only consumes time due to iterative calculations, but may also introduce significant errors due to the risk of falling into a local optimum. Section IV will detail how to address this problem.

IV. RELAVANT PARAMETERS IN CFS OF BFR MODEL

The key to parameter definition lies in the eigenvalue solution. In fact, the eigen-analysis method is a fundamental and powerful technique for the eigenvalue solution, with mature applications in power system stability and control such as in small-signal stability analysis. Therefore, the eigen-analysis method can also be introduced into frequency stability analysis, enabling the analytical solution of eigenvalues, and subsequently defining the expressions of relevant parameters in the CFS.

A. State Equation for Small-signal Stability Analysis

Small disturbances continuously impact synchronous generators that operate in parallel through transmission lines, causing relative swings between the rotors of generators and resulting in sustained low-frequency oscillations [40]. To analyze whether the power system can maintain in synchronization under small disturbances, the power system can be de-

scribed by its state equations, and dynamic analysis can be conducted using the eigen-analysis method [41].

The state equation for small-signal stability analysis can be expressed as:

$$\begin{bmatrix} \Delta \dot{\boldsymbol{\delta}}_g \\ \Delta \dot{\mathbf{f}}_g \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \omega_0 \bar{\mathbf{I}} \\ -\bar{\mathbf{M}}^{-1} \bar{\mathbf{B}}_s & -\bar{\mathbf{M}}^{-1} \bar{\mathbf{C}} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\delta}_g \\ \Delta \mathbf{f}_g \end{bmatrix} \quad (35)$$

Equation (35) can be rewritten as:

$$\bar{\mathbf{M}} \Delta \boldsymbol{\delta}_g s^2 + \bar{\mathbf{C}} \Delta \boldsymbol{\delta}_g s + \bar{\mathbf{K}} \Delta \boldsymbol{\delta}_g = \mathbf{0} \quad (36)$$

Compared with (16), (36) is a homogeneous equation. This distinction arises because under small disturbances, the power system remains in or transitions to a near-steady operating state. As a result, the overall FR dynamics of the system are not considered, with the focus placed exclusively on the frequency oscillations between generators. However, under a large disturbance, the system deviates significantly from its original stable state, necessitating a comprehensive analysis of both the overall FR dynamics and frequency oscillations.

B. Parameter Definition by Eigen-analysis Method

In practical engineering applications, the eigen-analysis method is commonly employed for small-signal stability analysis. Its function involves obtaining the eigenroots λ_i of the state equation, and subsequently assessing whether the power system is in a small-signal stability state based on the distribution of the real parts of these eigenroots [4].

The corresponding characteristic equation for (36) is:

$$|\bar{\mathbf{M}} \lambda^2 + \bar{\mathbf{C}} \lambda + \bar{\mathbf{K}}| = 0 \quad (37)$$

Due to the system in neutral equilibrium, (37) is a $2N^{\text{th}}$ -order polynomial of λ with a zero root and $N-1$ pairs of complex conjugate roots:

$$\begin{cases} \lambda_1 = 0 \\ (\lambda_i, \lambda_i^*) = -\zeta_i \omega_i \pm j \sqrt{1 - \zeta_i^2} \omega_i \quad i = 2, 3, \dots, N \end{cases} \quad (38)$$

In vibration dynamics, ω_i and λ_i determine the free vibration modals of undamped and damped systems, respectively. Under the influence of damping, the free vibration of the system changes from the original simple-harmonic oscillation to damped oscillation with the damping ratio ζ [37]. Therefore, the relationship between ω and λ is given by:

$$\omega_i = |\lambda_i| = |\lambda_i^*| \quad i = 2, 3, \dots, N \quad (39)$$

Since the eigen-analysis method can provide abundant valuable information related to the system dynamic stability, it is currently the most commonly used method for small-signal stability analysis of modern power systems. The mainstream power system simulation tools in various countries such as PSASP, PSS/E, and DIGSILENT all embed stable and efficient eigenroot solution tools for small-signal analysis [5]. This paper focuses on the FR under large disturbances. Therefore, we assume that the bulk power system is stable under small disturbances; otherwise, the system would not be practicable. Consequently, we consider that the values of λ_i have already been obtained through these simulation tools in prior small-signal stability analyses, accounting for operational conditions such as load and generation.

With λ_i established, the analytical solution to ω_i can be obtained based on (39). When ω_i is treated as known quantity, (18) transforms into a set of N^{th} -order singular linear system equations, allowing the analytical solution for ϕ_i to be obtained, and subsequently defining the expressions for $(\omega_{n,i}, \zeta_i, \omega_{r,i}, \alpha_i)$.

Due to the explicit definitions of all parameters in the CFS, a purely analytical solution can be achieved without relying on any simulation software or hardware investments.

V. CASE STUDIES AND SIMULATION RESULTS

In this section, the BFR model and its CFS are verified through three cases. Case A verifies the validity of the simplifications made in the CFS derivation based on the Western Systems Coordinating Council (WSCC) 9-bus system. Cases B and C verify the accuracy of the CFS based on the New England 39-bus system without fast FR resources and a real-world provincial power system with fast FR resources in China, respectively. Several statistical indices including absolute error, relative error, mean absolute percentage error (MAPE), root mean square error (RMSE), and coefficient of determination (R^2) are selected to evaluate the solution performance.

We focus on the FR curves over time and five frequency security indicators, namely the maximum frequency deviation $\Delta f_{\max}$, time to the maximum frequency deviation t_{nadir} , initial rate of change of frequency $\Delta f_{\text{initial}}$, quasi-steady-state frequency deviation Δf_{qss} , and frequency oscillation period T_{osc} . These indicators collectively indicate the severity of the disturbance and how “close” the system approaches to potential stability or operational limit [5].

A. Case A: WSCC 9-bus System

The WSCC 9-bus system consists of 9 buses, 6 transmission lines, 3 transformers, 3 loads, and 3 synchronous generators. More details regarding the system description and parameters are given in [42]. Case studies are conducted with a 60 MW sudden increase of load at Bus 9.

Three simplifications are made in the derivation of CFS: ① ignoring Δf_{osc} and approximating Δf_g by Δf_{coi} in Section II-C; ② ignoring non-diagonal elements of $\bar{C}_p$ in Section III-B; and ③ ignoring $\Delta P'_f$ to solve $\dot{x}_{p,i}(t)$ in Section III-B.

Regarding the verification of approximating Δf_g by Δf_{coi} , T_r is typically valued in the range of 6–14 s. By varying $T_{r,3}$ of G3 and keeping the remaining parameters constant, the comparison of $\Delta P_{f,3}$ and $\Delta P'_{f,3}$ curves within 0–5 s under a large disturbance is obtained. As shown in Fig. 5, the two curves almost overlap with very minimal error when $T_{r,3}$ is within 6–14 s. Further widening the value range, the maximum absolute error of the two curves is only 0.01 p.u. when the value of $T_{r,3}$ is less than 2.5 s. Therefore, the error of $\Delta P'_{f,3}$ relative to $\Delta P_{f,3}$ is negligible, and the validity of the first simplification can be verified.

Regarding the verification of ignoring the non-diagonal elements of $\bar{C}_p$ and $\Delta P'_f$, we focus on $\Delta f_{\max}$, the most concerned metric during frequency transients. By varying H_3 , $F_{h,3}$, $T_{r,3}$, and R_3 in their typical value ranges and keeping the remaining parameters constant, the absolute error of

$\Delta f_{\max}$ solved before and after the two simplifications can be obtained. As shown in Fig. 6, the above two simplifications result in errors of no more than 0.02 Hz. Therefore, the validity of the second and third simplifications can be verified.

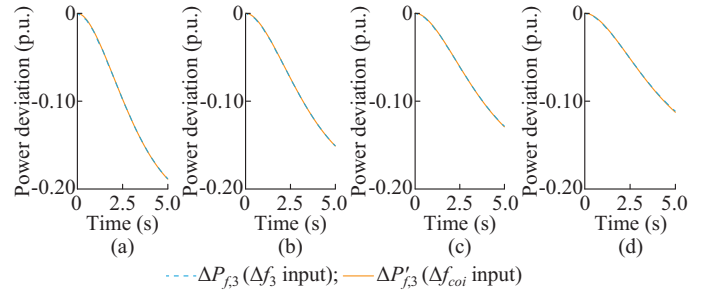


Fig. 5. Comparison of $\Delta P_{f,3}$ and $\Delta P'_{f,3}$ curves within 0–5 s under a large disturbance for $T_{r,3}$ values of 6 s, 8 s, 10 s, and 12 s. (a) $T_{r,3}=6$ s. (b) $T_{r,3}=8$ s. (c) $T_{r,3}=10$ s. (d) $T_{r,3}=12$ s.

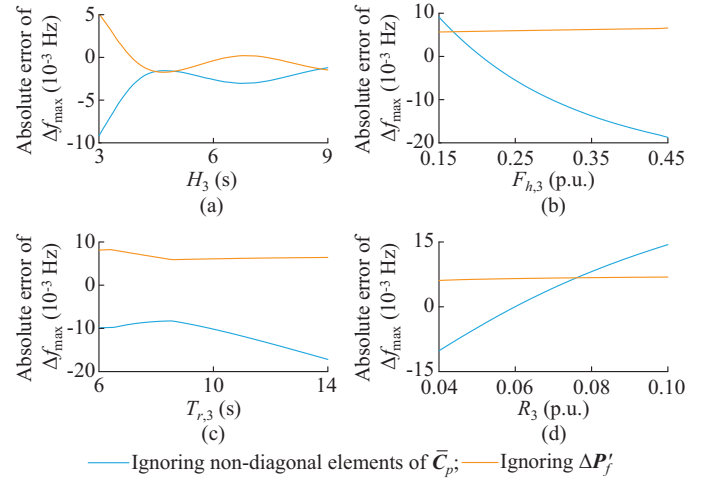


Fig. 6. Absolute error of $\Delta f_{\max}$ solved before and after simplification. (a) Varying H_3 . (b) Varying $F_{h,3}$. (c) Varying $T_{r,3}$. (d) Varying R_3 .

From an overall perspective, the validity of the three simplifications is also verified by comparing the FR curves solved by BFR model and its CFS. As shown in Fig. 7 and Table I, the maximum absolute error of two curves is 0.0098 Hz at 0.71s, and the relative errors of five frequency security indicators are within 2%, demonstrating very high accuracy. Therefore, the simplifications made can be considered reasonable, and the solution performances of BFR model and its CFS are almost identical.

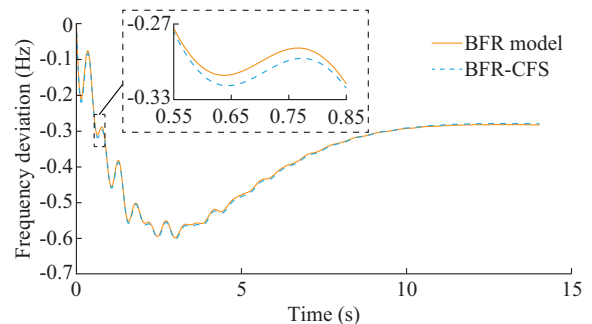


Fig. 7. Comparison of FR curves between BFR model and CFS.

TABLE I
COMPARISON OF INDICATORS BETWEEN BFR MODEL AND CFS

Model	$\Delta f_{initial}$ (Hz/s)	Δf_{max} (Hz)	t_{nadir} (s)	Δf_{qss} (Hz)	T_{osc} (s)
BFR	-2.332	-0.599	2.99	-0.284	0.42
CFS	-2.364	-0.603	2.99	-0.281	0.42
Relative error (%)	1.35	0.66	0	1.07	0

B. Case B: New England 39-bus System Without Fast FR Resources

The New England 39-bus system consists of 39 buses, 34 transmission lines, 12 transformers, 19 loads, and 10 synchronous generators. More details regarding the system description and parameters are given in [42]. Case studies are conducted in a scenario with 1000 MW generator tripping at Bus 9. The benchmark is the precise FR numerical solutions provided by Power System Analysis Software Package (PSASP), which uses the 6th-order GENROU generator model, the IEEE T1 AVR model, and the IEEE G1 governor model. The solution accuracy of the BFR model is verified by comparing it with that of the SFR model.

The solution results including the FR curves, Δf_{max} , and t_{nadir} for all 39 buses are provided in [43], solved by PSASP, BFR, and SFR models. As shown in Fig. 8, despite the fact that the New England 39-bus system is highly meshed and equipped with well-distributed synchronous generators, leading to the less significant FSDC, the relative error of the BFR model remains much lower than that of the SFR model, especially for t_{nadir} .

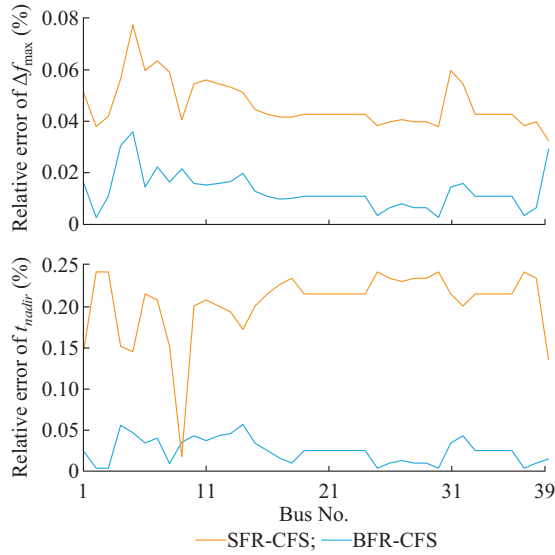


Fig. 8. Comparison of relative errors of Δf_{max} and t_{nadir} between BFR and SFR models in Case B.

Since the BFR model shows the largest relative error in the results of Bus 5, we conduct the studies in more detail. The FR curves of Bus 5 solved by various methods are obtained, as shown in Fig. 9. The FR curve solved by PSASP indicates that there is severe frequency oscillation during frequency transients, especially during the initial period. The FR curve solved by the SFR model indicates that it can only describe the FR of COI, which will cause large errors when

the system shows significant FSDCs. The FR curve solved by the BFR model indicates that it can precisely capture both the significant FSDC and the FR of COI. Since the BFR model ignores some smaller time constants in the governor model, the FR curve solved by PSASP lags behind that of the BFR model, resulting in a minor phase shift. The values of frequency security indicators are given in Table II. Compared with the PSASP results, the relative errors of these indicators are given in Table III. It can be observed that the BFR model shows a much higher solution accuracy than the SFR model.

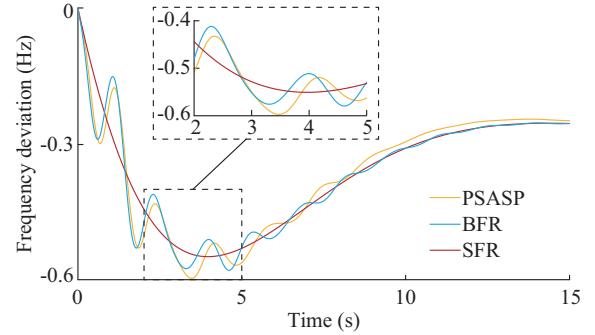


Fig. 9. Comparison of FR curves among PSASP, BFR, and SFR models in Case B.

TABLE II
VALUES OF FREQUENCY SECURITY INDICATORS

Model	$\Delta f_{initial}$ (Hz/s)	Δf_{max} (Hz)	t_{nadir} (s)	Δf_{qss} (Hz)	T_{osc} (s)
BFR	-0.475	-0.576	3.31	-0.255	1.53
SFR	-0.297	-0.551	3.97	-0.255	
PSASP	-0.446	-0.597	3.47	-0.249	1.61

TABLE III
RELATIVE ERRORS OF INDICATORS COMPARED WITH PSASP

Model	Relative error (%)				
	$\Delta f_{initial}$	Δf_{max}	t_{nadir}	Δf_{qss}	T_{osc}
BFR	6.50	3.52	4.61	2.41	4.97
SFR	33.41	7.71	14.41	2.41	Inf

C. Case C: A Provincial Power System with Fast FR Resources in China

A real-world provincial power system in China consists of 48 buses, 123 transmission lines, 313 loads, 10 conventional power plants with a total of 44 synchronous generators, and 5 wind power plants with a total of 73 doubly-fed induction generators. The proportion of renewable energy generation is 24%. More details regarding the system description are given in [43]. Case studies are conducted with a 3.90% power deficit scenario to verify the practicality of the BFR in a bulk power system. Numerical solutions provided by the PSASP model are used as the benchmark, which are topologically accurate and approximate the dynamics of this provincial power system. The simulation time length is 15 s, which is the same as that in Case B.

The FR solution results for all 48 buses solved by PSASP, BFR, and SFR models are given in [43]. The initial 0-5 s

following the disturbance is the period when the frequency drops from the rated value to its nadir, which shows the most significant FSDC. Using the PSASP results as a benchmark, the MAPE, RMSE, and R^2 in 0-5 s of BFR and SFR models are compared in Fig. 10, where MAPE and RMSE are used to quantify the solution errors, and R^2 is used to quantify the overall approximation of several FR curves.

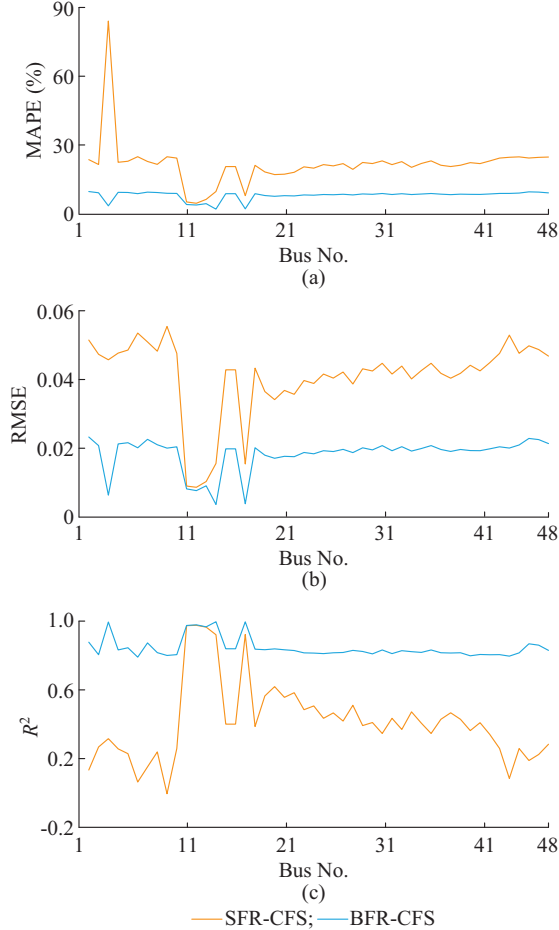


Fig. 10. Comparison of MAPE, RMSE, and R^2 between BFR and SFR models in 0-5 s. (a) MAPE. (b) RMSE. (c) R^2 .

Since the BFR model shows the largest MAPE in the results of Bus 9, we conduct a more detailed analysis of this case. The FR curves of Bus 9 solved by different models are shown in Fig. 11. It can be observed from Fig. 11 and Table IV that the errors of the BFR model increase with the system-scale growth, but still keep within an acceptable range. In contrast, the SFR model fails to accurately capture the FR dynamics. Additionally, the R^2 of the BFR model is significantly closer to 1, indicating a stronger alignment with the “shape” of the PSASP results. Based on these findings, the practicality of the BFR and its simplifications during the solution process have been validated in the provincial power system.

The solution time of PSASP, BFR, and SFR models is 2.375 s, 8.434 ms, and 2.049 ms, respectively. While advancements in computing power have reduced the solution time of simulation methods to the order of seconds, they still fall short of meeting the requirements for online calculations

and large-scale computations. The SFR model, although being sufficiently fast, exhibits significant errors. In contrast, the proposed reduced-order BFR model achieves a solution speed in the millisecond range while maintaining acceptable error levels.

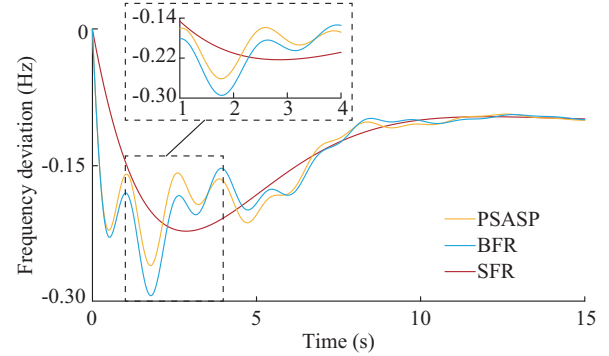


Fig. 11. Comparison of FR curves among PSASP, BFR, and SFR models in Case C.

TABLE IV
COMPARISON OF STATISTICAL INDICES IN CASE C

Model	MAPE (%)	RMSE (Hz)	R^2
BFR	8.891	0.020	0.800
SFR	24.838	0.055	-0.005

VI. DISCUSSIONS

A. Data Acquisition for BFR

For any active power disturbance scenario, the data required for the proposed reduced-order BFR model and its CFS include the admittance of each bus, the dynamic parameters of each generator, and the disturbance location along with its power deficit. It is important to note that identifying the exact cause of the disturbance is not necessary.

During massive computations such as reserve planning and unit commitment, the admittance of each bus can be derived from the system network topology parameters, the dynamic parameters of each generator can be obtained from their rated values, and the disturbance location and its power deficit can be manually configured. In these scenarios, data acquisition does not rely on any measurement devices, making it immune to issues such as noise, data loss, or delays.

During online computations such as emergency control, the admittance of each bus and the dynamic parameters of each generator can still be obtained using the same method, with dynamic updates based on changes in operational conditions. However, the disturbance location and its power deficit must be measured in real time using measurement devices such as supervisory control and data acquisition (SCADA) and phasor measurement unit (PMU). Therefore, data acquisition may be affected by data contamination such as noise, data loss, or delays.

In the real-world provincial power system, when faults such as communication interruptions occur, the communication information system employs emergency control strategies such as network self-healing protection, rapid switching to backup channels, and ring network switching to achieve

fast recovery of data communication. For faults that cannot be resolved immediately, corrective control measures such as automatic rerouting and emergency communication are implemented to ensure temporary data communication continuity.

Given the robustness of the communication information system described above, and considering that both large disturbances and communication interruptions are rare events, the performances of the proposed reduced-order BFR model and its CFS are generally unaffected by data communication issues in practical engineering applications.

B. Prospects for Further Research

Due to the inherent limitations of analytical methods, this paper only focuses on the primary influencing factors of the FR and ignores secondary factors such as reactive power-voltage characteristics, nonlinearities of deadband and limiter, and internal dynamics of fast FR resources. As the power grid evolves and technology advances, secondary influencing factors may become primary, and more advanced dispatch and control methods for fast FR resources may emerge in future bulk power systems. Future work will focus on enhancing the proposed reduced-order BFR model to address these limitations and adapt to the evolving power system landscape.

1) To address the limitation of ignoring reactive power-voltage characteristics, we plan to separately solve the FR caused by active power and reactive power variations. By applying the superposition theorem, we aim to derive the full bus frequency response, ensuring that the model more comprehensively captures the coupling effects of frequency and voltage.

2) To address the limitation of ignoring certain nonlinear components such as deadband and limiter, we plan to model these components using piecewise linearization. By incorporating these nonlinear effects into the analytical solution of FR, we aim to provide a more precise representation of system dynamics, particularly in scenarios where these nonlinearities are significant.

3) To address the limitation of ignoring the internal dynamics of fast FR resources, we plan to explore a data-driven method to characterize the relationship between the frequency deviation and the power variation of various types of fast FR resources. Additionally, we will investigate the integration of data-driven methods with physical models to develop a hybrid modeling framework suitable for analytical solution. This combination of data-driven and physics-based methods aims to enhance the accuracy and applicability of the model while maintaining its tractability for analytical solution.

VII. CONCLUSION

To accurately preserve the significant FSDCs in bulk power systems through analytical methods, this paper proposes a reduced-order BFR model and derives its CFS. Theoretical analysis and simulation results demonstrate several advantages of the proposed reduced-order BFR model summarized as follows.

1) Compared with simulation models, it can efficiently solve the FR of all buses in the system, reducing the computational time to milliseconds while maintaining acceptable error margins.

2) Compared with uniform-frequency models, it can capture both the FR dynamics of the system COI and the inter-generator oscillation FR induced by the FSDC, and any frequency oscillation can be characterized as a damped sinusoid, providing higher solution accuracy.

3) Compared with other distinct-frequency models, the explicit expressions for all its parameters are clearly defined, thereby enabling a purely analytical solution, reducing reliance on sample data, and showing stronger robustness.

As a result, the proposed reduced-order BFR model is not only suitable for research requiring high computational speed such as frequency stability emergency control, but also able to provide valuable frequency security constraints for optimization problems related to reserve planning and unit commitment.

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