Machine Learning Based Uncertainty-alleviating Operation Model for Distribution Systems with Energy Storage

Xi Lu, *Member*, *IEEE*, Xinzhe Fan, Haifeng Qiu, *Member*, *IEEE*, Wei Gan, *Member*, *IEEE*, Wei Gu, *Senior Member*, *IEEE*, Shiwei Xia, *Senior Member*, *IEEE*, and Xiao Luo

***Abstract*—In this paper, an operation model for distribution systems with energy storage (ES) is proposed and solved with the aid of machine learning. The model considers ES applications with uncertainty realizations. It also considers ES applications for economy and security purposes. Considering the special features of ES operations under day-ahead decision mechanisms of distribution systems, an ES operation scheme is designed for transferring uncertainties to later hours through ES to ensure the secure operation of distribution system. As a result, uncertainties from different time intervals are assembled and may counteract each other, thereby alleviating the uncertainties. As different ES applications rely on ES flexibility (in terms of charging and discharging) and interact with each other, by coordinating different ES applications, the proposed operation model achieves efficient exploit of ES flexibility. To shorten the computation time, a long short-term memory recurrent neural network is used to determine the binary variables corresponding to ES status. The proposed operation model then becomes a convex optimization problem and is solved precisely. Thus, the solving efficiency is greatly improved while ensuring the satisfactory use of ES flexibility in distribution system operation.**

***Index Terms*—Distribution system, energy storage, machine learning, uncertainty.**

Manuscript received: August 30, 2023; revised: November 9, 2023; accepted: December 30, 2023. Date of CrossCheck: December 30, 2023. Date of online publication: April 24, 2024.

This work was supported in part by the National Natural Science Foundation of China (No. 52107078), in part by the Fundamental Research Funds for the Central Universities (No. 2242022R40051), and in part by High Level Personnel Project of Jiangsu Province (No. JSSCBS20220125).

X. Lu, X. Fan, W. Gu, and X. Luo are with School of Electrical Engineering, Southeast University, Nanjing 210096, China (e-mail: xlu@seu. edu. cn; 220222706@seu.edu.cn; wgu@seu.edu.cn; acepilotxl@foxmail.com).

H. Qiu is with School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore (e-mail: haifeng.qiu@ntu.edu.sg).

W. Gan is with School of Engineering, Cardiff University, Cardiff CF24 3AA, U.K. (e-mail: ganw4@cardiff.ac.uk).

S. Xia (corresponding author) is with School of Electrical and Electronic Engineering, North China Electric Power University, Beijing 102206, China (e-mail: s.w.xia@ncepu.edu.cn).

DOI: 10.35833/MPCE.2023.000613

NOMENCLATURE

A. Sets

N	Set of nodes in distribution system
$N_b(i)$, $N_a(i)$	Sets of parent and child nodes of node i
N_{ES}	Set of nodes with energy storage (ES)
N_{RES}	Set of nodes with renewable energy sources (RESs)
$S(\mu)$	Ambiguity set

B. Parameters

λ_i^{ch}, λ_i^{dis}	Charging and discharging efficiencies of ES i
$\mu_{t,i}$	Forecasting error of uncertainty in renewable energy output at node i in hour t
μ	All involved uncertainties
ω	Expectation of μ
Θ	Covariance of μ
Δt	Time interval (an hour)
d_μ	Distribution of μ
E_i^0	State of charge of ES i at beginning of day
$E_i^{\max}$, $E_i^{\min}$	Allowed maximum and minimum states of charge of ES i
$l_{t,i}^p$, $l_{t,i}^q$	Active and reactive load demands of node i in hour t
m_t	Electricity price in hour t
m_i^{c}, m_i^{d}	Charging and discharging cost coefficients of ES i
m_t^+, m_t^-	Cost coefficients for positive and negative deviations of electricity purchase
$p_i^{\text{c,max}}$, $p_i^{\text{d,max}}$	Limits on charging and discharging power of ES i
$p_{t,i}^{\text{f}}$	Forecasted RES output at node i in hour t
$r_{i,j}$, $x_{i,j}$	Feeder resistance and reactance between nodes i and j

T Number of hours considered in model

v_0 Base voltage

$v_{t,i}$ Voltage of node i in hour t

C. Decision Variables

$\theta_{t,i}$ Status indicator of ES i in hour t

e_t^{p} Purchased electricity for hour t

e_t^{a} Actual imported electricity in hour t

$E_{t,i}$ State of charge of ES i in hour t

$p_{t,i,j}^{\mathrm{line}}$, $q_{t,i,j}^{\mathrm{line}}$ Active and reactive power in feeder from node i to j in hour t

$p_{t,i}^{\mathrm{c}}$, $p_{t,i}^{\mathrm{d}}$ ES charging and discharging power at node i in hour t

I. Introduction

In recent years, renewable energy sources (RESs) have played a vital role in distribution systems because of their advantages in terms of environmental protection. However, as RESs are affected by weather [1], [2], the generated power has nonnegligible uncertainties [3], which complicate the distribution system operation. Major thermal power technologies sometimes struggle to cope with the unexpected changes in RES outputs that are widely scattered in different distribution systems [4]. Therefore, the distribution system requires flexibility to ensure its stable operation. As a major option of flexibility, energy storage (ES) can improve the reliability and flexibility of distribution systems [5]. Some studies have focused on distribution system operation based on ES flexibility. In [6], the loads are shifted using ES flexibility to reduce the operation costs. Reference [7] proposes a coordinated voltage control based on ES flexibility. Various types of ES have also been used simultaneously [8]. However, these studies have established deterministic models without considering the effects of the uncertainties incurred by RESs, which may result in unsatisfactory outcomes.

Because of its flexibility, ES can reduce the effects of RES uncertainties [9]. References [10] and [11] study reserve scheduling and unit commitment problems in transmission systems involving ES and RES uncertainties, respectively. However, unlike the transmission system operations considered in [10] and [11], day-ahead energy consumption plans for distribution system operation must be made typically. This strengthens the intertemporal correlations and complicates the ES operation in distribution systems. References [12] and [13] use ES to ensure the operation feasibility against uncertainties but assume that the distribution system is not responsible for energy balancing and that the utility grid will compensate for any power mismatch caused by uncertainties. Consequently, in [12] and [13], ES flexibility is not fully exploited in terms of uncertainty alleviation. Reference [14] considers microgrid operations with ES when the day-ahead market is involved, but assumes that the actual values of uncertainties in all hours are known simultaneously, which is impractical and may lead to overly optimistic solutions. ES is used for peak-shaving and voltage regulation in [15] and uncertainty alleviation in [16]. However, [15] fixes the active power of ES in real-time operation, and [16] determines the ES operation by allocating its flexibility evenly to each hour. These inflexible schemes impede the efficient use of ES flexibility. In short, although many researchers have studied ES operations influenced by uncertainties, research gaps in terms of systematic theories remain regarding the use of ES flexibility considering the features of distribution system operations. Operations involving ES are complicated because of the intertemporal constraints derived from the ES state of charge (SOC), and the complexity is strengthened by the interaction between uncertainties from different time intervals caused by the day-ahead decision mechanism of distribution systems. Current studies generally do not give sufficient attention to the interaction between different uncertainties and regard uncertainty alleviation in different hours as a separate task, which means that ES flexibility cannot be arranged from a holistic perspective.

Based on the aforementioned research gaps, this paper proposes an operation model to comprehensively utilize the ES flexibility in distribution system operation. The realized uncertainties differ from other known deterministic information since the day-ahead stage, and still act like uncertainties even though their actual values are known. This is because they still cause deviations from the day-ahead energy consumption plans of the distribution system. As a result, uncertainties travel in the time domain through the ES, which will be further discussed in Section II. Based on this background, the ES operation is accurately modeled by distinguishing the operations that are influenced by uncertainty realizations from those that are not influenced, while tracking each uncertainty during the entire time horizon. Special attention is given to the interaction between uncertainties from different time intervals caused by the transfer of uncertainties. The fact that uncertainties are able to counteract each other when they have positive and negative realizations is focused and utilized. Accordingly, a tailored ES operation scheme is proposed to assemble as many uncertainties in RES outputs as possible using ES flexibility, as this provides a greater possibility of counteracting different uncertainties. In addition to the influence of uncertainties, ES applications must also be considered for two distinct purposes. One type of ES application ensures the secure operation of distribution system, and is achieved using robust optimization techniques. The other aims to improve the economic performance of the distribution system, and is achieved using distributionally robust optimization. The ES flexibility works through the ES operation and is restricted by the charging and discharging rates and ES capacity. As a result, the aforementioned ES applications are mutually affected. To fully exploit ES flexibility, it is necessary to coordinate between different ES applications. This is achieved by the proposed operation model under uncertainties.

To avoid simultaneous charging and discharging of an ES, binary variables are required to restrict ES status. In some research works including [17] and [18], the binary variables can be relaxed. However, the proposed operation model in this paper alleviates uncertainties by using ES and considers the costs incurred by uncertainties. Consequently, simultaneous charging and discharging is beneficial for ES in terms of alleviating uncertainty, which means that binary variables for restricting ES status are indispensable. However, it is time-consuming to solve the proposed operation model when using binary variables. In addition, it is crucial to properly consider the probabilistic information, particularly the correlations of uncertainties, because the proposed operation model attempts to utilize the counteraction between uncertainties, which is enabled by using distributionally robust optimization, and introduces second-order conic constraints into the proposed operation model. As a result, the complexity of solving the proposed operation model derived from binary variables is aggravated. To improve the efficiency of determining binary variables in optimization problems, traditional methods usually perform computations without accumulating experience [19]. With the rapid improvement in computational performance and big data techniques, neural networks have rapidly advanced in recent years, and have been used in distribution system operations. However, the performance of neural networks degrades when the corresponding problem occurs on a large scale. Based on the strong intertemporal features of the proposed operation model and the interaction between uncertainties from different time intervals, a long short-term memory (LSTM) neural network (as opposed to an ordinary neural network) is used to determine the binary variables corresponding to the ES status. Compared with ordinary neural networks, LSTM neural networks can process sequential data more effectively, and capture the mutual influence of decisions in different time intervals. However, this paper attempts to take advantage of the computation speed of the LSTM neural network while simultaneously reducing the effects of its errors. Accordingly, instead of directly using the LSTM neural network to make all decisions about distribution system operations, this paper uses the LSTM to determine the binary variables that control the ES charging or discharging status because the binary variables are the chief contributors to the time-consuming complexity of the proposed operation model.

The major contributions of this paper are as follows.

1) As the uncertainties travel in time domain through the ES in distribution system operations under day-ahead decision mechanisms, a tailored ES operation scheme that assembles uncertainties in RES outputs from different time intervals is proposed, which utilizes the possibility of uncertainties counteracting each other to alleviate them with secure operation of distribution system as a prerequisite. In addition, through elaborate modeling of ES operations, different ES applications are coordinated for both security and economic purposes, and ES flexibility is efficiently utilized.

2) To fulfill the tailored ES operation scheme that focuses on the interaction between uncertainties, binary variables that determine the ES status are treated as indispensable for complicated ES operations, and probabilistic information of uncertainties is considered using distributionally robust optimization, which amplifies the complexity derived from the binary variables. In view of the unique features of the proposed operation model, it is solved with the aid of neural networks.

3) To properly capture the strong mutual influence between distribution system operations at different hours under the proposed ES operation scheme, LSTM neural networks are trained. To utilize the advantages of LSTM neural networks and reduce the negative effects of their errors, LSTM neural networks are used only to determine the binary variables that determine the ES status, and the proposed operation model becomes convex and is solved precisely to the global optimal solution. Case studies show that the proper use of LSTM neural networks significantly reduces the computation time and only slightly affects the efficient utilization of ES flexibility.

II. Proposed Operation Model

We first discuss the distribution system operation with ES. The proposed operation model for distribution system is then presented with a tailored ES operation scheme, which will be described in detail in Section III.

A. Distribution System Operation with ES

This paper investigates the distribution system operation in which the ES offers flexibility and the RES offers uncertainty in its outputs (i.e., forecasting errors). We assume that the ES can be directly operated by the distribution system operator. The distribution system operation is conducted in two stages: day-ahead and real-time. In the day-ahead stage, the distribution system purchases electricity that is imported from the transmission system [20]. Due to the uncertainties in RES output, the actual imported power in the real-time stage generally deviates from the electricity purchase. As in [21], we assume that the distribution system must be responsible for its uncertainties and pay corresponding penalties to the transmission system operator for its energy deviations, as uncertainties from distribution systems can destabilize transmission system operation. Note that the proposed operation model in this paper is also applicable to other settings as long as uncertainties are unfavorable and the distribution system is programmed to alleviate its uncertainties. For example, the proposed operation model operates when the energy price is higher for purchasing than for selling in the real-time stage. The aim of the problem is to use ES flexibility for ensuring distribution system security and minimizing the total operation costs including energy purchasing cost, ES operation cost as well as penalties for energy deviations. The proposed operation model can be extended by considering a case in which the distribution system sells its excess energy. Here, the designed ES operation scheme will remain applicable.

As an increasing number of uncertainties are involved in power system operations, it is necessary to design tailored operation schemes according to the specific operation environment. During the operation of a general transmission system with an ES, the power of the ES and other system components is determined by the economic dispatch conducted during the day. Under these circumstances, earlier uncertainties are incorporated into the deterministic parameters when the operation decisions are made again at the next time slot.

This means that earlier uncertainties are not different from other deterministic information after they are observed. However, when day-ahead energy purchases or plans are made for the distribution system operation, the realized uncertainties differ from other known deterministic information since the day-ahead stage, because they still cause deviations from the day-ahead energy plan. In other words, due to the day-ahead decision mechanism of the distribution system operation, the realized uncertainties still act as uncertainties. As a result, the ES not only simply transfers energy in the time domain, but also transfers uncertainties to later hours, which leads to the direct interplay between uncertainties from different time intervals. Note that this is usually ignored in the literature. Therefore, to achieve the efficient exploitation of ES flexibility, it is necessary to clearly track each uncertainty during operation over the entire time horizon, and the interaction between uncertainties must be properly considered. Based on this background, a tailored ES operation scheme that exploits the counteraction between uncertainties is proposed with a operation model for distribution system that fulfills the ES operation scheme.

B. Formulation of Operation Model for Distribution System

The formulation of the proposed operation model is presented through the following equations.

$$\min \sum_{t=1,2,\ldots,T} m_t e_t^{\mathrm{p}} + \max_{d_\mu \in S(\mu)} \mathrm{E}\left[\sum_{i \in N_{\mathrm{ES}}} \sum_{t=1,2,\ldots,T} m_i^{\mathrm{c}} p_{t,i}^{\mathrm{c}} \lambda_i^{\mathrm{ch}} + \sum_{i \in N_{\mathrm{ES}}} \sum_{t=1,2,\ldots,T} m_i^{\mathrm{d}} p_{t,i}^{\mathrm{d}} / \lambda_i^{\mathrm{dis}} + \sum_{t=1,2,\ldots,T} \max\left\{c_1, c_2\right\}\right] \tag{1}$$

$$c_1 = m_t^{+}\left(e_t^{\mathrm{a}} - e_t^{\mathrm{p}}\right) \tag{2}$$

$$c_2 = m_t^{-}\left(e_t^{\mathrm{p}} - e_t^{\mathrm{a}}\right) \tag{3}$$

$$S(\mu) = \left\{d_\mu \middle| \mathrm{E}[\mu] = \omega, \mathrm{E}\left[(\mu - \omega)(\mu - \omega)'\right] = \Theta\right\} \tag{4}$$

Equation (1) expresses the objective of the problem. The operation costs presented in (1) can be divided into two parts. The first term $\sum_{t=1,2,\ldots,T} m_t e_t^{\mathrm{p}}$ represents the electricity purchasing cost in the day-ahead stage and is not directly influenced by uncertainties. The second part represents the costs within the operator $\max_{d_\mu \in S(\mu)} \mathrm{E}[\cdot]$, which is influenced by uncertainties realized in the real-time stage. More specifically, the first and second items within the operator $\max_{d_\mu \in S(\mu)} \mathrm{E}[\cdot]$ are the ES charging and discharging costs, respectively. The last item in the operator $\max_{d_\mu \in S(\mu)} \mathrm{E}[\cdot]$ is the penalty for power deviations, and is in the form of (2) when the deviation is positive and (3) when the deviation is negative, respectively. As discussed in Section II-A, the proposed operation model is also applicable to other settings in terms of costs derived from uncertainties. For example, the coefficient m_t^{-} in (3) is zero if the residual energy is allowed to be wasted by the distribution system. It is also possible for m_t^{-} to be a negative number whose absolute value is less than m_t^{+} if the distribution system can sell the residual energy at a lower price in the real-time stage. In either case, the cost or income incurred from uncertainties in the real-time stage can be uniformly expressed by the last item in the operator $\max_{d_\mu \in S(\mu)} \mathrm{E}[\cdot]$ in (1), which is a convex piecewise-linear function.

The operation costs of the distribution system are influenced by RES uncertainties and are not deterministic. In some studies including [13], the worst-case cost is optimized; however, this is too conservative. Other studies such as [12] optimize the costs in a forecasting scenario. However, as uncertainties become nonnegligible, optimizing the average operation costs is obviously more reasonable. In this regard, distributionally robust optimization (DRO) is advantageous and is thus adopted. With the ambiguity set in (4) established based on historical observations of uncertainties, DRO considers the probability distributions matching the statistical expectation ω and covariance Θ of the RES uncertainty μ. This means that decisions related to distribution system operation can be made by knowing the degree to which each uncertainty fluctuates and the correlation between different uncertainties. As the accurate uncertainty distribution cannot be known under limited information, the largest expectation of operation costs over the ambiguity set is computed through the operator $\max_{d_\mu \in S(\mu)} \mathrm{E}[\cdot]$ in (1), and is regarded as the objective of the proposed operation model. It can be observed that DRO not only properly uses the probabilistic information of uncertainties, but also guarantees robustness against the inevitable ambiguity in the uncertainty distribution. Through DRO, the objective in (1) is equivalently transformed into deterministic second-order conic forms, which means that the uncertainties are eliminated through mathematical transformation. Unlike the stochastic optimization, DRO avoids the cumbersome process of choosing representative scenarios, and its performance can be enhanced by accumulating information about uncertainties without increasing computational complexity. Further details on the adopted DRO and its transformation are available in [22].

The constraints of the proposed operation model are given by (5)-(15). Equation (5) provides the electricity imported from the distribution system. The voltage $v_{t,i}$ at each node is computed in (6) and is limited to 0.95-1.05 times of the base voltage v_0 in (7). Equations (8) and (9) calculate the active and reactive power flows. The binary variables $\theta_{t,i}$ in (10) restricts ES status to either charging or discharging. In (11) and (12), the ES charging power and discharging power are limited by their maximum values $p_i^{\mathrm{c,max}}$ and $p_i^{\mathrm{d,max}}$, respectively. An ES SOC must be at the same level E_i^0 at the beginning and end of the day according to (13), and is constrained from violating its upper and lower bounds (i.e., $E_i^{\max}$ and $E_i^{\min}$) in (14). Finally, (15) calculates the SOC of ES $E_{t,i}$ based on its operation. Similar to the objective of the proposed operation model, these constraints include uncertainties. To ensure the safe operation of distribution system, these constraints must be satisfied with respect to all possible uncertainty realizations and thus are actually robust constraints that are transformed using robust optimization [23].

$$e_t^{\mathrm{a}} = p_{t,1,2}^{\mathrm{line}} \Delta t \quad \forall t \tag{5}$$

$$v_{t,N_b(i)}=\frac{1}{v_0}\left(r_{N_b(i),i}p^{\text{line}}_{t,N_b(i),i}+x_{N_b(i),i}q^{\text{line}}_{t,N_b(i),i}\right)+v_{t,i}\quad \forall t,\forall i\in N/\{1\} \tag{6}$$

$$0.95v_0\le v_{t,i}\le 1.05v_0\quad \forall t,\forall i\in N \tag{7}$$

$$p^{\text{line}}_{t,N_b(i),i}+p^{\text{d}}_{t,i}+p^{\text{f}}_{t,i}+\mu_{t,i}=\sum_{j\in N_a(i)}p^{\text{line}}_{t,i,j}+p^{\text{c}}_{t,i}+l^{p}_{t,i}\quad \forall t,\forall i\in N/\{1\} \tag{8}$$

$$q^{\text{line}}_{t,N_b(i),i}=\sum_{j\in N_a(i)}q^{\text{line}}_{t,i,j}+l^{q}_{t,i}\quad \forall t,\forall i\in N/\{1\} \tag{9}$$

$$\theta_{t,i}\in\{0,1\}\quad \forall t,\forall i\in N_{\text{ES}} \tag{10}$$

$$0\le p^{\text{c}}_{t,i}\le\theta_{t,i}p^{\text{c,max}}_{i}\quad \forall t,\forall i\in N_{\text{ES}} \tag{11}$$

$$0\le p^{\text{d}}_{t,i}\le\left(1-\theta_{t,i}\right)p^{\text{d,max}}_{i}\quad \forall t,\forall i\in N_{\text{ES}} \tag{12}$$

$$E_{T,i}=E^{0}_{i}\quad \forall i\in N_{\text{ES}} \tag{13}$$

$$E^{\min}_{i}\le E_{t,i}\le E^{\max}_{i}\quad \forall t,\forall i\in N_{\text{ES}} \tag{14}$$

$$E_{t,i}=E_{t-1,i}-p^{\text{d}}_{t,i}/\lambda^{\text{dis}}_{i}\Delta t+p^{\text{c}}_{t,i}\lambda^{\text{ch}}_{i}\Delta t\quad \forall t,\forall i\in N_{\text{ES}} \tag{15}$$

In the proposed operation model, variables exist in both the day-ahead stage (such as electricity purchased for different hours) and real-time stage (such as ES charging and discharging power). After ES flexibility is exploited, the ES SOC is changed and ES flexibility must be made available again. Therefore, when an ES is used to alleviate uncertainties, its operation is not simply influenced by uncertainties from the current hour. More specifically, an uncertainty such as the forecasting error of RES output in one hour can influence the ES operation in several later hours, and the ES operation in one hour can be influenced by uncertainties from several earlier hours. We should also note that the ES operates without knowing later uncertainty realizations, which implies that ES operation must comply with the time sequence of uncertainties, and this complicates ES operation in the distribution system.

In terms of modeling multi-period optimization problems, two-stage robust and scenario-based stochastic models are often used. However, both ignore the time sequences of uncertainties and decisions, and achieve overly optimistic solutions [11], [24]. The models using scenario trees meet the requirements incurred from time sequences, but are computationally expensive [25]. Therefore, an effective and computationally efficient method, specifically linear decision rule (LDR), is adopted in this paper to track uncertainties, embody their time sequences, and set real-time variables to affine functions of earlier uncertainties [26]. Although the performance of the basic LDR degrades when the studied problem has strong nonlinear characteristics, improved versions of LDR exist that can achieve more accurate modeling. For example, the segregated LDR splits uncertainties into segments and assumes linear relationships based on segregated uncertainties [27], whereas lifted LDR projects uncertainties into a lifted space and assumes affine functions of the lifted uncertainties [28]. An example based on the discharging power of the ES at node i in hour t under LDR is given in (16). The deterministic components $cons^{\text{dis}}_{t,i}$ and linear coefficients $cons^{\text{dis}}_{t,i}$ for all real-time variables under LDR are not fixed parameters but are in fact decision variables, which are determined by solving the proposed operation model. Once the deterministic components and linear coefficients of LDR are determined and the uncertainty realizations are known, real-time operation can be conducted. Instead of considering LDR as a purely mathematical method, this paper uses it to establish an ES operation scheme by analyzing each component with respect to the specific problem of distribution system operation. This is further discussed in Section II-C.

$$p^{\text{d}}_{t,i}=cons^{\text{dis}}_{t,i}+\sum_{\bar{t}=1,2,\ldots,t}\sum_{j\in N_{\text{RES}}}\beta^{\bar{t},j}_{t,i}\mu^{\text{RES}}_{\bar{t},j} \tag{16}$$

C. ES Operation Under Uncertainties

The ES operation is embodied in charging and discharging. Similar to the ES discharging power expressed in (16), the ES charging power is also in the form of an affine function under the LDR. Instead of mixing deterministic operations with uncertainty-responsive operations, as in many other studies, they are separately and clearly modeled in this paper. More specifically, the basic operation plan of the distribution system in the day-ahead stage is reflected by the deterministic components of the LDR, and how the distribution system should respond to uncertainty realizations in the use of ES in the real-time stage is determined by linear coefficients. As discussed in Section I, this paper adopts two perspectives to classify ES operations. One considers whether the ES operation is influenced by uncertainty realizations. The other considers ES operations for economy and security purposes. Under the proposed operation model, three ES flexibility applications are selected for detailed analysis, which are designed to reduce the electricity purchasing costs, ensure safe node voltage, and alleviate uncertainties, respectively. Other ES applications can also be incorporated into the proposed operation model. However, to achieve a more concise and clear analysis, they are not discussed in this paper. Figure 1 shows the selected ES applications in the proposed operation model.

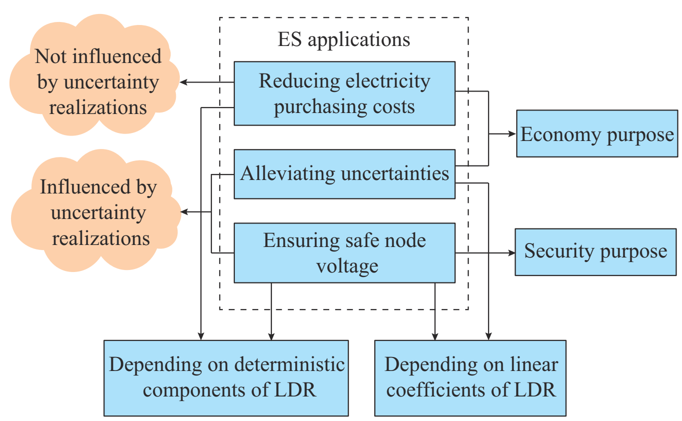

Fig. 1. ES applications in proposed operation model.

The ES application for reducing electricity purchasing costs depends on the deterministic components of ES power under LDR, which determines the electricity purchase of the distribution system for each hour. By contrast, the ES application for uncertainty alleviation depends on the linear coefficients of the LDR. Finally, the ES application for ensuring safe node voltage is influenced by both the deterministic components and the linear coefficients of the LDR, which means that a proper day-ahead arrangement and a real-time adjustment are both needed.

In these three ES applications, the alleviation of uncertainty is not straightforward; its theory is further explained here. For example, regardless of the realization of uncertainty in the RES output, the ES can adjust its discharging power by the same magnitude but in the opposite direction. Consequently, no variation or uncertainty exists in terms of electricity imported from the distribution system. In this case, the ES compensates for the variation in RES outputs, and it can be observed that ES absorbs the RES uncertainty. Although this application is based on the discharging status of ES, the situation is similar when the ES is charged. However, although the ES can absorb uncertainties, as previously discussed, its SOC changes after it absorbs uncertainties and must be restored, as the ES capacity is limited. In other words, the absorbed uncertainties must be released from the ES, and this can be achieved by adjusting the ES charging or discharging power. Figure 2 illustrates the use of an ES to alleviate uncertainties, where only one RES and one ES are assumed to exist in the distribution system, and the RES uncertainties are μ_{t_1} and μ_{t_2}, respectively. The ES absorbs uncertainties through charging or discharging in t_1, and thus no uncertainty exists in terms of the distribution system. The uncertainties absorbed in t_1 by the ES are released in t_2, and therefore, the uncertainties from the two hours are now collected in the same hour and can counteract each other when they have positive and negative realizations. As Fig. 2 shows, the process of absorbing and releasing uncertainties using ES is similar to allocating the relevant uncertainties to later hours. Therefore, the LDR coefficients for ES operation in the proposed operation model actually determine how uncertainties are transferred in the time domain and enable the accurate tracking of uncertainties. The theory also works when more uncertainties and hours are involved and explains why uncertainties can be alleviated in the proposed operation model.

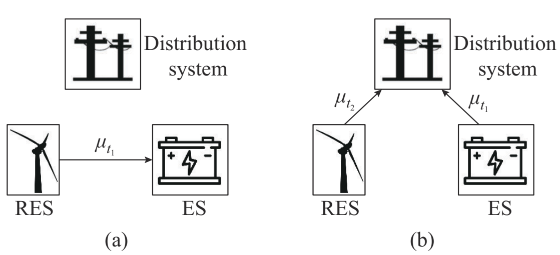

Fig. 2. Use of ES to alleviate uncertainties. (a) $t=t_1$. (b) $t=t_2$.

Note that the case illustrated in Fig. 2 is not trivial. For example, this case does not hold for the general operation problem of distribution system. This is because μ_{t_1} becomes deterministic information after it is transferred to t_2, which means that μ_{t_1} and μ_{t_2} incur costs in different manners, and no counteraction will occur between uncertainties. Even in a distribution system operation, if the worst-case cost or the cost in the predicted scenario is minimized, as in [12] and [13], the interplay between uncertainties will not be given sufficient attention because its influence on the operation model will be limited. In fact, the proposed ES operation scheme is enabled by an in-depth analysis of ES operations in distribution systems and the integration of LDR into the problem investigated in this paper. The designed ES operation scheme as well as the theory and interpretation of ES operation is original, and do not derive directly from the LDR. In addition to the studies based on LDR, the proper modeling of the problem is also crucial. The interaction between uncertainties cannot be properly analyzed or controlled if the model does not optimize the average operation costs or if the correlations of uncertainties are not effectively considered, as discussed in Section II-B. In summary, this paper does not simply apply or combine LDR and other methods. Instead, it conducts a systematic investigation of the proposed operation model and the solution method, with the latter further discussed in Section III.

III. LSTM Neural Network

With the help of LDR, robust optimization, and DRO, the proposed operation model is transformed into a deterministic mixed-integer second-order conic program, which can be directly solved by commercial solvers. However, it is time-consuming. We then present an LSTM neural network that accelerates the computation of the distribution system operation model.

A. Recurrent Neural Network (RNN) and LSTM Neural Network

An RNN is a type of neural network with sequential data used as both input and output. Unlike ordinary neural networks, an RNN considers not only the current input but also the input and output at previous time, which means that it can store and utilize previous information [29]. However, a traditional RNN may encounter a vanishing gradient problem during reverse propagation, and the corresponding possibility increases when the sequence is too long. If the gradient value becomes very small, the neural network stops learning because of insufficient weight changes. In the proposed operation model, parameters exist for 24 hours and act as input information for the neural networks to determine the binary variables corresponding to the ES status. Given the long sequence involved in the proposed operation model, LSTM neural networks are adopted, rather than ordinary RNNs. The LSTM neural network is a special RNN with one more hidden unit that effectively solves the gradient vanishing problem [30]. Because of the increased number of hidden units, LSTM neural networks can easily carry information over a long distance and are therefore more suitable for the 24-hour problem considered in this paper.

B. Generation and Preprocessing of Training Data

The LSTM neural network must determine the binary variable related to the ES status for charging and discharging. The binary variable is also the label for the training samples.

The evaluation of the correlations between ES status and parameters for distribution system operation is accomplished by computing Pearson coefficients. Accordingly, electricity prices, forecasted RES outputs, and load demands are the major influencing factors, which are thus used as the features of the LSTM neural network. Simulated operation parameters are generated based on 20% random variations in electricity prices and forecasted RES outputs and 10% random variations in load demands. Directly solving the proposed operation model is feasible although time-consuming. Therefore, the training samples are obtained by solving the proposed operation model under the simulated parameters. In real-world applications, training samples can be collected based on the actual operation parameters and the corresponding solutions provided by the proposed operation model. In addition, to guarantee the performance of the LSTM neural network and reduce the training time, different features are normalized prior to training. In addition, as the chosen test system has 32 nodes with both active and reactive loads, the load data in each hour are 64-dimensional, which increases the complexity and difficulty of training the LSTM neural network. Therefore, principal component analysis is adopted, which uses uncorrelated variables, albeit less, to obtain as much information as possible about the original data, as it reveals the internal structure of the data by transforming the observation perspective [31]. Specifically, the eigenvectors and eigenvalues of the covariance matrix are computed. The eigenvalues represent the amount of information that the new features carry and are sorted from the largest to the smallest, the sum of which is set to be larger than 0.98 times that of all eigenvalues. Through principal component analysis, a set of 20-dimensional data is used to represent the 64-dimensional load data for each hour in the LSTM neural network.

C. Application of LSTM Neural Network

After training is completed, the input information including electricity prices, load demands, and forecasted RES outputs, can be provided to the trained LSTM neural network, whose outputs are the charging and discharging statuses for each EV and are used as decisions in the proposed operation model. After the binary variables corresponding to the ES status are replaced with the decisions made by the LSTM neural network, the proposed operation model becomes a convex optimization problem and is then solved precisely to the globally optimal solution by off-the-shelf solvers for making other decisions about distribution system operations.

The flexibility of the proposed operation model is not affected because ES statuses are assumed to be fixed in real-time operations to avoid damage caused by unexpected changes in ES statuses. Although the optimality of the proposed operation model is influenced by LSTM errors, the LSTM neural network can generally achieve satisfactory outcomes due to the advantages previously discussed. In addition, to address the ES task of uncertainty alleviation, we must know how the ES should respond to uncertainties. This response actually plays a more important role than its statuses because the ES can alleviate uncertainties regardless of whether it is charging or discharging. Therefore, LSTM errors have a limited influence on the optimality of the proposed operation model, and the performance of the proposed operation model when the LSTM neural network is applied is demonstrated through conducted case studies, which are described in Section IV-B.

Other operation decisions are made by solving the operation model for the globally optimal solution after the binary variables are decided using LSTM neural network. Therefore, the operation constraints can be strictly met, and there will be no unexpected infeasibility in real-time operation. A small possibility exists that improper ES statuses provided by the LSTM neural network may cause the proposed operation model to have no feasible solution. In this case, the proposed operation model can be solved directly without using the LSTM neural network to eliminate the effects of LSTM errors.

IV. Cases Studies and Discussion

This section demonstrates the ES flexibility in the proposed operation model, and then describes the performance of the LSTM neural networks in the proposed operation model. Case studies are based on the standard IEEE 33-node distribution network but with the ES and RES connected to nodes 22 and 32 and to nodes 13 and 30, respectively. Table I lists the parameters of the ES. The time horizon is 24 hours unless otherwise stated.

TABLE I
Parameters of ES

Parameter	Value
Charging/discharging efficiency	0.95
The maximum charging/discharging rate (kW)	300.00
Initial SOC (kWh)	500.00
The minimum SOC (kWh)	50.00
The maximum SOC (kWh)	900.00

A. ES Flexibility in Distribution System Operation

1) Uncertainty Alleviation

This part discusses the effectiveness of the proposed operation model in uncertainty alleviation. To eliminate the effects of other ES applications, electricity prices are set to be constant for each hour. Table II lists the changes in operation costs due to exploiting ES flexibility. All uncertainty-affected costs are presented as average values based on 10000 sets of uncertainty realizations. The deviation penalty decreases significantly with ES flexibility, indicating the effectiveness of the proposed operation model in alleviating uncertainty. Although other costs increase, the total cost decreases due to ES flexibility. Figure 3 presents the average deviation penalty with and without ES flexibility. With ES flexibility, the uncertainties from different hours can be obtained. Therefore, no deviation penalty is incurred for many hours, and the total deviation penalty in the day is reduced because of the counteraction between uncertainties. As illustrated in Section II, the ES power can be split into deterministic components and linear functions of uncertainties under LDR. Figure 4 shows the deterministic components of the

ES power under ES flexibility, where positive and negative values correspond to the ES charging power and discharging power, respectively. As the electricity prices in different hours are set to be constant, the aim of ES operation is to alleviate uncertainties. Figure 4 shows that simultaneous charging and discharging of the two ESs occur because the ES can alleviate uncertainties regardless of whether it is charging or discharging. In addition, the ES has greater charging and discharging power in the last several hours because it tends to assemble uncertainties in the last several hours. This requires greater ES power and is consistent with the higher deviation penalties in the last several hours, as shown in Fig. 3.

TABLE II
CHANGES IN OPERATION COSTS WITH USE OF ES FLEXIBILITY

Cost	Value (cent)
Electricity purchasing cost	611.3
Average deviation penalty	–3747.1
Average ES operation cost	1277.6
Average total cost	–1858.3

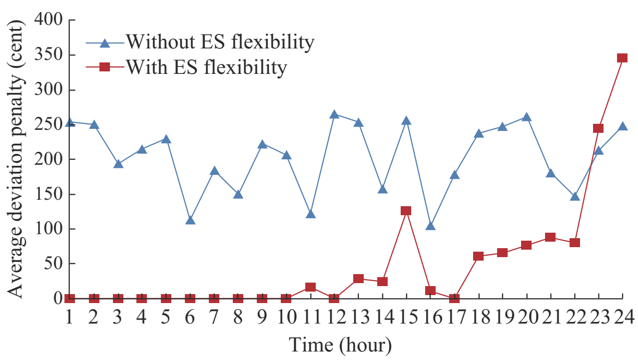

Fig. 3. Average deviation penalty with and without ES flexibility.

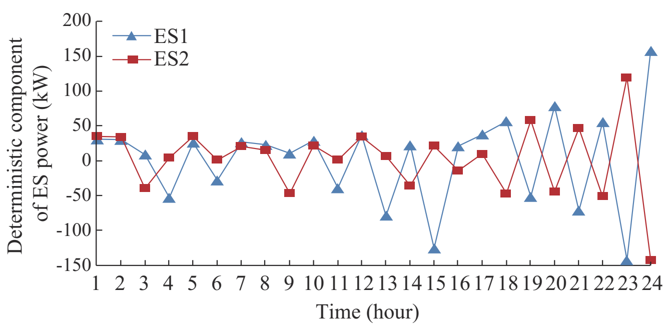

Fig. 4. Deterministic components of ES power under ES flexibility.

2) Influence of Network Constraint on ES Operation

The two settings for the safe range of the node voltage are adopted, as shown in Table III, and the time horizon is assumed to contain 10 rather than 24 hours to achieve a clearer demonstration. Case studies are conducted by adopting the electricity prices listed in Table IV and setting the load demand in the last two hours to be relatively higher. According to Table IV, electricity prices are significantly lower during the last two hours. Therefore, the load demands are shifted to the last two hours through the ES to reduce the electricity purchasing costs, and ES charges in the last two hours.

TABLE III
TWO SETTINGS FOR SAFE RANGE OF NODE VOLTAGE

Setting	Safe range of node voltage (p.u.)
A	0.95-1.05
B	0.96-1.04

TABLE IV
ELECTRICITY PRICES ADOPTED IN SECTION IV-A-2)

Time (hour)	Electricity price (cent/kWh)	Time (hour)	Electricity price (cent/kWh)
1	6.6	6	6.5
2	6.7	7	6.3
3	6.8	8	6.2
4	6.9	9	4.6
5	6.7	10	4.4

The average deviation penalties under the two settings are shown in Fig. 5, and Table V shows the average total deviation penalties. It can be observed that the deviation penalty has different profiles under the two settings, and the penalty is lower under Setting A, the reasons for which are as follows. As corroborated by the deviation penalty curve shown in Fig. 5, uncertainties are mainly assembled in the last two hours to counteract each other when the constraint on node voltage is not very strict under Setting A. However, when the constraint on node voltage becomes stricter under Setting B, the limitations on ES charging power become stronger as well. This means that the available ES flexibility is reduced in the last two hours under Setting B. With reduced ES flexibility, uncertainties cannot be assembled intensively in the last two hours and are allocated more dispersedly from the 7th to the 10th hour under Setting B. This results in positive average deviation penalties during these hours, as shown in Fig. 5. Because the uncertainties are more poorly assembled, less opportunity exists for them to counteract, and the average total deviation penalty listed in Table V is higher under Setting B.

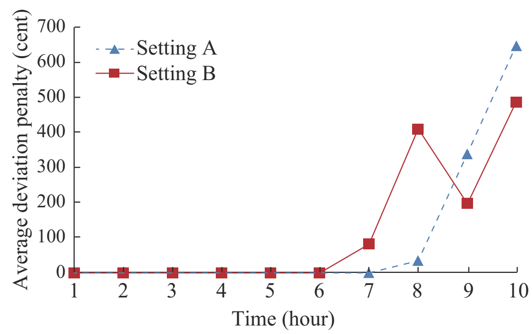

Fig. 5. Average deviation penalties under two settings.

TABLE V
AVERAGE TOTAL DEVIATION PENALTIES UNDER TWO SETTINGS

Setting	Average total deviation penalty (cent)
A	1021.1
B	1174.8

3) Different Applications of ES Flexibility in Different Time Intervals

The electricity prices listed in Table VI are next adopted, and the load demands are set relatively higher in the last two hours. The penalty coefficient for the electricity deviations is set to be zero to exclude the influence of uncertainty alleviation. This case study is conducted based on three settings, i. e., Settings A and B as presented in Table III, and Setting B with constant electricity prices. The deterministic components of the ES power under the three settings are presented in Fig. 6, where positive and negative values correspond to the ES charging power and discharging power, respectively. Figure 6 shows that ES discharges in the 3rd and 4th hours and charges in the 5th and 6th hours under Setting A. These occur because the ES shifts loads to reduce the electricity purchasing costs and thus to make use of the electricity prices listed in Table VI. When the electricity prices in different hours are set to be constant, the ES will not operate and this reduces electricity purchasing costs. However, Fig. 6 shows that the ES still discharges in the last two hours under Setting B but with constant electricity prices. This is because the stricter network constraint in Setting B requires ES discharge to decrease the power flow on certain feeders. Finally, when the electricity price given in Table VI and the stricter network constraint in Setting B are adopted, the ES first discharges, then charges, and finally discharges again, as shown in Fig. 6. This occurs because the ES first shifts loads to reduce electricity purchasing costs and later discharges to ensure a safe node voltage. This case shows that the proposed operation model can properly allocate ES flexibility to different applications in different time intervals to fully exploit the ES.

TABLE VI
ELECTRICITY PRICES

Time (hour)	Electricity price (cent/kWh)	Time (hour)	Electricity price (cent/kWh)
1	6.6	6	4.4
2	6.7	7	5.4
3	6.8	8	5.6
4	6.9	9	5.8
5	4.6	10	6.0

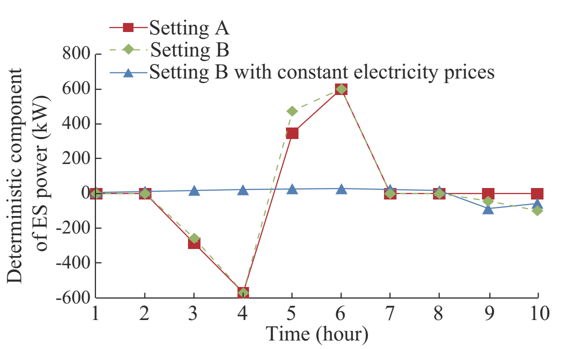

Fig. 6. Deterministic components of ES power under different settings.

4) Comparison with an Existing Operation Model

To illustrate the superiority of the proposed operation model, we compare it with the existing operation model in [16]. Instead of a designed scheme that clearly tracks uncertainties during the entire ES operation while optimizing ES flexibility in terms of all-day operation (as with the proposed method), the existing operation model allocates the corresponding ES flexibility equally to all hours while using the ES to alleviate uncertainties. This is unreasonable because the levels of uncertainties in different hours are not the same. Thus, the existing operation model regards uncertainty alleviation in each hour as a separate task and thus does not consider the interplay between uncertainties from different hours. This means it is short-sighted and cannot achieve the globally optimal arrangement of ES flexibility. Figure 7 shows that the deviation penalty curve under the existing operation model is similar to that without uncertainty alleviation. This is because under the existing method, ES flexibility is improperly restricted and cannot be fully used to alleviate uncertainties. Therefore, the proposed operation model achieves greater savings in the use of ES flexibility as compared with existing operation models, as shown in Table VII. Figure 8 shows that when the ES is not used to alleviate uncertainty, it operates only during certain hours to exploit the differences between electricity prices in different hours by shifting loads. The two curves shown in Fig. 8 reveal that under the proposed and existing operation models, a larger charging or discharging power generally exists under the proposed operation model. This is because ES flexibility can be used to a greater extent to alleviate uncertainties. However, this is different for the 6th and 10th hours. As the use of ES flexibility in uncertainty alleviation is restricted under the existing operation model, the ES allocates more of its flexibility to load shifting by charging during the 6th hour, and thus has higher deterministic components in charging power under the existing operation model than under the proposed operation model. In the 10th hour, the deterministic ES discharging power is higher under the existing operation model than under the proposed operation model. This is because the ES has a greater need to restore its flexibility from load shifting to alleviate uncertainties under the rigid scheme of the existing operation model.

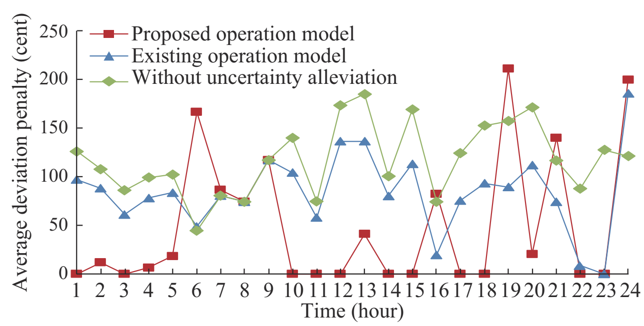

Fig. 7. Average deviation penalties under different models when ES flexibility is exploited.

TABLE VII
CHANGES IN OPERATION COSTS WITH USE OF ES FLEXIBILITY UNDER DIFFERENT OPERATION MODELS

Operation model	Electricity purchasing cost (cent)	Average deviation penalty (cent)	Average ES operation cost (cent)	Average total cost (cent)
Proposed	–1122.7	–1637.8	1107.9	–1652.5
Existing	–1128.9	–797.0	822.4	–1103.3

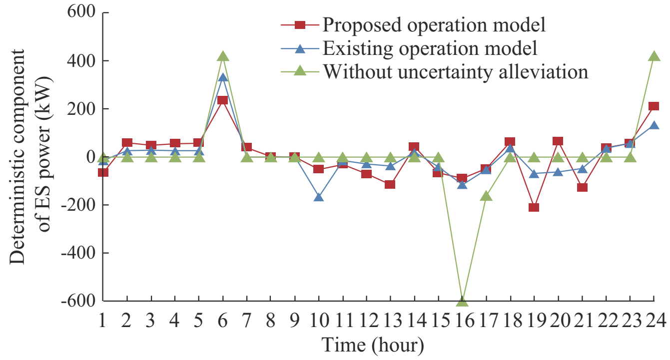

Fig. 8. Deterministic components of ES power under different models when ES flexibility is exploited.

B. Performance of LSTM Neural Networks in Solving Proposed Operation Model

The LSTM neural networks are used to accelerate the computation speed of the proposed operation model, and its relevant performance will be demonstrated in the first part of this subsection. The solution method using the LSTM neural network is then compared with the precise solution in the second part and with the solution method using an ordinary RNN in the third part. All of this enables us to assess the outcomes of the proposed operation model.

1) Reduction in Computation Time Derived from LSTM Neural Network

Based on 100 sets of randomly generated parameters, the proposed operation model is solved in two scenarios, i. e., with and without the LSTM neural network. The computation time is listed in Table VIII. The table shows that compared with the scenario in which the proposed operation model is directly solved, the computation time is significantly reduced and is practically negligible when the LSTM neural network is used. This occurs because, after the ES status of charging and discharging is fixed by the LSTM neural network, the proposed operation model is transformed from a mixed-integer optimization problem into a typical second-order conic programming problem, and the solving difficulty is greatly reduced. The training time of the LSTM neural network is approximately 1700 s, which is sufficient because the training is conducted offline, and the model can be repeatedly used once it is trained. In addition, the training time is derived from using a PC with Core i7 processor and can be further shortened if high-performance workstations are used.

TABLE VIII
COMPUTATION TIME USING AND WITHOUT USING LSTM NEURAL NETWORK BASED ON 100 SETS OF RAMDOMLY GENERATED PARAMETERS

Scenario	The maximum computation time (s)	The minimum computation time (s)	Average computation time (s)
Using LSTM neural network	18.0	9.6	11.9
Without using LSTM neural network	13428.5	1574.0	3524.5

2) Influence on ES Flexibility Derived from LSTM Neural Network

With ES flexibility, the electricity purchasing costs and penalties for electricity deviations can be reduced. When an LSTM neural network is used, its error may impede the ES flexibility from being effectively used and may result in fewer cost reductions for the distribution system. To study the effects of the LSTM neural network on the proposed operation model, 50 sets of parameters are generated. The proposed operation model is then solved using these parameters along with the LSTM neural network, and it is also solved precisely without using the LSTM. Figure 9 shows the ratio of cost reductions when LSTM is used to cost reductions when LSTM is not used, both under 50 sets of parameters. As some operation costs of the distribution system are influenced by the realization of RES uncertainty, the uncertainty-influenced costs are the average values calculated based on 10000 randomly generated uncertainty realizations.

Figure 9 shows that the effect of the LSTM neural network on the proper use of ES flexibility is acceptable, particularly when considering the significant reduction in computation time. The ratio should be less than or equal to 1. However, according to Fig. 9, it is shown to be greater than 1 in some datasets because the presented results are based on finite uncertainty realizations and may show minor deviations from the actual situation. Table IX presents further comparisons. It can be observed that the reductions in deviation penalties are nearly the same under the two solution methods because the ES statuses are determined by the difference between the two solution methods, and the ES can alleviate uncertainties regardless of whether it is charging or discharging.

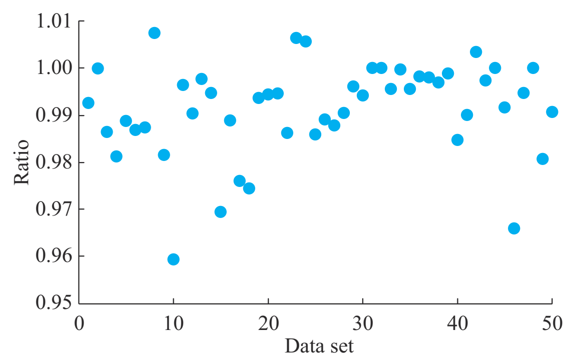

Fig. 9. Ratio of cost reductions when LSTM is used to cost reductions when LSTM is not used.

TABLE IX
COST REDUCTION COMPARISONS UNDER PRECISE SOLUTION METHOD AND SOLUTION METHOD USING LSTM NEURAL NETWORK

Solution method	Reduction in electricity purchasing cost brought by ES flexibility (cent)	Average reduction in deviation penalty brought by ES flexibility (cent)	Average ES operation cost (cent)	Average cost reduction brought by ES flexibility (cent)
Precise	2815.2	3943.3	1805.8	4952.8
Using LSTM neural network	2750.6	3945.0	1785.8	4909.9

3) Comparison of LSTM Neural Network and RNN

The binary variables in the proposed operation model are determined using LSTM neural network and RNN. Table X shows that the RNN performs similar to the LSTM neural network when the time horizon includes 10 hours. However, when the problem becomes more complicated as the time horizon increases to 24 hours, the advantage of the LSTM neural network over the RNN in considering the mutual influence of decisions in different time intervals becomes more evident. As the LSTM neural network can make better decisions for binary variables in determining the ES status, ES flexibility achieves greater cost savings when the LSTM neural network is used, as shown in Table XI. To further illustrate the advantage of applying the LSTM neural network rather than the RNN in the proposed operation model, the average deviation penalties of the distribution system when integer variables are determined in different manners are shown in Fig. 10. It can be observed that the curve of the original model approximates that of the LSTM neural network compared with that of RNN, and the total deviation penalty over the day under the LSTM neural network is also lower than that under the RNN.

TABLE X
ACCURACIES OF RNN AND LSTM NEURAL NETWORK IN DETERMINING BINARY VARIABLES

Time horizon (hour)	Accuracy of RNN (%)	Accuracy of LSTM neural network (%)
10	92.9	93.3
24	77.8	80.5

TABLE XI
COST REDUCTION COMPARISONS USING RNN AND LSTM NEURAL NETWROK

Network	Reduction in electricity purchasing cost brought by ES flexibility (cent)	Average reduction in deviation penalty brought by ES flexibility (cent)	Average ES operation cost (cent)	Average cost reduction brought by ES flexibility (cent)
Using RNN	2734.1	3685.7	1791.8	4628.0
Using LSTM neural network	2745.2	3914.9	1771.9	4888.2

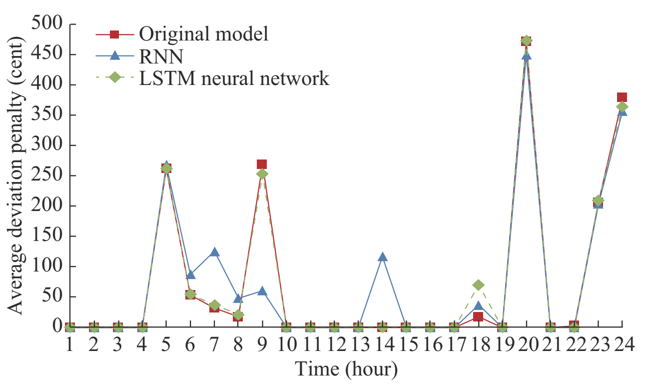

Fig. 10. Average deviation penalties when integer variables are determined in different manners.

V. CONCLUSION

An operation model is proposed in this paper for distribution systems to use ES flexibility in alleviating uncertainties, reducing electricity purchasing costs, and ensuring a secure node voltage. Based on the complexity of the proposed operation model, an LSTM neural network is used to determine the ES charging and discharging statuses and is combined with off-the-shelf optimization solvers in the proposed operation model. It is shown that the proposed operation model could achieve a significant reduction in computation time while simultaneously maintaining satisfactory outcomes through case studies. The effectiveness of the proposed operation model in utilizing ES flexibility in different manners is also verified, and the operation costs are reduced to ensure the safe operation of distribution system. Because of their aggregation through the ES, uncertainties from the RES are significantly alleviated. In addition, we demonstrate that with the proposed operation model, different applications of ES flexibility can be conducted simultaneously and can directly influence each other. They can also operate in different time intervals. Overall, ES flexibility is properly allocated among the different applications using the proposed operation model.

REFERENCES

[1] Y. Yang, W. Wu, B. Wang *et al.*, "Optimal decomposition of stochastic dispatch schedule for renewable energy cluster," *Journal of Modern Power Systems and Clean Energy*, vol. 9, no. 4, pp. 711-719, Jul. 2021.

[2] S. Zhang and J. Yu, "Bayesian deep learning for dynamic power system state prediction considering renewable energy uncertainty," *Journal of Modern Power Systems and Clean Energy*, vol. 10, no. 4, pp. 913-922, Jul. 2022.

[3] S. Liu, T. Zhang, Z. Lin *et al.*, "Controlled islanding strategy considering uncertainty of renewable energy sources based on chance-constrained model," *Journal of Modern Power Systems and Clean Energy*, vol. 10, no. 2, pp. 471-481, Mar. 2022.

[4] S. Weitemeyer, D. Kleinhans, T. Vogt *et al.*, "Integration of renewable energy sources in future power systems: the role of storage," *Renewable Energy*, vol. 75, pp. 14-20, Mar. 2015.

[5] T. Ha, Y. Xue, K. Lin *et al.*, "Optimal operation of energy hub based micro-energy network with integration of renewables and energy storages," *Journal of Modern Power Systems and Clean Energy*, vol. 10, no. 1, pp. 100-108, Jan. 2022.

[6] M. Uddin, M. F. Romlie, M. F. Abdullah *et al.*, "A novel peak shaving algorithm for islanded microgrid using battery energy storage system," *Energy*, vol. 196, p. 117084, Apr. 2020.

[7] X. Sun, J. Qiu, Y. Yi *et al.*, "Cost-effective coordinated voltage control in active distribution networks with photovoltaics and mobile energy storage systems," *IEEE Transactions on Sustainable Energy*, vol. 13, no. 1, pp. 501-513, Jan. 2022.

[8] A. X. Y. Mah, W. Ho, M. Hassim *et al.*, "Optimization of photovoltaic-based microgrid with hybrid energy storage: a P-graph approach," *Energy*, vol. 233, p. 121088, Oct. 2021.

[9] X. Yan, H. Zhang, C. Gu *et al.*, "Truncated strategy based dynamic network pricing for energy storage," *Journal of Modern Power Systems and Clean Energy*, vol. 11, no. 2, pp. 544-552, Mar. 2023.

[10] N. G. Cobos, J. M. Arroyo, N. Alguacil *et al.*, "Robust energy and reserve scheduling considering bulk energy storage units and wind uncertainty," *IEEE Transactions on Power Systems*, vol. 33, no. 5, pp. 5206-5216, Sept. 2018.

[11] Y. Zhou, Q. Zhai, and L. Wu, "Multistage transmission-constrained unit commitment with renewable energy and energy storage: implicit and explicit decision methods," *IEEE Transactions on Sustainable Energy*, vol. 12, no. 2, pp. 1032-1043, Apr. 2021.

[12] Z. Guo, W. Wei, L. Chen *et al.*, "Distribution system operation with renewables and energy storage: a linear programming based multistage robust feasibility approach," *IEEE Transactions on Power Systems*, vol. 37, no. 1, pp. 738-749, Jan. 2022.

[13] Y. Luo, Q. Nie, D. Yang *et al.*, "Robust optimal operation of active distribution network based on minimum confidence interval of distributed energy beta distribution," *Journal of Modern Power Systems and Clean Energy*, vol. 9, no. 2, pp. 423-430, Mar. 2021.

[14] G. Liu, Y. Xu, and K. Tomsovic, "Bidding strategy for microgrid in day-ahead market based on hybrid stochastic/robust optimization," *IEEE Transactions on Smart Grid*, vol. 7, no. 1, pp. 227-237, Jan. 2016.

[15] Y. Guo, Q. Zhang, and Z. Wang, "Cooperative peak shaving and voltage regulation in unbalanced distribution feeders," *IEEE Transactions on Power Systems*, vol. 36, no. 6, pp. 5235-5244, Nov. 2021.

[16] A. Saint-Pierre and P. Mancarella, "Active distribution system management: a dual-horizon scheduling framework for DSO/TSO interface under uncertainty," *IEEE Transactions on Smart Grid*, vol. 8, no. 5, pp. 2186-2197, Sept. 2017.

[17] Z. Li, Q. Guo, H. Sun *et al.*, "Sufficient conditions for exact relaxation of complementarity constraints for storage-concerned economic dispatch," *IEEE Transactions on Power Systems*, vol. 31, no. 2, pp. 1653-1654, Mar. 2016.

[18] N. Nazir and M. Almassalkhi, "Guaranteeing a physically realizable battery dispatch without charge-discharge complementarity constraints," *IEEE Transactions on Smart Grid*, vol. 14, no. 3, pp. 2473-2476, May 2023.

[19] A. J. Ardakani and F. Bouffard, "Identification of umbrella constraints in DC-based security-constrained optimal power flow," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 3924-3934, Nov. 2013.

[20] R. Palma-Behnke, J. L. Cerda A., L. S. Vargas *et al.*, "A distribution company energy acquisition market model with integration of distributed generation and load curtailment options," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 1718-1727, Nov. 2005.

[21] W. Leterme, F. Ruelens, B. Claessens *et al.*, "A flexible stochastic optimization method for wind power balancing with PHEVs," *IEEE Transactions on Smart Grid*, vol. 5, no. 3, pp. 1238-1245, May 2014.

[22] K. Natarajan, M. Sim, and J. Uichanco, "Tractable robust expected utility and risk models for portfolio optimization," *Mathematical Finance*, vol. 20, no. 4, pp. 695-731, Oct. 2010.

[23] D. Bertsimas, D. B. Brown, and C. Caramanis, "Theory and applications of robust optimization," *SIAM Review*, vol. 53, no. 3, pp. 464-501, Jan. 2011.

[24] A. Lorca and X. A. Sun, "Multistage robust unit commitment with dynamic uncertainty sets and energy storage," *IEEE Transactions on Power Systems*, vol. 32, no. 3, pp. 1678-1688, May 2017.

[25] V. Dvorkin, D. Mallapragada, and A. Botterud, "Multi-stage decision rules for power generation & storage investments with performance guarantees," *IEEE Transactions on Power Systems*, vol. 39, no. 1, pp. 755-769, Jan. 2024.

[26] A. Lorca, X. Sun, E. Litvinov *et al.*, "Multistage adaptive robust optimization for the unit commitment problem," *Operations Research*, vol. 64, no. 1, pp. 32-51, Jan.-Feb. 2016.

[27] R. A. Jabr, "Segregated linear decision rules for inverter Watt-VAr control," *IEEE Transactions on Power Systems*, vol. 36, no. 3, pp. 2702-2708, May 2021.

[28] X. Xu, Y. Gao, H. Wang *et al.*, "Distributionally robust optimization of photovoltaic power with lifted linear decision rule for distribution system voltage regulation," *IEEE Transactions on Sustainable Energy*, vol. 15, no. 2, pp. 758-772, Apr. 2024.

[29] A. C. Tsoi, "Recurrent neural network architectures – an overview," in *Adaptive Processing of Sequences and Data Structures*, C. L. Giles and M. Gori, Eds., Berlin: Springer-Verlag, 1998, pp. 1-26.

[30] J. Chung, C. Gulcehre, K. H. Cho *et al.* (2014, Dec.). Empirical evaluation of gated recurrent neural networks on sequence modeling. [Online]. Available: https://arxiv.org/abs/1412.3555

[31] A. I. Khuri and S. Mukhopadhyay, *Wiley Interdisciplinary Reviews: Computational Statistics*. Washington: Wiley, 2010.

Xi Lu received the B.Eng. degree in electrical engineering from North China Electric Power University, Beijing, China, in 2015, and the Ph.D. degree in electrical engineering from The Hong Kong Polytechnic University, Hong Kong, China, in 2020. He is currently a Lecturer with School of Electrical Engineering, Southeast University, Nanjing, China. His research interest includes optimization of power system operation with uncertainties.

Xinzhe Fan received the B.E. degree in electrical engineering from Southeast University, Nanjing, China, in 2022. She is currently pursuing the M.Sc. degree in electrical engineering in Southeast University, Nanjing, China. Her research interests include operation optimization and uncertainty analysis of power systems and integrated energy systems of electricity and heat.

Haifeng Qiu received the B.Eng. degree from Nanjing Normal University, Nanjing, China, in 2015, and the Ph.D. degree from Southeast University, Nanjing, China, in 2021, both in electrical engineering. From 2019 to 2020, he was a Visiting Scholar with Cornell University, Ithaca, USA. He is currently a Postdoctoral Research Fellow with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. His research interests include energy system planning, operation, and market trading.

Wei Gan received the B.S. and Ph.D. degrees in electrical engineering from Huazhong University of Science and Technology, Wuhan, China, in 2016 and 2021, respectively. From 2018 to 2019, he was a visiting Ph.D. student with China Electric Power Research Institute, Beijing, China. From 2019 to 2020, he was a visiting Ph.D. student with the Robert W. Galvin Center for Electricity Innovation, Illinois Institute of Technology, Chicago, USA. He is currently a Postdoctoral Research Associate with the School of Engineering, Cardiff University, Cardiff, UK. His research interests include peer-to-peer energy trading, coordinated optimization of power-traffic systems, and multiple energy system integration.

Wei Gu received the B.S. and Ph.D. degrees in electrical engineering from Southeast University, Nanjing, China, in 2001 and 2006, respectively. From 2009 to 2010, he was a Visiting Scholar at the Department of Electrical Engineering, Arizona State University, Tempe, USA. He is currently a Professor with the School of Electrical Engineering, Southeast University. He is the Director of the Institute of distributed generations and active distribution networks. His research interests include distributed generations, microgrids, and integrated energy systems.

Shiwei Xia received the Ph. D. degree in power systems from The Hong Kong Polytechnic University, Hong Kong, China, in 2015. Currently, he is with the State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, School of Electrical and Electronic Engineering, North China Electric Power University, Beijing, China. His research interests include security and risk analysis for power systems with renewable energies, and distributed optimization and control of multiple sustainable energy sources in smart grid.

Xiao Luo received the B.E. degree in electrical engineering from Southeast University, Nanjing, China, in 2020, and the M.Sc. degree in software engineering in Southeast University, Suzhou, China, in 2023. His research interest includes optimization of distribution system operation.