Multi-stage Robust Scheduling for Community Microgrid with Energy Storage

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Abstract—**Energy storage devices can effectively balance the uncertain load and significantly reduce electricity costs in the community microgrids (C-MGs) integrated with renewable energy sources. Scheduling of energy storage is a multi-stage decision problem in which the decisions must be guaranteed to be nonanticipative and multi-stage robust (all-scenario-feasible). To satisfy these two requirements, this paper proposes a method based on a necessary and sufficient feasibility condition of scheduling decisions under the polyhedral uncertainty set. Unlike the most popular affine decision rule (ADR) based multi-stage robust optimization (MSRO) method, the method proposed in this paper does not require the affine decision assumption, and the feasible regions (the set of all feasible solutions) are not reduced, nor is the solution quality affected. A simple illustrative example and real-scale scheduling cases demonstrate that the proposed method can find feasible solutions when the ADR-based MSRO fails, and that it finds better solutions when both methods succeed. Comprehensive case studies for a real system are performed and the results validate the effectiveness and efficiency of the proposed method.**

Index Terms—**Community microgrids, short-term scheduling, energy storage, nonanticipativity, polyhedral uncertainty set.**

Nomenclature

A. Indices and Sets

Ω	Uncertainty set of net load
Ω_t	Conditional uncertainty set of net load
i, j, m	Indices of budget constraints on load demand, renewable energy, and net load
$[1:I]$, $[1:J]$, $[1:M]$	Sets of budget constraints on load demand, renewable energy, and net load
k_1, k_2	Vertex indices of uncertainty set of load demand and renewable energy
$[1:K_1]$, $[1:K_2]$	Vertex sets of uncertainty set of load demand and renewable energy
R	Real set
t	Index of time period
$[1:T]$	Set of time periods

B. Parameters

τ	Length of each time period
ε	Tightness of budget constraints
λ^{buy}	Unit price of buying electricity energy from grid
λ^{sell}	Unit price of selling electricity energy to grid
η_d, η_c	Discharging and charging efficiencies of energy storage system
a, α	Budget constraint coefficients of load demand
A, B	Affine coefficients
b, β	Budget constraint coefficients of renewable energy
c, q	Budget constraint coefficients of net load
$\bar{d}$, $\underline{d}$	Upper and lower bounds of load demand
$\bar{\tilde{d}}$, $\underline{\tilde{d}}$	Upper and lower bounds of net load
$\tilde{d}_t^{\text{exp}}$	Expected net load
$\bar{E}$, $\underline{E}$	Upper and lower bounds of energy storage level
$\bar{g}$, $\underline{g}$	Upper and lower bounds of power transfer with main grid
$\bar{p}^{\text{dis}}$, $\bar{p}^{\text{ch}}$	The maximum discharging and charging power of energy storage
$\bar{r}$, $\underline{r}$	Upper and lower bounds of power outputs of renewable energy sources

C. Uncertain Variables

d	Uncertain load demand
$\tilde{d}$	Uncertain net load
r	Uncertain power output of renewable energy sources

D. Unfolded (Realized) Uncertain Variables

$\tilde{\boldsymbol{d}}=(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_T)$	A group of possible realizations of uncertain net load from time periods 1 to T

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$\tilde{\boldsymbol{d}}_{[t]}=(\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_t)$ Realized uncertain net load up to time period t

E. Decision Variables

$\bullet^*$ Optimal solution of a quantity

E Energy storage level of energy storage system

$\bar{E}^{\text{true}}$, $\underline{E}^{\text{true}}$ Safe upper and lower bounds of energy storage level. It should be noted that the safe bounds are variables and are different from $\bar{E}$, $\underline{E}$

g Power injection of the community microgrid from (positive) or to (negative) grid

p Charging (negative)/discharging (positive) power of energy storage

F. Functions

$h(x)$ An auxiliary function with respect to x and $x\in[\underline{x},\bar{x}]$

$h^{-1}(y)$ Inverse of auxiliary function with respect to y and $y\in[\underline{y},\bar{y}]$

I. Introduction

PROMOTION to community microgrids (C-MGs) provides new opportunities for existing community networks such that the penetration ratio of renewable energy sources (RESs) can be increased to reduce carbon emissions [1]-[3]. However, the uncertainty of RESs and load demand still challenges the operation safety of C-MGs [4]. To deal with the uncertainties and to improve the stability of C-MGs, energy storage systems (ESSs) have drawn great attention, which can be used to balance/reduce the uncertainties arising from load demand and RESs so as to reduce the electricity cost [5]-[7].

The scheduling of energy storage is a typical multi-stage decision process. In the real application, it is found that nonanticipativity and multi-stage robustness should be considered for multi-stage scheduling problems in recent years and relevant literatures have given some examples to illustrate the serious consequences if these two requirements are not satisfied [8]-[10]. To better explain the two concepts, we note that there is a salient feature in the multi-stage scheduling process that makes the problem completely different from the traditional deterministic optimization.

In the short-term scheduling of energy storage with uncertainties, the decision-making and the realization of uncertainties are repeated alternately and iteratively during the whole time horizon. For example, we must determine the charging/discharging power level of the ESS at time period 1 immediately after we know the net load information at this time period. Then, we must repeat this process at time periods 2, 3, …, T.

According to this feature of the scheduling process, the definitions of nonanticipativity and multi-stage robustness are naturally obtained.

At time period t, the decisions of this time period must be made based only on the unfolded (realized) random variables up to time period t and the set of all possible realizations at the future time periods (rather than any specific realization of the uncertainties at the future time periods). In other words, the decisions made at time period t must be the same for all possible future realizations of the uncertainties. This also implies that the current decision must be formulated as a function with respect to the unfolded (realized) random variables. This requirement is defined as nonanticipativity.

According to the nonanticipativity, the decision made at time period t is independent of the specific uncertainty realizations at the future time periods. Then, the subsequent question is whether the current decision is always feasible in the future when there are coupling constraints on the decisions at different time periods. For example, the energy storage levels at different time periods are coupled by charging/discharging power limits and the energy storage dynamic equation. Then, the subsequent question is how to guarantee that a feasible solution will always be found in the future based on historical decisions. In other words, the current decision must be carefully chosen such that the feasible region at the future time periods is not empty. This is a very complicated requirement.

For the multi-stage scheduling process mentioned above, the definitions of nonanticipativity and multi-stage robustness accurately reflect the actual physical process and the actual operation needs, respectively, and they are two key issues in the multi-stage scheduling problem. Moreover, we say that multi-stage robustness (all-scenario-feasibility) is guaranteed by a method if and only if the nonanticipativity is guaranteed and, at the same time, infeasibility at future time periods does not arise from inappropriate historical decisions. Here, it is seen that nonanticipativity is the prerequisite of multi-stage robustness.

On the basis of the mentioned features, it is worthwhile to classify and give a brief overview of the existing methods for solving the scheduling problem in chronological order.

1) Methods without guaranteeing nonanticipativity or multi-stage robustness

A traditional deterministic optimization problem rather than the original stochastic program is solved in the chance-constrained-based methods [11]-[13]. This is completed by formulation transformation of the original scheduling problem based on the probability distribution information of the related random variables and the given confidence level. Nonanticipativity can be satisfied in chance-constrained-based methods. However, multi-stage robustness is not guaranteed because they allow the existence of violation within a certain confidence level (some possible future realizations will be excluded by the chance constraints).

Scenario-based methods [14]-[16] formulate the uncertainties by several representative scenarios and then a deterministic optimization problem is solved with all these scenarios considered in a unified formulation. Since the decision-making process does not conform to the sequential logic in scenario-based methods, nonanticipativity cannot be guaranteed. Further, multi-stage robustness cannot be satisfied too.

Similar to the scenario-based methods, scenario-tree-based methods [17]-[19] also formulate the uncertainties by several scenarios. However, the scenarios are organized in a tree structure according to the time order. Coupling constraints among different scenarios are considered and then a unified formulation is established and solved. Nonanticipativity is guaranteed by the scenario-tree-based method (only for the scenarios included in the scenario tree) because it follows a time-causal policy. However, the multi-stage robustness is not guaranteed since some possible future realizations are not included in the tree structure.

In the two-stage robust optimization methods [20] - [23], all decision variables are separated into two groups. The optimal values of the first group variables are determined without any information on the realization of the random variables while those of the second group variables are determined based on full information about the realization of all the random variables. The second stage violates the sequential logic that uncertainty realization and decision making are events that appeared alternatively and repeatedly. Consequently, nonanticipativity is not satisfied in two-stage robust optimization methods and counterexamples have been given in [24]. Meanwhile, multi-stage robustness cannot be satisfied either.

2) Methods with nonanticipativity and multi-stage robustness guaranteed

To satisfy both the nonanticipativity and multi-stage robustness requirements, new methods have been proposed in the literature. Among these, the affine decision rule (ADR) based multi-stage robust optimization (MSRO) methods [9], [25], [26], and the all-scenario-feasible (ASF) methods [27], [28] are the most successful.

The ADR-based MSRO method is an effective way to solve the general form of MSRO problems. In the ADR-based MSRO methods, it is assumed that the decisions at time period t are affine functions with respect to the unfolded uncertainties up to time period t. In this way, nonanticipativity is naturally satisfied. Then, all original constraints are transformed into constraints on affine coefficients and therefore the multi-stage robustness is guaranteed when a group of feasible affine coefficients is found. However, ADR-based MSRO methods suffer from some serious limitations: for example, the affine function assumption may reduce the solution space and there may be no feasible affine coefficients even if the original problem has feasible solutions.

A different idea is adopted in the ASF method. In this method, a scenario-based formulation is established based on several carefully chosen scenarios. Meanwhile, some auxiliary variables and constraints are introduced such that both the nonanticipativity and multi-stage robustness requirements can be satisfied by solving a single-level mixed-integer linear programming (MILP) problem. The ASF method has been successfully used in solving security-constrained unit commitment [27], [28]. However, the auxiliary variables and constraints in this method are designed based on the special structures of the ramp-rate constraints.

The method in [27], [28] cannot be applied to solve the scheduling problem with ESS due to different constraint structures. For the unit commitment scheduling problem in [27], [28], the key variables are the power outputs of thermal units. The main complex time-coupling constraint is the ramp-rate constraint, which is a linear constraint. Differently, the key variables in the scheduling problem with ESS are the energy storage levels of the storage system. The main complex time-coupling constraints involve nonlinear constraints, i. e., the relationship between the change of storage levels (at two continuous time periods) and the charging/discharging power. Consequently, it is noted the structure of constraints in the formulation of the energy storage scheduling problem is different from that of thermal units and is much more complicated. Therefore, the ASF method cannot be directly adopted.

It should be noted that the structure and formulation of the uncertainty set are also important factors in the scheduling of C-MGs with ESS. An interesting result given in [29] suggests that when the uncertain net loads at all time periods are formulated by a box region (box uncertainty set), a simple necessary and sufficient feasibility condition for guaranteeing the nonanticipativity and multi-stage robustness requirements of the decisions can be established. Then, an efficient solution method is proposed based on this condition. However, the box uncertainty set is too conservative and the method given in [29] may even fail to find a feasible solution when the general uncertainty set with polyhedral constraints is considered. It has been found that the economic performance of the scheduling solution will be greatly improved with the polyhedral uncertainty set introduced [30], [31].

With the definitions of nonanticipativity and multi-stage robustness, the current decisions should depend only on the realization of uncertainties up to the current time, meanwhile, the current decisions should guarantee the feasibility of future decisions. It is very challenging, especially when there are time-coupling constraints in the problem formulation (e. g., the evolution equation of the energy storage levels).

Consequently, the problems are: how to solve the scheduling problem with the general uncertainty set (e.g., polyhedral uncertainty set), and whether is it possible to guarantee nonanticipativity and multi-stage robustness without the traditional ADR assumption.

A method is proposed in this paper to address these problems. The main contributions are as follows.

First, specific robust feasible regions (the maximum permissible ranges) of the energy storage level are defined based on the analysis of constraints of the scheduling problem (particularly the evolution equation of the energy storage level), so that the original problem can be decoupled at each time period. In fact, the result is a necessary and sufficient condition for ensuring the nonanticipativity and multi-stage robustness of scheduling decisions under the general polyhedral uncertainty set.

Second, based on the necessary and sufficient condition mentioned above, an efficient method is established for solving the scheduling problem without ADR assumption. More than that, the (approximated) expectation of the total cost rather than the worst-case cost is minimized and therefore the solution is less conservative than the robust optimization-based methods.

Moreover, an interesting example is given to show that ADR may fail due to the affine assumption on the decisions (the proposed method is still valid for this example). Extensive numerical testing is performed and the results suggest that the proposed method is promising.

The paper is organized as follows. Basic formulation of the scheduling problem is given in Section II. Then, Section III provides a simple example to emphasize the necessity of nonanticipativity and multi-stage robustness in scheduling C-MGs with ESS. Next, another example is also given to show the limitations of the ADR-based MSRO method. To deal with the problem, in Section IV, the proposed method provides the feasible regions for the decision variables such that the constraints can be decoupled at each time period and the solution quality will not be affected. Consequently, the original problem can be directly solved by using the proposed method, as given in Section V. Numerical results are analyzed in Section VI and the paper is concluded in Section VII.

II. Basic Formulation of Scheduling Problem

A. Formulation of Uncertainty Set

In robust optimization-based methods, uncertainty is modeled through an uncertainty set. Generally, for a robust optimization problem, the robustness of the solution is closely related to and mainly determined by the uncertainty set.

In this paper, uncertainties considered in C-MGs include the load demand and renewable power outputs. In robust optimization, a direct uncertainty set of uncertainties can be given by the box uncertainty set. However, due to the temporal correlations between the load demands (or renewable power outputs) in consecutive hours, the sample points will not spread everywhere in the box region. Therefore, if the whole box region is used as the uncertainty set, the robust scheduling solution obtained will be somewhat conservative since it includes many impossible scenarios.

Consequently, it is necessary to consider and exploit the historical correlations in the uncertainty set formulation to avoid impossible scenarios in practice and reduce the conservativeness of the solution. As an efficient way, budget constraints can be included in the uncertainty set formulation to give a much tighter and better description of the uncertainty set by a set of linear/nonlinear constraints.

In this paper, the set of uncertain load demand can be described by:

$$\underline{d}_t \le d_t \le \bar{d}_t \quad t=1,2,\ldots,T \tag{1}$$

$$\sum_{t=1}^{T} a_{i,t} d_t \le \alpha_i \quad i=1,2,\ldots,I \tag{2}$$

Formula (1) determines a box set, and (2) corresponds to the linear budget constraints on load demand.

Based on the budget constraint and the box uncertainty set, a new set containing all historical points of load demand can be constructed. And the new set can be controlled by adjusting the budget constraint coefficients $a_{i,t}$ and α_i.

Similarly, uncertainty set of renewable power outputs can be described as:

$$\underline{r}_t \le r_t \le \bar{r}_t \quad t=1,2,\ldots,T \tag{3}$$

$$\sum_{t=1}^{T} b_{j,t} r_t \le \beta_j \quad j=1,2,\ldots,J \tag{4}$$

Formula (3) determines a box set and (4) is the budget constraint. And $b_{j,t}$ and β_j can be obtained by the data-driven method based on historical data [32], [33].

To simplify the presentation, load demand and renewable power outputs are aggregated as net load $\tilde{d}_t = d_t - r_t$. And a proposition is given here.

Proposition 1: all possible net loads can be represented by convex combinations of a finite number of vectors and therefore the set can also be determined by a group of linear inequalities based on basic linear algebra.

The proof of proposition 1 is given in the Appendix A.

Then, the uncertainty set of net load can also be formulated as:

$$\underline{\tilde{d}}_t \le \tilde{d}_t \le \bar{\tilde{d}}_t \quad t=1,2,\ldots,T \tag{5}$$

$$\sum_{t=1}^{T} c_{m,t} \tilde{d}_t \le q_m \quad m=1,2,\ldots,M \tag{6}$$

where $\bar{\tilde{d}}_t$ and $\underline{\tilde{d}}_t$ are set as $\bar{d}_t - \underline{r}_t$ and $\underline{d}_t - \bar{r}_t$, respectively.

The uncertainty set of the net load is denoted as:

$$\Omega = \{\tilde{\boldsymbol{d}} \in \mathbf{R}^{\mathrm{T}} | (5), (6)\} \tag{7}$$

An important concept is the conditional uncertainty set. In time period t, when $\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_t$ are known, the budget constraints imply that a more accurate range of $\tilde{d}_{t+1}, \tilde{d}_{t+2}, \ldots, \tilde{d}_T$ can be obtained as:

$$\Omega_t(\tilde{\boldsymbol{d}}_{[t]}) = \{(\tilde{d}_{t+1}, \tilde{d}_{t+2}, \ldots, \tilde{d}_T) | \tilde{\boldsymbol{d}} \in \Omega\} \tag{8}$$

For simplicity, $\Omega_t(\tilde{\boldsymbol{d}}_{[t]})$ is abbreviated as Ω_t in the latter part of this paper. If the decisions at time period t are made based on this more accurate range of uncertainties, the economic performance of the decisions will be better and this is why the concept is introduced.

B. Model of Scheduling Problem

C-MGs can enhance the reliability and economics of the community power supply. As shown in Fig. 1, the typical structure of a C-MG often includes uncertain power generations, uncertain load demands, and an ESS. Generally, C-MGs are grid-connected and can have power exchange with the main grid.

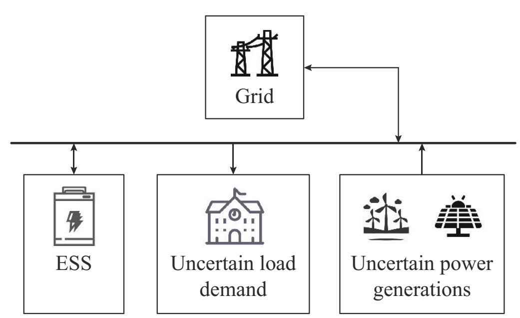

Fig. 1. Typical structure of a C-MG.

The scheduling of C-MGs with ESS under uncertainties is a multi-stage decision problem involving the decision variables p_t (charging/discharging power) and g_t (transfer power). The sequential decision-making process can be illustrated in Fig. 2. In time period t, p_t and g_t are determined based on the realized decisions p_{t-1} and g_{t-1}, and the observed information $(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_t)$, while $(\tilde{d}_{t+1}, \tilde{d}_{t+2}, \ldots, \tilde{d}_T)$ is unknown at this time.

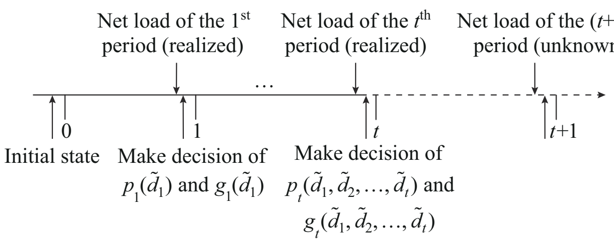

Fig. 2. Sequential decision-making process.

Consequently, decisions in time period t must be formulated as functions with respect to the unfolded uncertainties (net load) up to time period t, as shown in (9), and that is nonanticipativity as explained in Section I.

$$\begin{cases} p_t = p_t(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_t) \\ g_t = g_t(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_t) \end{cases} \tag{9}$$

Let $\tilde{\boldsymbol{d}}_{[t]} = (\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_t)$ be the row vector of net load up to time period t, then (9) can be rewritten as:

$$\begin{cases} p_t = p_t(\tilde{\boldsymbol{d}}_{[t]}) \\ g_t = g_t(\tilde{\boldsymbol{d}}_{[t]}) \end{cases} \tag{10}$$

With the above explanations, the formulation of the scheduling problem can be established now. Without losing of any generality, we consider a scheduling problem from time period t_0 to time period T. Time period t_0 is equal to 1 at the beginning and will increase by 1 after each stage is completed.

Conceptually, the complete formulation of the scheduling problem can be given as below.

The objective function of the scheduling problem during $[t_0, T]$ can be expressed as:

$$\min \sum_{t=t_0}^{T} \tau(\lambda_t^{\text{buy}} \max\{g_t(\tilde{\boldsymbol{d}}_{[t]}), 0\} + \lambda_t^{\text{sell}} \min\{g_t(\tilde{\boldsymbol{d}}_{[t]}), 0\}) \tag{11}$$

The constraints in the scheduling problem are listed as:

$$\underline{E}_t \le E_t(\tilde{\boldsymbol{d}}_{[t]}) \le \bar{E}_t \quad \forall t, \tilde{\boldsymbol{d}} \in \Omega_{t_0} \tag{12}$$

$$\underline{g}_t \le g_t(\tilde{\boldsymbol{d}}_{[t]}) \le \bar{g}_t \quad \forall t, \tilde{\boldsymbol{d}} \in \Omega_{t_0} \tag{13}$$

$$g_t(\tilde{\boldsymbol{d}}_{[t]}) + p_t(\tilde{\boldsymbol{d}}_{[t]}) = \tilde{d}_t \quad \forall t, \tilde{\boldsymbol{d}} \in \Omega_{t_0} \tag{14}$$

$$-\bar{p}_t^{\text{ch}} \le p_t(\tilde{\boldsymbol{d}}_{[t]}) \le \bar{p}_t^{\text{dis}} \quad \forall t, \tilde{\boldsymbol{d}} \in \Omega_{t_0} \tag{15}$$

$$E_t(\tilde{\boldsymbol{d}}_{[t]}) - E_{t-1}(\tilde{\boldsymbol{d}}_{[t-1]}) = \begin{cases} -\tau\eta_c p_t(\tilde{\boldsymbol{d}}_{[t]}) & p_t(\tilde{\boldsymbol{d}}_{[t]}) \le 0, \forall t, \tilde{\boldsymbol{d}} \in \Omega_{t_0} \\ -\dfrac{\tau}{\eta_d} p_t(\tilde{\boldsymbol{d}}_{[t]}) & p_t(\tilde{\boldsymbol{d}}_{[t]}) > 0, \forall t, \tilde{\boldsymbol{d}} \in \Omega_{t_0} \end{cases} \tag{16}$$

The objective function (11) in the scheduling problem is to minimize the total electricity cost under the time-of-use prices. Constraint (12) is the bound limit of energy storage level, and it is usually required that $\underline{E}_T = \bar{E}_T = E_0$ in real operation. Constraint (13) is the power exchange limit [28], which limits the lower and upper bounds of the power exchange between the C-MG and the main grid. Constraint (14) is the power balance constraint and it requires that the algebraic sum of the transfer power and the charging/discharging power must be equal to the uncertain net load. Constraint (15) limits the bounds on discharging and charging power of the ESS. Constraint (16) describes the evolution equation of energy storage level.

Compared with the deterministic model, the variables in (11)-(16) are actually unknown decision functions. In addition, the model in this paper follows the nonanticipativity requirement due to the introduction of (10). Then, we investigate the multi-stage robustness of the problem in the next section.

III. Necessity of Multi-stage Robustness in Scheduling with ESS and Defects of Affine Assumption

A. Necessity of Multi-stage Robustness

In Fig. 2, decisions p_t and g_t at time period t should guarantee that there will always be feasible solutions p_s and g_s for any realization of $\tilde{d}_s$ $(s = t+1, t+2, \ldots, T)$ within the uncertainty set, as mentioned in the multi-stage robustness definition. Consequently, scheduling C-MGs with ESS is to obtain a group of feasible/optimal functions defined in (10) such that the constraints in (12)-(16) are satisfied.

Multi-stage robustness is very important in scheduling problems with ESS [34]. To show the necessity of multi-stage robustness, a 3-period case is given here, where the initial state, upper and lower bounds of the energy storage level are 6 MWh, 8 MWh, and 4 MWh, respectively. And other parameters are given in Table I.

TABLE I
Parameters of C-MGs

T (hour)	τ (hour)	$[\underline{g}, \bar{g}]$ (MW)	$\bar{p}^{\text{dis}}$ (MW)	$\bar{p}^{\text{ch}}$ (MW)	η_d (%)	η_c (%)
3	1	[3.2, 3.5]	1	2.2	80	80

The uncertainty sets of net load at each time period are [2.1, 3.1]MW, [2.8, 4.5]MW, and [2.2625, 4.3]MW, respectively. The solution regions with and without nonanticipativity and multi-stage robustness are depicted in Fig. 3. It should be noted that only the feasible regions of the energy storage levels are shown in Fig. 3. The blue region corresponds to the solutions without the nonanticipativity and multi-stage robustness, i. e., where the net load information of all time periods is known before making a decision. For example, $\boldsymbol{p} = [p_1, p_2, p_3] = [-0.1, -0.4, -0.9]$MW and $\boldsymbol{g} = [g_1, g_2, g_3] = [3.2, 3.2, 3.2]$MW are the optimal solutions for the scenario $\tilde{\boldsymbol{d}} = [\tilde{d}_1, \tilde{d}_2, \tilde{d}_3] = [3.1, 2.8, 2.3]$MW.

However, only the observed $\tilde{d}_1$ is available for g_1 and p_1 at $t=1$, while $\tilde{d}_2$ and $\tilde{d}_3$ are unknown. Consequently, if g_1 and p_1 are decided under the above presumption ($g_1 = 3.2$

MW, $p_1=-0.1$ MW), there will be no feasible solutions for $\tilde{d}_2=4.5$ MW and $\tilde{d}_3=4.3$ MW.

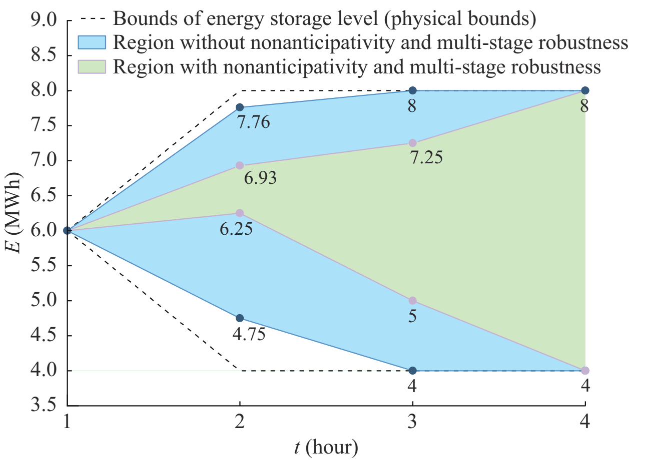

Fig. 3. Solution regions with and without nonanticipativity and multi-stage robustness.

The green region corresponds to the feasible solutions with the nonanticipativity and multi-stage robustness. If the decisions are within this region, any realization within the uncertainty set can be handled. For example, the optimal solution in the green region is $p_1=-0.3125$ MW, $g_1=3.4125$ MW for $\tilde{d}_1=3.1$ MW and there will always be a feasible solution for any $\tilde{d}_2\in[2.8, 4.5]$MW and $\tilde{d}_3\in[2.2625, 4.3]$MW. Consequently, the green region can be applied in the actual operations.

B. Effects of Uncertainty Set and Limitations of Affine Assumption

It is very challenging to guarantee the nonanticipativity and robustness of the solutions in the scheduling problem with ESS under polyhedral uncertainties because the optimal functions or the feasible regions of g_t and p_t are hard to be formulated. The inter-time coupling of uncertainties and the evolution equation of energy storage levels are among the main reasons for this difficulty.

To deal with the multi-stage scheduling problem, ADR-based MSRO method is regarded as an available way. However, some studies have pointed out that the solution region will be reduced under the affine assumption [8]. A general example is constructed in this subsection to illustrate the performance and the limitations of ADR.

In ADR-based MSRO, the unknown decision functions are assumed to be affine. For example, we can assume that

$$E_t-E_{t-1}=A_t\tilde{d}_t+B_t \quad \forall t \tag{17}$$

Equation (17) is a simplified version of ADR usually adopted in literature, i. e., the decision is an affine function with respect to $\tilde{d}_t$ only [25]. Based on (17), the way to solve the original scheduling problem is to find a group of affine coefficients A_t and B_t with the minimal total electricity cost.

Now consider an example where $E_0=6$ MWh, $[\underline{E}_1, \bar{E}_1]=[3.75, 7.74]$MWh, and $[\underline{E}_2, \bar{E}_2]=[2.5, 9.5]$MWh. Other parameters are the same as those given in Table I. There are two time periods in this example, and two uncertainty sets of the net load are given as:

$$\Omega^{\text{box}}=\{(\tilde{d}_1, \tilde{d}_2)|\tilde{d}_1=3.5, 0.5\le\tilde{d}_2\le 6.5\} \tag{18}$$

$$\Omega^{\text{poly}}=\{(\tilde{d}_1, \tilde{d}_2)|\tilde{d}_1=3.5, 0.5\le\tilde{d}_2\le 6.5, 4.5\le\tilde{d}_1+\tilde{d}_2\le 8\} \tag{19}$$

Though the two sets are given in two time periods, they are in fact interval (one-dimensional) uncertainty sets since the net load in time period 1 is constant. However, these two simple uncertainty sets are enough to show some important features of the existing methods and to investigate the performance of ADR.

For the box uncertainty set Ω^{box}, a possible scenario is $(\tilde{d}_1, \tilde{d}_2)=(3.5, 6.5)$MW. Total electricity energy demand of this scenario is obtained as $3.5\text{ MW}\times 1\text{ hour}+6.5\text{ MW}\times 1\text{ hour}=10\text{ MWh}$. The maximum energy bought from the main grid is $2\bar{g}\tau=7$ MWh. The maximum energy can be obtained by discharging of the ESS, which is $(E_0-\underline{E}_2)\eta_d=2.8$ MWh. It holds that $7+2.8=9.8<10$ and this means no feasible solution can be found under the box uncertainty set case.

For the polyhedral uncertainty set Ω^{poly}, it will be shown in Section VI that feasible solutions can be found by the method proposed in this paper. However, no feasible solution can be found by ADR and this will be explained below.

Consider the problem in time period 2. We analyze the exact lower/upper bounds in which E_2-E_1 must lie to keep the decision feasible. The analysis is based on (12)-(16).

$$(12)\Leftrightarrow \underline{E}_2-E_1\le E_2-E_1\le\bar{E}_2-E_1 \tag{20}$$

$$(13)\Leftrightarrow \underline{g}\le g_2\le\bar{g} \tag{21}$$

$$(14)\Leftrightarrow g_2+p_2=\tilde{d}_2 \tag{22}$$

$$(15), (16)\Leftrightarrow -\tau\bar{p}^{\text{dis}}/\eta_d\le E_2-E_1\le\tau\eta_c\bar{p}^{\text{ch}} \tag{23}$$

Substitute (22) into (21), then we can obtain that

$$\underline{g}\le\tilde{d}_2-p_2\le\bar{g}\Leftrightarrow\tilde{d}_2-\bar{g}\le p_2\le\tilde{d}_2-\underline{g} \tag{24}$$

The energy balance equation of the ESS is as follows.

$$E_2-E_1=\begin{cases}-\tau\eta_c p_2 & p_2\le 0\\ -\tau\dfrac{p_2}{\eta_d} & p_2>0\end{cases} \tag{25}$$

Equation (25) implies that E_2-E_1 is uniquely determined by p_2 and vice versa. Based on (24) and (25) we can obtain:

$$\begin{cases}-\tau(\tilde{d}_2-\underline{g})/\eta_d\le E_2-E_1\le-\tau(\tilde{d}_2-\bar{g})/\eta_d & \tilde{d}_2\ge\bar{g}\\ -\tau\eta_c(\tilde{d}_2-\underline{g})\le E_2-E_1\le-\tau\eta_c(\tilde{d}_2-\bar{g}) & \tilde{d}_2\le\underline{g}\\ -\tau(\tilde{d}_2-\underline{g})/\eta_d\le E_2-E_1\le-\tau\eta_c(\tilde{d}_2-\bar{g}) & \underline{g}<\tilde{d}_2<\bar{g}\end{cases} \tag{26}$$

In fact, (25) means that function E_2-E_1 is positive/negative when p_2 is negative/positive. Therefore, if $\tilde{d}_2\ge\bar{g}$, the ESS must work in discharging state and, in this case, (24) and the second case in (25) together corresponds to the first case in (26). The case of $\tilde{d}_2\le\underline{g}$ in (26) is then obtained by (24) and the first case in (25). The third case in (26) can be obtained similarly.

The above analysis will be generalized in Section IV and it is one of the key steps in the proposed method.

Equations (20), (23), and (26) reveal the complicated relationship between E_2-E_1 and $\tilde{d}_2$. It is depicted in Fig. 4 with the parameters given in Table I.

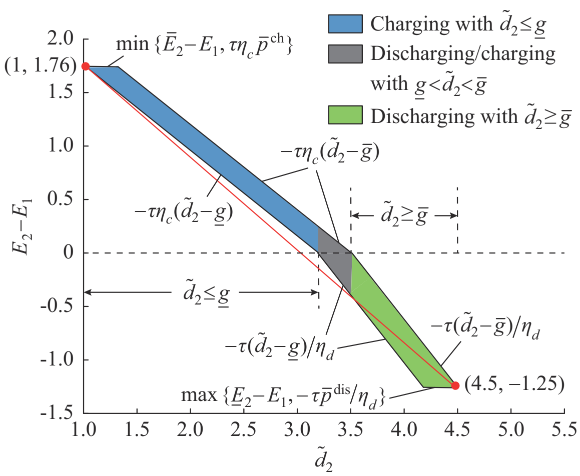

Fig. 4. Relationship between $E_2 - E_1$ and $\tilde{d}_2$.

As observed from Fig. 4, no feasible solutions can be found by ADR for this example. In fact, it can be observed that there exists only one feasible $E_2 - E_1$ for $\tilde{d}_2 = 1$ and $\tilde{d}_2 = 4.5$, respectively (the two red points). Then, the two points must be on the line determined by any feasible ADR. The line is therefore unique, but some points of the line are not in the feasible region, which means that ADR fails for this example. In Section VI, we will show that feasible solutions for this example can be found by the proposed method.

IV. Robust Feasible Regions for Solving Scheduling Problems of C-MGs with ESS

In a completely different way, this paper explores an implicit method that defines specific robust feasible regions for the decision variables rather than restricts the explicit relationship between uncertainties and decision variables like ADR-based methods. To this end, an auxiliary function is introduced first.

A. An Auxiliary Function and Formulation Transformation

Equation (16) can be simplified by introducing the following auxiliary function [29].

$$h(x) = -\frac{\tau}{\eta_d}\max\{x, 0\} - \tau\eta_c \min\{x, 0\} \tag{27}$$

$$h^{-1}(y) = -\frac{\eta_d}{\tau}\min\{y, 0\} - \frac{1}{\eta_c \tau}\max\{y, 0\} \tag{28}$$

It can be observed from Fig. 5 that auxiliary function $h(x)$ and its inverse $h^{-1}(y)$ both are monotonous decreasing. Then, (16) can be replaced by:

$$E_t - E_{t-1} = h(p_t) \tag{29}$$

Based on (14), (15), and (29), we can obtain:

$$p_t = h^{-1}(E_t - E_{t-1}) \tag{30}$$

$$g_t = \tilde{d}_t - h^{-1}(E_t - E_{t-1}) \tag{31}$$

$$-\bar{p}_t^{\mathrm{ch}} \leq h^{-1}(E_t - E_{t-1}) \leq \bar{p}_t^{\mathrm{dis}} \tag{32}$$

Based on (31), (13) is thus transformed into:

$$\underline{g}_t \leq \tilde{d}_t - h^{-1}(E_t - E_{t-1}) \leq \bar{g}_t \tag{33}$$

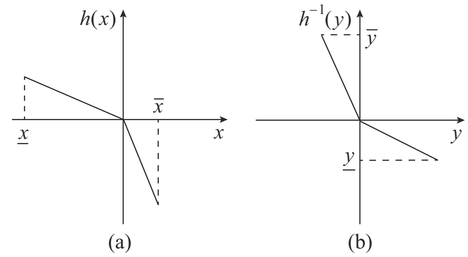

Fig. 5. Auxiliary function and its inverse. (a) Auxiliary function. (b) Inverse of auxiliary function.

Equation (11) can also be rewritten based on (31). The original formulation is finally transformed into (11), (12), (32), and (33), which are closely related to the variable E_t. In other words, we can use E_t as the only variable and all other variables are eliminated.

B. Robust Feasible Regions in One Time Period

A proposition is given below to determine the feasible regions of the decision variables in each time period.

Proposition 2: in one time period, if (34) is satisfied and $[\underline{E}_{t-1}^{\mathrm{true}}, \bar{E}_{t-1}^{\mathrm{true}}]$ is non-empty, we can always find a feasible solution for E_t if and only if $E_{t-1} \in [\underline{E}_{t-1}^{\mathrm{true}}, \bar{E}_{t-1}^{\mathrm{true}}]$.

$$\begin{cases} h(\bar{\tilde{d}}_t - \bar{g}_t) \geq h(\bar{p}_t^{\mathrm{dis}}) \\ h(-\bar{p}_t^{\mathrm{ch}}) \geq h(\underline{\tilde{d}}_t - \underline{g}_t) \end{cases} \tag{34}$$

$$\begin{cases} \underline{E}_{t-1}^{\mathrm{true}} = \max\limits_{\tilde{d}_t}\{\underline{E}_t^{\mathrm{true}} - \bar{f}_t(\tilde{d}_t), \underline{E}_{t-1}\} \\ \bar{E}_{t-1}^{\mathrm{true}} = \min\limits_{\tilde{d}_t}\{\bar{E}_t^{\mathrm{true}} - \underline{f}_t(\tilde{d}_t), \bar{E}_{t-1}\} \end{cases} \tag{35}$$

Proof: as has been explained by (10), the decisions are functions with respect to the uncertainties. Then, after the formulation transformation, the decision rule should be in the form as follows:

$$E_t - E_{t-1} = f_t(\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_t) \tag{36}$$

Moreover, (12), (32), and (33) are in fact all bound limits and thus they can be aggregated to give a more compact bound limit formulation.

$$(12) \Leftrightarrow \underline{E}_t - E_{t-1} \leq E_t - E_{t-1} \leq \bar{E}_t - E_{t-1} \tag{37}$$

$$(32) \Leftrightarrow h(\bar{p}_t^{\mathrm{dis}}) \leq E_t - E_{t-1} \leq h(-\bar{p}_t^{\mathrm{ch}}) \tag{38}$$

$$(33) \Leftrightarrow h(\tilde{d}_t - \underline{g}_t) \leq E_t - E_{t-1} \leq h(\tilde{d}_t - \bar{g}_t) \tag{39}$$

In time period t, the decision on E_{t-1} has been put into practice and cannot be changed. We must determine E_t based on the realization information of $\tilde{d}_t$. Formulas (37)-(39) imply that if the decision on E_{t-1} is inappropriate, then no feasible E_t can be found. Therefore, an important problem is, in time period t, in order to guarantee that there are feasible decisions on E_t in its lower/upper bounds, how to determine the maximum permissible range of E_{t-1}. This problem is also very interesting since though the decision on E_t is made after the decision on E_{t-1}, the permissible range of E_{t-1} must be determined based on the permissible range of E_t. The maximum permissible range of E_{t-1} can be easily obtained based on (37)-(39).

To simplify the presentation, two functions are defined as:

$$\bar{f}_t(\tilde{d}_t)=\min\{h(-\bar{p}_t^{\mathrm{ch}}),h(\tilde{d}_t-\bar{g}_t)\} \tag{40}$$

$$\underline{f}_t(\tilde{d}_t)=\max\{h(\bar{p}_t^{\mathrm{dis}}),h(\tilde{d}_t-\underline{g}_t)\} \tag{41}$$

Equations (40) and (41) are related to the right-/left-hand sides of (38) and (39). Then, (38) and (39) can be replaced by:

$$\underline{f}_t(\tilde{d}_t)\le E_t-E_{t-1}\le\bar{f}_t(\tilde{d}_t) \tag{42}$$

It should be noted that, based on (42), there are feasible decisions on E_t only when $\underline{f}_t(\tilde{d}_t)\le\bar{f}_t(\tilde{d}_t)$ holds. This is a necessary condition and will be used later.

Formulas (37) and (42) are both interval constraints on E_t-E_{t-1}. There are feasible decisions if and only if the intersection of the two intervals is nonempty, i.e.,

$$\max\{\underline{f}_t(\tilde{d}_t),\underline{E}_t-E_{t-1}\}\le\min\{\bar{f}_t(\tilde{d}_t),\bar{E}_t-E_{t-1}\} \tag{43}$$

When $\underline{f}_t(\tilde{d}_t)\le\bar{f}_t(\tilde{d}_t)$ is satisfied, (43) holds if and only if

$$\begin{cases}\underline{E}_t-E_{t-1}\le\bar{f}_t(\tilde{d}_t)\\ \underline{f}_t(\tilde{d}_t)\le\bar{E}_t-E_{t-1}\end{cases} \tag{44}$$

Formula (44) is equivalent to

$$E_{t-1}\in[\underline{E}_t-\bar{f}_t(\tilde{d}_t),\bar{E}_t-\underline{f}_t(\tilde{d}_t)] \tag{45}$$

Therefore, the permissible range of E_{t-1} (to guarantee there are feasible decisions on E_t) can be obtained based on (45) and the original lower and upper bounds on E_{t-1} are as follows:

$$\begin{cases}\underline{E}_{t-1}^{\mathrm{true}}=\max\{\underline{E}_t-\bar{f}_t(\tilde{d}_t),\underline{E}_{t-1}\}\\ \bar{E}_{t-1}^{\mathrm{true}}=\min\{\bar{E}_t-\underline{f}_t(\tilde{d}_t),\bar{E}_{t-1}\}\end{cases} \tag{46}$$

Equation (46) is still not the final result for two reasons. Firstly, (46) is an equation related to time periods $t-1$ and t and the same argument still holds for time periods t and $t+1$. Therefore, $\underline{E}_t$ and $\bar{E}_t$ in (46) should be replaced by $\underline{E}_t^{\mathrm{true}}$ and $\bar{E}_t^{\mathrm{true}}$, and thus a recursive equation is established. Secondly, according to the nonanticipativity, $\tilde{d}_t$ is unknown when we make a decision on E_{t-1}, which means that (46) must hold for all possible $\tilde{d}_t$. Therefore, the lower bound $\underline{E}_{t-1}^{\mathrm{true}}$ must be the maximum of all lower bounds and $\bar{E}_{t-1}^{\mathrm{true}}$ must be the minimum of all upper bounds.

$$\begin{cases}\underline{E}_{t-1}^{\mathrm{true}}=\max\max\limits_{\tilde{d}_t}\{\underline{E}_t^{\mathrm{true}}-\bar{f}_t(\tilde{d}_t),\underline{E}_{t-1}\}\\ \bar{E}_{t-1}^{\mathrm{true}}=\min\min\limits_{\tilde{d}_t}\{\bar{E}_t^{\mathrm{true}}-\underline{f}_t(\tilde{d}_t),\bar{E}_{t-1}\}\end{cases} \tag{47}$$

The final recursive equation of the exact lower and upper bounds is then obtained as:

$$\begin{cases}\underline{E}_{t-1}^{\mathrm{true}}=\max\limits_{\tilde{d}_t}\{\underline{E}_t^{\mathrm{true}}-\bar{f}_t(\tilde{d}_t),\underline{E}_{t-1}\}\\ \bar{E}_{t-1}^{\mathrm{true}}=\min\limits_{\tilde{d}_t}\{\bar{E}_t^{\mathrm{true}}-\underline{f}_t(\tilde{d}_t),\bar{E}_{t-1}\}\end{cases} \tag{48}$$

Based on the analysis, (48) gives the exact lower and upper bounds on E_{t-1}. In other words, we can always find a feasible E_t if and only if $E_{t-1}\in[\underline{E}_{t-1}^{\mathrm{true}},\bar{E}_{t-1}^{\mathrm{true}}]$.

Moreover, it has been pointed out (see the discussion after (42)) that $\underline{f}_t(\tilde{d}_t)\le\bar{f}_t(\tilde{d}_t)$ is a necessary condition for the above analysis. Based on (40) and (41), we know this inequality holds if and only if (49) holds for all $\tilde{d}_t$.

$$\begin{cases}h(\tilde{d}_t-\bar{g}_t)\ge h(\bar{p}_t^{\mathrm{dis}})\\ h(-\bar{p}_t^{\mathrm{ch}})\ge h(\tilde{d}_t-\underline{g}_t)\end{cases} \tag{49}$$

Together with the fact that $h(\cdot)$ is monotonous decreasing, this implies that

$$\begin{cases}h(\bar{\tilde{d}}_t-\bar{g}_t)\ge h(\bar{p}_t^{\mathrm{dis}})\\ h(-\bar{p}_t^{\mathrm{ch}})\ge h(\underline{\tilde{d}}_t-\underline{g}_t)\end{cases}\quad t=1,2,\dots,T \tag{50}$$

The proposition is related to one time period and it will be further extended to the whole time period as follows.

C. Calculation of $\underline{E}_{t_0}^{\mathrm{true}}$ and $\bar{E}_{t_0}^{\mathrm{true}}$

The scheduling problem of C-MGs with ESS is a typical multi-stage process and the decision process has a natural rolling horizon framework. Consequently, in each current time period t_0, only $\underline{E}_{t_0}^{\mathrm{true}}$ and $\bar{E}_{t_0}^{\mathrm{true}}$ are needed. The details of the method will be given in the next section and here we only explain the method for calculating $\underline{E}_{t_0}^{\mathrm{true}}$ and $\bar{E}_{t_0}^{\mathrm{true}}$.

For $t=T$, it is clear that $\underline{E}_T^{\mathrm{true}}=\underline{E}_T$ and $\bar{E}_T^{\mathrm{true}}=\bar{E}_T$. For $t=T-1$, based on (35) we can obtain:

$$\underline{E}_{T-1}^{\mathrm{true}}=\max_{\tilde{d}_T}\{\underline{E}_T^{\mathrm{true}}-\bar{f}_T(\tilde{d}_T),\underline{E}_{T-1}\}=\max_{\tilde{d}_T}\{\underline{E}_T-\bar{f}_T(\tilde{d}_T),\underline{E}_{T-1}\} \tag{51}$$

Similarly, for $t=T-2$, we can obtain:

$$\begin{aligned}&\underline{E}_{T-2}^{\mathrm{true}}=\max_{\tilde{d}_{T-1}}\{\underline{E}_{T-1}^{\mathrm{true}}-\bar{f}_{T-1}(\tilde{d}_{T-1}),\underline{E}_{T-2}\}=\\ &\max_{\tilde{d}_{T-1}}\left\{\max_{\tilde{d}_T}\{\underline{E}_T-\bar{f}_T(\tilde{d}_T),\underline{E}_{T-1}\}-\bar{f}_{T-1}(\tilde{d}_{T-1}),\underline{E}_{T-2}\}\right\}=\\ &\max_{\tilde{d}_{T-1}}\left\{\max_{\tilde{d}_T}\{\underline{E}_T-\bar{f}_T(\tilde{d}_T)-\bar{f}_{T-1}(\tilde{d}_{T-1}),\underline{E}_{T-1}-\bar{f}_{T-1}(\tilde{d}_{T-1}),\underline{E}_{T-2}\}\right\}=\\ &\max_{\tilde{d}_{T-1},\tilde{d}_T}\{\underline{E}_T-\bar{f}_T(\tilde{d}_T)-\bar{f}_{T-1}(\tilde{d}_{T-1}),\underline{E}_{T-1}-\bar{f}_{T-1}(\tilde{d}_{T-1}),\underline{E}_{T-2}\}\end{aligned} \tag{52}$$

Through derivation, for $j=T-1,T-2,\dots,t_0$, it holds that

$$\underline{E}_j^{\mathrm{true}}=\max_{\tilde{d}_{j+1},\tilde{d}_{j+2},\dots,\tilde{d}_T}\left\{\underline{E}_T-\sum_{i=j+1}^{T}\bar{f}_i(\tilde{d}_i),\dots,\underline{E}_m-\sum_{i=j+1}^{m}\bar{f}_i(\tilde{d}_i),\dots,\underline{E}_j\right\} \tag{53}$$

The optimization problem defined by (53) can be transformed into a group of simpler optimization problems.

$$\underline{E}_j^{\mathrm{true}}=\max_{j\le m\le T}\left\{\max_{\tilde{d}_{j+1},\tilde{d}_{j+2},\dots,\tilde{d}_T}\left\{\underline{E}_m-\sum_{i=j+1}^{m}\bar{f}_i(\tilde{d}_i)\right\}\right\} \tag{54}$$

Equation (54) means that we only need to solve $T-j+1$ problems and, then we can obtain $\underline{E}_j^{\mathrm{true}}$ as the maximum of the $T-j+1$ optimal objective values. The subproblem with index m in (54) can be further transformed as:

$$\max_{(\tilde{d}_{j+1},\tilde{d}_{j+2},\dots,\tilde{d}_T)\in\Omega_j}\left\{\underline{E}_m-\sum_{i=j+1}^{m}\bar{f}_i(\tilde{d}_i)\right\}\Leftrightarrow\min_{(\tilde{d}_{j+1},\tilde{d}_{j+2},\dots,\tilde{d}_T)\in\Omega_j}\sum_{i=j+1}^{m}\bar{f}_i(\tilde{d}_i) \tag{55}$$

According to (27) and (40), $\bar{f}_i(\tilde{d}_i)$ is a piecewise linear and nonconvex function with only one variable. By using the standard and well-known formulation skills in MILP [35], (55) can be transformed into a simple MILP and thus be solved very efficiently by commercial solvers.

The upper bound $\bar{E}_j^{\text{true}}$ can be calculated by using a similar procedure and the details are omitted due to the length limit.

V. Solution Methodology

The scheduling problem has a typical multi-stage decision process structure. In this process, the decisions in time period 1 are made and put into effect immediately when the uncertainties in time period 1 are known (unfold). At this moment, the specific decisions in time period 2 are still not determined (the ADR only gives the function relationship between decisions and the realization of uncertainties, and the specific decisions can only be determined when the uncertainties are realized). Then, in time period 2, the decisions in this time period are made and put into effect and the future decisions are still not determined.

The above analysis suggests that the decision process has a natural rolling horizon framework, i. e., in each current time period t_0, we only need to determine the specific decisions that must be made in this time period. However, nonanticipativity and multi-stage robustness must be guaranteed for future periods. Since the maximum permissible range of the ESS storage level has been obtained by solving (54) and (55) for $j=t_0$, the decisions in time period t_0 can be obtained by solving the problem given by (54).

$$\begin{cases} \min\limits_{E_t, g_t} \sum\limits_{t=t_0}^{T} \tau(\lambda_t^{\text{buy}} \max\{g_t, 0\} + \lambda_t^{\text{sell}} \min\{g_t, 0\}) \\ \text{s.t. } \underline{E}_{t_0}^{\text{true}} \le E_{t_0} \le \bar{E}_{t_0}^{\text{true}} \\ \quad \underline{E}_t \le E_t \le \bar{E}_t \quad \forall t \ge t_0+1 \\ \quad \underline{g}_t \le g_t \le \bar{g}_t \quad \forall t \ge t_0 \\ \quad g_{t_0} = \tilde{d}_{t_0} - h^{-1}(E_{t_0} - E_{t_0-1}) \\ \quad g_t = \tilde{d}_t^{\text{exp}} - h^{-1}(E_t - E_{t-1}) \quad \forall t \ge t_0+1 \\ \quad -\bar{p}_t^{\text{ch}} \le h^{-1}(E_t - E_{t-1}) \le \bar{p}_t^{\text{dis}} \quad \forall t \ge t_0 \end{cases} \tag{56}$$

The following comments on (56) are important and necessary.

1) Formula (56) is a traditional deterministic single-level optimization problem rather than a stochastic/robust programming problem. In other words, no uncertainties are included in (56).

2) The formulation transformation proposed in Section IV is used in (56) and some variables are eliminated (for example, the charging/discharging power of the ESS).

3) The expected uncertain net loads $\tilde{d}_t^{\text{exp}}$ are considered in (56) (for $t \ge t_0+1$) to improve the economic performance of the solution (approximate the expectation of the cost).

4) When the optimal solution to (56) (E_t^* and g_t^*) is obtained, only $E_{t_0}^*$ and $g_{t_0}^*$ are used in time period t_0. For all $t \ge t_0+1$, E_t^* and g_t^* are discarded.

5) Formula (56) will be solved iteratively for $t_0 = 1, 2, \ldots, T$ and the conditional uncertainty set Ω_{t_0} (in solving (54) and (55)) is updated each time when t_0 increases. In this way, the realization information of the uncertainties is fully utilized and therefore the solution obtained is less conservative.

6) Nonanticipativity is naturally guaranteed for decisions on $E_{t_0}^*$ and $g_{t_0}^*$ since no uncertainty realization information in future time periods is used. Multi-stage robustness is also guaranteed since $\underline{E}_{t_0}^{\text{true}} \le E_{t_0}^* \le \bar{E}_{t_0}^{\text{true}}$.

7) The proposed method is different from ADR-based methods. Firstly, the lower/upper bounds (feasible region) of the decision variables given in Section IV are nonlinear functions of the uncertainties. Theoretically, this means the decision rule is not a linear expression. Secondly, it is proved that no linear decision rule based solution exists for the simple example given in Section III. However, for this example, a solution is found by using the proposed method. This means (by numerical testing) that the proposed method is not based on the linear decision rule. Thirdly, it is proved in Section IV that the feasible region given in this paper is the necessary and sufficient condition for a feasible solution. Therefore, any feasible solution obtained based on the linear decision rule must be included in this region. This means the feasible region is enlarged (compared with the linear decision rule based solutions).

VI. Numerical Simulations

The proposed method is implemented with MATLAB R2020b environment using Gurobi 9.0.2.

A. System Parameters

A C-MG scheduling problem with 24 periods (1 hour per period) is tested. The parameters are given in Table II and Table III. The batteries are aggregated as one large battery.

TABLE II
Parameters of C-MG

T (hour)	τ (hour)	$[\underline{E}, \bar{E}]$ (MWh)	E_0 (MWh)	$[\underline{g}, \bar{g}]$ (MW)
24	1	[12.5, 47.5]	30	[15, 28.5]
$\bar{p}^{\text{dis}}$ (MW)	$\bar{p}^{\text{ch}}$ (MW)	η_d (%)	η_c (%)	
8	8	90	90	

TABLE III
Time-of-use Prices

Time period	λ^{buy} (\$/MWh)	λ^{sell} (\$/MWh)
01:00-07:00	50.8	21.7
08:00-12:00, 18:00-22:00	181.6	173.3
13:00-17:00, 23:00-24:00	109.0	86.6

The uncertainty set of the net load is a polyhedral uncertainty set as defined by:

$$(\tilde{d}_t^{\text{exp}} - \tilde{d}_{t-1}^{\text{exp}}) - \varepsilon \le \tilde{d}_t - \tilde{d}_{t-1} \le (\tilde{d}_t^{\text{exp}} - \tilde{d}_{t-1}^{\text{exp}}) + \varepsilon \tag{57}$$

$$\underline{\tilde{d}}_t \le \tilde{d}_t \le \bar{\tilde{d}}_t \tag{58}$$

Formula (57) is the linear budget constraint as suggested in [36]. The positive parameter ε denotes the tightness of the budget constraint and a smaller ε means the budget constraint is tighter.

The lower and upper bounds on the net loads are given in (58). The lower and upper bounds and the expected net load are also shown in Fig. 6.

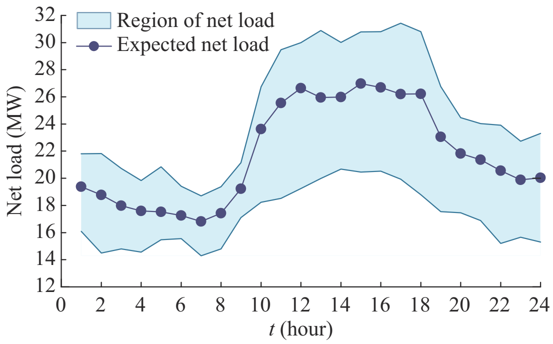

Fig. 6. Lower and upper bounds on net loads and expected net load.

B. Solutions in Expected Net Load Scenario

The solutions in the expected net load scenario are depicted in Fig. 7, which shows the performance of the proposed method. Based on Fig. 7 and Table II, it can be observed that the power exchange level g_t, energy storage level E_t, and charging/discharging power level p_t are all within their bound limits. Therefore, (12), (13), and (15) are satisfied. Equation (14) is also satisfied as shown in Fig. 7 (the value of $\tilde{d}_t$ coincides with the sum of p_t and g_t). The purple bars and the curve of E_t in Fig. 7 suggest that (16) is satisfied.

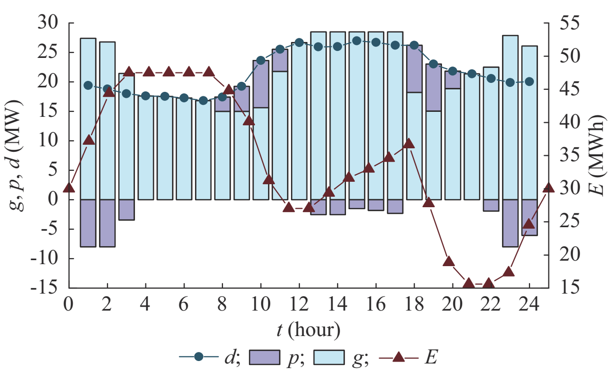

Fig. 7. Solutions in expected net load scenario.

Meanwhile, it can be observed that the battery is charged in time periods of 1-5, 23-24, and 13-17 hours and is discharged in time periods of 8-12 and 18-21 hours. The charging/discharging decisions are reasonable according to the price levels given in Table III.

C. Effects of Uncertainty Sets

The uncertainty set has an important influence on the solution of the scheduling problem and the influence is analyzed in this subsection. Solutions under different uncertainty sets are shown in Table IV.

In Table IV, the average cost of 500 scenarios generated by Monte Carlo simulation is calculated. It is observed that the cost increases when ε increases and this is reasonable since a larger ε means a larger uncertainty set.

TABLE IV
SOLUTIONS UNDER DIFFERENT UNCERTAINTY SETS

Uncertainty set	Tightness (MW)	Cost in expected net load scenario ($)	Average cost ($)
Polyhedral uncertainty set with linear budget constraint	0.1	63138	63647
	1.0	63277	63763
	10.0	63563	63933
Box uncertainty set		63524	63946

The box uncertainty set in Table IV means only (58) is adopted to define the uncertainty set ((57) is omitted). In this case, the uncertainty set is still large and is nearly equal to the case of $\varepsilon = 10$ MW.

To further illustrate the influence of the uncertainty set, the feasible regions of energy storage levels under different uncertainty sets are shown in Fig. 8. As shown in Fig. 8, the purple shade region is a subset of the blue region, which means the more exact the uncertainty set, the larger the feasible region. Thus, the solution under polyhedral uncertainty set with $\varepsilon = 1$ MW will be less conservative than that under box uncertainty set. It suggests that more efforts should be made to establish a more exact uncertainty set.

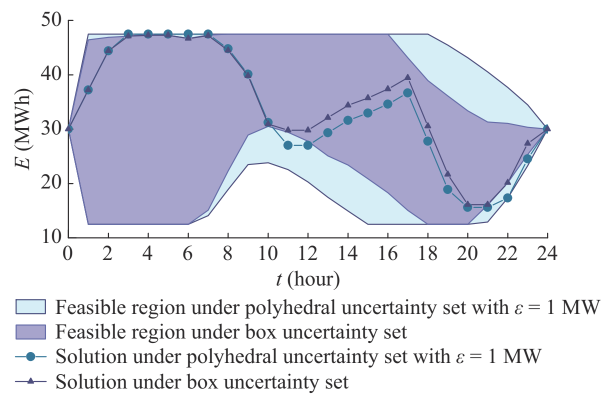

Fig. 8. Feasible regions and solutions of energy storage levels under different uncertainty sets.

In addition, it can be observed from Fig. 8 that though the solution profiles of the energy storage levels under the two uncertainty sets are similar, the charging (discharging) power levels in time periods of 11-17 hours under the polyhedral uncertainty set with $\varepsilon = 1$ MW are much larger than those under the box uncertainty set. According to the time-of-use electricity prices in these time periods, a more economic scheduling solution is obtained under the polyhedral uncertainty set with $\varepsilon = 1$ MW.

D. Comparison with ADR-based MSRO Method

The proposed method is also compared with the ADR-based MSRO method. It is noted that the objective of ADR-based MSRO method is settled to minimize the operation cost in the expected net load scenario so as to make the comparison fair.

The solutions obtained by the ADR-based MSRO method and the proposed method are shown in Fig. 9. It can be observed that both the solutions imply that the battery is charged in low price periods and is discharged in high price periods. However, the optimal cost obtained by the proposed method ($63277) is smaller than that obtained by the ADR-based MSRO method ($63892). This is because the feasible region is reduced by the affine assumption.

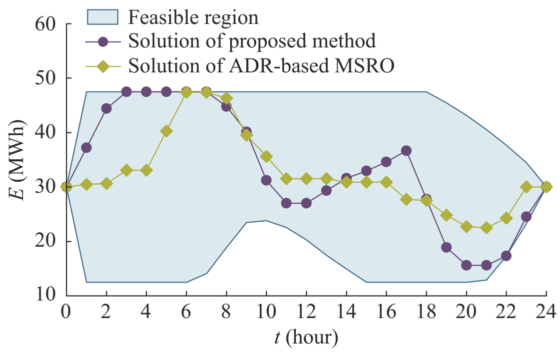

Fig. 9. Performance comparison of ADR-based MSRO method and proposed method.

Except for the economic performance of the solutions, it is also found that in numerical testing the proposed method successfully finds the optimal solutions when the ADR-based MSRO method fails in some cases.

Again, consider the example given in Section III. It has been pointed out that no feasible solution can be found by the ADR-based MSRO method for this example. However, it can be found that (59) is the feasible solutions with the guaranteed nonanticipativity and multi-stage robustness.

$$\begin{cases} E_1 = 6 \text{ MWh} \\ p_1 = 0 \text{ MW} \\ g_1 = 3.5 \text{ MW} \\ E_2 \le 6 + \min\{1.76, 2.8 - 0.8\tilde{d}_2, 4.375 - 1.25\tilde{d}_2\} \text{MWh} \\ E_2 \ge 6 + \max\{-1.25, 2.56 - 0.8\tilde{d}_2, 4 - 1.25\tilde{d}_2\} \text{MWh} \end{cases} \tag{59}$$

All other variables such as p_2 and g_2 can be obtained by (30) and (31) if the specific value of E_2 is determined. A real scale example is also found and the main parameters of this example are given in Table V. The performance of the proposed method in this real scale example is shown in Fig. 10 and the optimal cost is $64147. However, there is still no feasible solution in the ADR-based MSRO method in this example.

TABLE V
PARAMETERS OF A REAL SCALE EXAMPLE

T (hour)	τ (hour)	$[\underline{E}, \bar{E}]$ (MWh)	E_0 (MWh)	$[\underline{g}, \bar{g}]$ (MW)
24	1	[25, 95]	60	[17, 29]
$\bar{p}^{\text{dis}}$ (MW)	$\bar{p}^{\text{ch}}$ (MW)	η_d (%)	η_c (%)	
8	2	90	90	

It is concluded that the proposed method performs well in finding feasible solutions for cases where the ADR-based MSRO method fails. Meanwhile, when both these two methods succeed, the proposed method can get a better solution.

VII. Conclusion

Nonanticipativity and multi-stage robustness must be guaranteed in the scheduling problem of C-MGs with ESS.

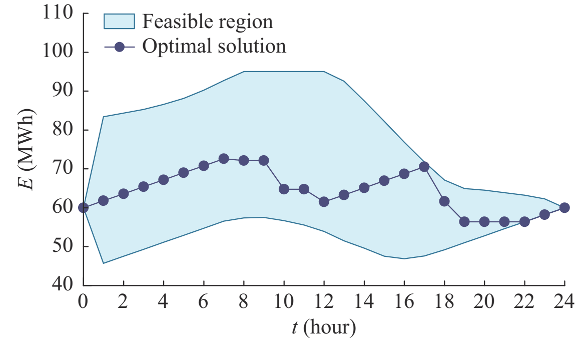

Fig. 10. Feasible region and optimal solutions in a real scale example.

The maximum permissible range of the energy storage levels in each time period is obtained based on analysis of the constraints' structure of the scheduling problem (particularly the energy storage level evolution equation), which is a necessary and sufficient condition for guaranteeing nonanticipativity and multi-stage robustness. That is, if the current decision is within this range, a feasible decision can always be obtained for any future realizations of uncertainty. Based on this condition, an efficient method is established, and the approximated expectation of the total cost is minimized. Real-world scheduling examples are provided to compare the proposed method with the most successful ADR-based MSRO method. The results show that the proposed method can find feasible solutions when the ADR-based MSRO method fails, and it finds better solutions when both methods succeed. Future work will generalize the proposed method and use it for solving the security-constrained unit commitment problem with energy storage.

Appendix A

Equations (1) and (2) or (3) and (4) imply that there exist a finite number of load demand scenarios $\boldsymbol{d}^{(1)}, \boldsymbol{d}^{(2)}, \ldots, \boldsymbol{d}^{(K_1)}$ (the vertices of the convex uncertainty set of load demands) and renewable power output scenarios $\boldsymbol{r}^{(1)}, \boldsymbol{r}^{(2)}, \ldots, \boldsymbol{r}^{(K_2)}$ (the vertices of the convex uncertainty set of renewable power outputs) such that for any possible $\boldsymbol{d}$ and $\boldsymbol{r}$, (A1) and (A2) hold.

$$\boldsymbol{d} = \sum_{k_1=1}^{K_1} \lambda_{k_1} \boldsymbol{d}^{(k_1)} \tag{A1}$$

$$\boldsymbol{r} = \sum_{k_2=1}^{K_2} \lambda_{k_2} \boldsymbol{r}^{(k_2)} \tag{A2}$$

where $\lambda_{k_1} \ge 0$; $\lambda_{k_2} \ge 0$; $\sum_{k_1=1}^{K_1} \lambda_{k_1} = 1$; and $\sum_{k_2=1}^{K_2} \lambda_{k_2} = 1$.

Therefore, $\tilde{\boldsymbol{d}}$ can be rewritten as:

$$\tilde{\boldsymbol{d}} = \boldsymbol{d} - \boldsymbol{r} = \sum_{k_1=1}^{K_1} \lambda_{k_1} \boldsymbol{d}^{(k_1)} - \sum_{k_2=1}^{K_2} \lambda_{k_2} \boldsymbol{r}^{(k_2)} = \sum_{k_1=1}^{K_1} \left(\sum_{k_2=1}^{K_2} \lambda_{k_2} \right) \lambda_{k_1} \boldsymbol{d}^{(k_1)} - \sum_{k_2=1}^{K_2} \left(\sum_{k_1=1}^{K_1} \lambda_{k_1} \right) \lambda_{k_2} \boldsymbol{r}^{(k_2)} = \sum_{k_1=1}^{K_1} \sum_{k_2=1}^{K_2} \lambda_{k_1} \lambda_{k_2} (\boldsymbol{d}^{(k_1)} - \boldsymbol{r}^{(k_2)}) \tag{A3}$$

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