

# Inertia Market: Mechanism Design and Its Impact on Generation Mix

Jingwei Hu, Zheng Yan, Xiaoyuan Xu, and Sijie Chen

**Abstract**—Increasing penetration of renewable energy generation poses a challenge to power system inertia adequacy. It is vital to provide long-term incentive signals to induce a generation mix with adequate inertia supply. However, existing literature rarely studies inertia incentive mechanisms or considers inertia constraints when making generation investment decisions. Thus, we propose an inertia market to quantify the value of inertia and to remunerate inertia provision. To examine the impacts of the inertia market on generation mix, we then propose a stochastic bilevel generation investment equilibrium model that depicts a multi-leader and multi-follower Stackelberg game. The lower level of the model considers the proposed inertia market, along with the energy, reserve, and capacity markets. The upper level considers multiple profit-maximizing strategic producers, and each producer is able to build gas-fired generators, wind generators, and energy storages. Numerical experiments demonstrate that a generation mix with adequate inertia supply can be induced with the proposed inertia market whereas there can be inertia shortage without the inertia market. Interestingly, considering carbon taxes, it is more cost-competitive to invest in wind resources with virtual inertia facilities than to substitute wind resources by thermal generators. Correspondingly, the introduction of an inertia market does not significantly reduce wind generation shares but boosts virtual inertia facility penetration. Our findings imply a future power system powered by fully decarbonized power resources with adequate inertia.

**Index Terms**—Generation mix, generation investment equilibrium, inertia market, Stackelberg game, virtual inertia.

## NOMENCLATURE

### A. Sets

|                                |                                                           |
|--------------------------------|-----------------------------------------------------------|
| $\Phi_n$                       | Set of buses adjacent to bus $n$                          |
| $\Psi_G^S, \Psi_W^S, \Psi_B^S$ | Sets of candidate thermal units, wind units, and storages |

|                                               |                                                                                                                   |
|-----------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| $\Psi_G^E, \Psi_W^E, \Psi_B^E$                | Sets of existing thermal units, wind units, and storages                                                          |
| $\Psi_{G(n)}^S, \Psi_{W(n)}^S, \Psi_{B(n)}^S$ | Sets of candidate thermal units, wind units, and storages located at bus $n$                                      |
| $\Psi_{G(n)}^E, \Psi_{W(n)}^E, \Psi_{B(n)}^E$ | Sets of existing thermal units, wind units, and storages located at bus $n$                                       |
| $\Psi_N$                                      | Set of buses                                                                                                      |
| $\Xi^U$                                       | Set of upper-level variables                                                                                      |
| $\Xi^{\text{EnR}}, \Xi^C, \Xi^H$              | Sets of lower-level variables, including those in energy and reserve markets, capacity market, and inertia market |
| $\Omega_y$                                    | Set of generators of producer $y$                                                                                 |
| $I$                                           | Set of capacity demand curve segments                                                                             |
| $N$                                           | Set of nodes                                                                                                      |
| $S$                                           | Set of stochastic scenarios                                                                                       |
| $T$                                           | Set of time intervals                                                                                             |

### B. Parameters

|                                              |                                                                                                   |
|----------------------------------------------|---------------------------------------------------------------------------------------------------|
| $\gamma_w^{\text{SW}}, \gamma_b^{\text{SB}}$ | Ratios of firm power to rated power of candidate wind unit $w$ and storage $b$ in capacity market |
| $\gamma_w^{\text{EW}}, \gamma_b^{\text{EB}}$ | Ratios of firm power to rated power of existing wind unit $w$ and storage $b$ in capacity market  |
| $\Delta\tau$                                 | Time duration of each interval (hour)                                                             |
| $\eta_b^{\text{Sc}}, \eta_b^{\text{Sd}}$     | Charging and discharging efficiencies of candidate storage $b$                                    |
| $\eta_b^{\text{Ec}}, \eta_b^{\text{Ed}}$     | Charging and discharging efficiencies of existing storage $b$                                     |
| $\pi_s$                                      | Probability of scenario $s$                                                                       |
| $\rho$                                       | Power-to-energy ratio of storage (hour)                                                           |
| $\sigma_t$                                   | Number of days in a year represented by time $t$                                                  |
| $B_{nm}$                                     | Susceptance of transmission line $(n,m)$ (p.u.)                                                   |
| $B_i^{\text{Cap}}$                           | Bid of capacity demand for segment $i$ (\$/(MW·year <sup>-1</sup> ))                              |
| $C_g^{\text{SG}}, C_w^{\text{SW}}$           | Marginal operation costs of candidate thermal unit $g$ and wind unit $w$ (\$/MWh)                 |
| $C_g^{\text{EG}}, C_w^{\text{EW}}$           | Marginal operation costs of existing thermal unit $g$ and wind unit $w$ (\$/MWh)                  |
| $C_b^{\text{Sd}}, C_b^{\text{Sc}}$           | Marginal discharging and charging costs of candidate storage $b$ (\$/MWh)                         |

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|                                                           |                                                                                                                              |                                                                       |                                                                                                                                              |
|-----------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| $C_b^{\text{Ed}}, C_b^{\text{Ec}}$                        | Marginal discharging and charging costs of existing storage $b$ (\$/MWh)                                                     | $e_{tbs}^S, e_{tbs}^E$                                                | States of charges of candidate and existing storage $b$ at time $t$ for scenario $s$ (MWh)                                                   |
| $C_g^{\text{SGR}}, C_b^{\text{SBR}}$                      | Offers of candidate thermal unit $g$ and storage $b$ for reserve market (\$/MWh)                                             | $f_s^E, f_s^R$                                                        | Profits from energy and reserve markets for scenario $s$ (\$)                                                                                |
| $C_g^{\text{EGR}}, C_b^{\text{EBR}}$                      | Offers of existing thermal unit $g$ and storage $b$ for reserve market (\$/MWh)                                              | $f^C, f^H$                                                            | Profits from capacity and inertia markets (\$)                                                                                               |
| $D_{ins}$                                                 | Load demand of bus $n$ at time $t$ for scenario $s$ (MW)                                                                     | $h_g^{\text{SG}}, h_w^{\text{SW}}, h_b^{\text{SB}}$                   | Scheduled inertia of candidate thermal unit $g$ , wind unit $w$ , and storage $b$ (s)                                                        |
| $D_i^C$                                                   | The maximum capacity demand for segment $i$ (MW)                                                                             | $h_g^{\text{EG}}, h_w^{\text{EW}}, h_b^{\text{EB}}$                   | Scheduled inertia of existing thermal unit $g$ , wind unit $w$ , and storage $b$ (s)                                                         |
| $e_b^{\text{Emax}}$                                       | Rated capacity of existing storage $b$ (MWh)                                                                                 | $\hat{h}_g^{\text{SG}}, \hat{h}_w^{\text{SW}}, \hat{h}_b^{\text{SB}}$ | Inertia provisions of candidate thermal unit $g$ , wind unit $w$ , and storage $b$ (s)                                                       |
| $F_{nm}^{\text{max}}$                                     | Capacity of transmission line $(n,m)$ (MW)                                                                                   | $\hat{h}_g^{\text{EG}}, \hat{h}_w^{\text{EW}}, \hat{h}_b^{\text{EB}}$ | Inertia provisions of existing thermal unit $g$ , wind unit $w$ , and storage $b$ (s)                                                        |
| $H_g^{\text{SG}}, H_w^{\text{SW}}, H_b^{\text{SB}}$       | Inertia constants of candidate thermal unit $g$ , wind unit $w$ , and storage $b$ (s)                                        | $l_{ins}^{\text{shed}}$                                               | Load shedding of node $n$ at time $t$ for scenario $s$ (MW)                                                                                  |
| $H_g^{\text{EG}}, H_w^{\text{EW}}, H_b^{\text{EB}}$       | Inertia constants of existing thermal unit $g$ , wind unit $w$ , and storage $b$ (s)                                         | $o_{tgs}^{\text{SG}}, o_{tgs}^{\text{EG}}$                            | Electricity offers of candidate and existing thermal unit $g$ at time $t$ for scenario $s$ (\$/MWh)                                          |
| $H^{\text{req}}$                                          | Inertia requirement (s)                                                                                                      | $o_g^{\text{SGCap}}, o_w^{\text{SWCap}}, o_b^{\text{SBCap}}$          | Capacity offers of candidate thermal unit $g$ , wind unit $w$ , and storage $b$ (\$/(MW · year <sup>-1</sup> ))                              |
| $K_g^G, K_w^W, K_b^P$                                     | Annual investment costs of thermal unit $g$ , wind unit $w$ , and storage $b$ (\$/(MW · year <sup>-1</sup> ))                | $o_g^{\text{SGIne}}, o_w^{\text{SWIne}}, o_b^{\text{SBIne}}$          | Inertia offers of candidate thermal unit $g$ , wind unit $w$ , and storage $b$ (\$/(s · year <sup>-1</sup> ))                                |
| $K_b^E$                                                   | Annual investment cost of storage $b$ (\$/(MW · year <sup>-1</sup> ))                                                        | $o_g^{\text{EGCap}}, o_w^{\text{EWCAP}}, o_b^{\text{EBCap}}$          | Capacity offers of existing thermal unit $g$ , wind unit $w$ , and storage $b$ (\$/(MW · year <sup>-1</sup> ))                               |
| $P_g^{\text{Gmax}}, P_w^{\text{Wmax}}, P_b^{\text{Bmax}}$ | Rated power of existing thermal unit $g$ , wind unit $w$ , and storage $b$ (MW)                                              | $o_g^{\text{EGIne}}, o_w^{\text{EWIne}}, o_b^{\text{EBIne}}$          | Inertia offers of existing thermal unit $g$ , wind unit $w$ , and storage $b$ (\$/(s · year <sup>-1</sup> ))                                 |
| $P^E$                                                     | Penalty price for load shedding (\$/MWh)                                                                                     | $p_{tgs}^{\text{SG}}, p_{tws}^{\text{SW}}, p_{tbs}^{\text{Sd}}$       | Scheduled power of candidate thermal unit $g$ , wind unit $w$ , and storage $b$ (discharging and charging) at time $t$ for scenario $s$ (MW) |
| $PF_{tws}^S$                                              | Ratio of scheduled power to rated power of candidate wind unit $w$ at time $t$ for scenario $s$                              | $p_{tbs}^{\text{Sc}}$                                                 | Scheduled power of candidate thermal unit $g$ , wind unit $w$ , and storage $b$ (discharging and charging) at time $t$ for scenario $s$ (MW) |
| $PF_{tws}^E$                                              | Ratio of scheduled power to rated power of existing wind unit $w$ at time $t$ for scenario $s$                               | $p_{tgs}^{\text{EG}}, p_{tws}^{\text{EW}}, p_{tbs}^{\text{Ed}}$       | Scheduled power of existing thermal unit $g$ , wind unit $w$ , and storage $b$ (discharging and charging) at time $t$ for scenario $s$ (MW)  |
| $R_g^{\text{S10}}, R_b^{\text{S10}}$                      | 10-min ramp rates of candidate thermal unit $g$ and storage $b$ (p.u.)                                                       | $p_{tbs}^{\text{Ec}}$                                                 | Scheduled reserves of candidate thermal unit $g$ and storage $b$ at time $t$ for scenario $s$ (MW)                                           |
| $R_g^{\text{E10}}, R_b^{\text{E10}}$                      | 10-min ramp rates of existing thermal unit $g$ and storage $b$ (p.u.)                                                        | $r_{tgs}^{\text{SG}}, r_{tbs}^{\text{SB}}$                            | Scheduled reserves of candidate thermal unit $g$ and storage $b$ at time $t$ for scenario $s$ (MW)                                           |
| $R_t^{\text{req}}$                                        | Reserve requirement at time $t$ (MW)                                                                                         | $r_{tgs}^{\text{EG}}, r_{tbs}^{\text{EB}}$                            | Scheduled reserves of existing thermal unit $g$ and storage $b$ at time $t$ for scenario $s$ (MW)                                            |
| <b>C. Variables</b>                                       |                                                                                                                              | $x_g^G, x_w^W, x_b^B$                                                 | Newly-built power of thermal unit $g$ , wind unit $w$ , and storage $b$ (MW)                                                                 |
| $\theta_{ins}$                                            | Voltage angle of bus $n$ at time $t$ for scenario $s$ (MW)                                                                   |                                                                       |                                                                                                                                              |
| $\lambda_{m,s}^E, \lambda_{m,w,s}^E, \lambda_{m,b,s}^E$   | Electricity prices of node $n$ with thermal unit $g$ , wind unit $w$ , and storage $b$ at time $t$ for scenario $s$ (\$/MWh) |                                                                       |                                                                                                                                              |
| $\lambda_{ts}^R$                                          | Reserve price at time $t$ for scenario $s$ (\$/MWh)                                                                          |                                                                       |                                                                                                                                              |
| $\lambda^C, \lambda^H$                                    | Capacity and inertia prices (\$/(MW · year <sup>-1</sup> ), \$(s · year <sup>-1</sup> ))                                     |                                                                       |                                                                                                                                              |
| $C^I$                                                     | Investment cost of all candidate units (\$)                                                                                  |                                                                       |                                                                                                                                              |
| $c_g^{\text{SG}}, c_w^{\text{SW}}, c_b^{\text{SB}}$       | Scheduled capacities of candidate thermal unit $g$ , wind unit $w$ , and storage $b$ (MW)                                    |                                                                       |                                                                                                                                              |
| $c_g^{\text{EG}}, c_w^{\text{EW}}, c_b^{\text{EB}}$       | Scheduled capacities of existing thermal unit $g$ , wind unit $w$ , and storage $b$ (MW)                                     |                                                                       |                                                                                                                                              |
| $d_i^C$                                                   | Capacity demand for segment $i$ (MW)                                                                                         |                                                                       |                                                                                                                                              |
| $e_b^{\text{Smax}}$                                       | Newly-built capacity of storage $b$ (MWh)                                                                                    |                                                                       |                                                                                                                                              |

## I. INTRODUCTION

A fundamental characteristic of decarbonization in the electricity industry is that rapidly-growing renewable energy resources are replacing traditional thermal power plants. High penetration of renewable generation, inherently featuring uncertainty and low inertia [1], urges the power system to strengthen its flexibility, firm capacity, and inertia provision [2]. Current market designs have incentives for energy [3], [4], reserve [5], [6], and firm capacity [7]–[10], but rarely incentivize and induce adequate inertia provision. The resulting reduced system inertia may cause system frequency problems. Some actual markets have already encountered

problems related to a lack of system inertia [11]. It is urgent to quantify the value of inertia provision and to augment generation investment to guarantee enough inertia provision in the long run [12].

In terms of industrial practice, the only inertia market was run in Australia, but it was canceled later [13]. This is likely because current inertia shortage is not severe and can be solved by building thermal generators and strengthening transmission lines connected with neighbor grids. However, this situation may not hold in future due to the rapidly growing penetration of renewable generation. In terms of academic studies, [14] proposes an inertia market mechanism via a Vickrey-Clarke-Groves payment rule. This mechanism can ensure truthful bidding but may be too complicated to implement. As a result, rare existing markets use Vickrey-Clarke-Groves payment.

Here, we propose an auction-based inertia market and derive the corresponding generation mix via a game-theoretical generation investment model. The main contributions of our work are as follows.

1) This paper proposes an inertia market to remunerate inertia provision and to guarantee long-term inertia adequacy. Inspired by existing capacity markets, we present an auction-based inertia market, where all generation assets receive payments based on the marginal inertia price. The proposed inertia market can be incorporated as a new product in existing electricity markets. In this paper, the energy and reserve markets are cleared jointly and hourly whereas the capacity and inertia markets are cleared annually.

2) To examine the impacts of the proposed inertia market on generation mix, this paper proposes a bilevel stochastic generation investment equilibrium model. The upper-level problem yields investment decisions of each strategic producer aiming to maximize its profit. Multiple generation technologies are equally accessible to each strategic producer, including gas-fired generation, wind generation, and grid-level energy storages. The lower-level problem clears the energy, reserve, capacity, and inertia markets. Table I compares our model with existing generation investment models.

TABLE I  
COMPARISONS OF PROPOSED MODEL AND EXISTING GENERATION INVESTMENT MODELS

| Reference      | Model type                      | Incentive product                                       |
|----------------|---------------------------------|---------------------------------------------------------|
| [15]           | Single-level equilibrium model  | Capacity market                                         |
| [16]           | Centralized least-cost planning | Scarcity pricing and capacity payments                  |
| [17]           | Bilevel equilibrium model       | Energy, reserve, and capacity markets                   |
| [18]           | Bilevel equilibrium model       | Energy and gas markets                                  |
| [19]           | Bilevel equilibrium model       | Energy, reserve, and capacity markets                   |
| [20]           | Single-level equilibrium model  | Energy-only market                                      |
| [21]           | Bilevel equilibrium model       | Energy and capacity payments                            |
| [22]           | Bilevel equilibrium model       | Energy market                                           |
| [23]           | Bilevel equilibrium model       | Energy market (day-ahead market and long-term contract) |
| [24]           | Single-level equilibrium model  | Energy market                                           |
| Proposed model | Bilevel equilibrium model       | Energy, reserve, capacity, and inertia markets          |

The rest of this paper is organized as follows. Section II gives a detailed market design of the proposed inertia market. In Section III, we present the mathematical formulation of the proposed stochastic generation investment equilibrium model. In Section IV, we give the solution techniques. In Section V, we run case studies on a 5-bus system and the IEEE 118-bus system and analyze the impact of the inertia market on the generation mix. We give the conclusions in Section VI.

## II. INERTIA MARKET DESIGN

We propose an auction-based inertia market. The inertia market aims to provide a clear incentive for producers to invest in generators with large inertia or inertia augmentation technologies. On top of this, the market can provide upfront payments to producers and stabilize the volatile incomes of generators. This market mechanism is equally applied to all types of generation technologies. The consumers pay for the inertia provision that keeps the system frequency stable.

In this section, we introduce the inertia market. And then, we give the inertia market clearing algorithm.

### A. Product

We first briefly introduce the concept of inertia and the inertia response period. Inertia is a kind of energy to hinder any changes from current states. As a response to a sudden power imbalance, it reduces the rate-of-change-of-frequency (RoCoF) to prevent frequency from dropping to an out-of-control level before the primary frequency response is activated. In a grid with all generators at a nominal frequency, inertia is usually quantified by normalized inertia constant  $H$  (s) due to ease of calculation [25]–[27]. This inertia constant  $H$  is derived from the rotational inertia, equaling the kinetic energy  $J\omega^2/2$  (Joule). The parameter  $J$  represents the moment of inertia for the spinning mass determined by its mass distribution and axis of rotation ( $\text{kg}\cdot\text{m}^2$ ). The variable  $\omega$  represents the angular velocity of the spinning mass determined by system frequency (rad/s). Since any generator synchronously connected to the grid operates at a nominal frequency, making  $\omega$  constant, inertia can be expressed by inertia constant  $H$  as  $J\omega^2/(2S)$  (s). The parameter  $S$  represents the rated apparent power of a generator (MVA). Physically, the inertia constant  $H$  means the time that a generator could gen-

erate at its rated power using only its stored rotational kinetic energy. Conventional inertia refers to the rotational inertia provided by synchronous machines. It comes from the stored kinetic energy at the synchronous speed in the rotating masses. Renewable generators are decoupled from the grid by electronic devices, thus not being able to directly provide inertia to the grid. With the development of control strategies, the converters of renewable resources can be controlled to contribute to frequency response. By monitoring system frequency deviation, the control system of converters can alter the energy exchange with the grid to mitigate frequency deviation [28]. This type of energy is called virtual inertia. If renewable generators are equipped with virtual inertia facilities, the overall system inertia will consist of traditional inertia and virtual inertia [29], [30]. This paper treats wind farms as ideal inertia sources, analogous to thermal generators. First, from a steady-state view, studies relevant to power system operational problems usually treat traditional thermal units and wind farms with analogous modeling approaches [31]–[36]. Compared with operational problems, power system planning personnel have less information about the real-time operating point or exact control strategies of wind farms [30], [37]. Second, some real cases treat wind farms as ideal inertia sources. For example, Hydro Quebec, a Canadian utility, sets the requirement that wind farms should have the ability to provide an inertia constant of 3.5 s [12].

Inertia response is the period up to a few seconds in response to a sudden power imbalance before primary frequency response, as shown in Fig. 1. During this period, system frequency regulations are not activated, and the frequency dynamics are determined by the overall system inertia.

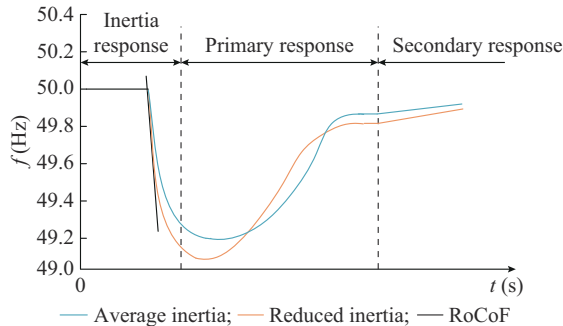


Fig. 1. System frequency response after a contingency.

Inertia, as a product to be auctioned, is quantified by the inertia constant  $H$ . This product has the following features.

1) For each generator, the level of inertia depends on its technical parameter: the inertia constant  $H$ . Once a generator has been built, its inertia constant is fixed and is irrelevant to its dispatched output. Also, once it is connected to the grid, its inertia can be absorbed by the power system.

2) For a power system, the overall system inertia is a sum of the inertia of all generators, weighted by rated power of each machine [37]. Each generator is paid based on its contribution to the overall system inertia.

3) The overall inertia demand is inelastic and is related to the RoCoF requirement of the grid. Usually, large-scale grids can withstand a relatively high RoCoF due to their adequate

flexibility to better conduct primary frequency regulation.

In terms of the inertia contributions from the demand side, this paper ignores them, leading to a conservative generation mix with adequate inertia provision. This is because the inertia provisions from the demand side have various types due to the complexity of load types. It is hard to model them, especially in a planning-scale horizon [27], [37].

This paper ignores PV installations as sources of inertia. Although both wind generation and PV equipped with energy storages can theoretically provide virtual inertia via advanced converter control, we usually only consider wind turbines to provide inertia. This is because wind turbines are rotational masses, resembling rotators of thermal generators, with kinetic energy to be released during frequency deviation. Compared with static masses such as PV, wind generation has more stored energy to contribute to frequency response. Also, besides the provision of virtual inertia, converters of PV are more suitable to provide many other functions to help stabilize the grid, such as voltage control through reactive power. Therefore, providing inertia might not be the best application of converters for PV [2].

### B. Participants

As sellers, different types of generators equally participate in the inertia market. Not only thermal generators with conventional rotating inertia, but also renewable generators and energy storage systems with virtual inertia facilities can participate.

As buyers, the independent system operator (ISO) or load-serving entities act on behalf of electricity consumers to pay for the inertia provision. This payment compensates for the installed capacity of thermal generators and the technical improvement of renewable generators for virtual inertia provision.

The regulator defines the total required system inertia based on a desired RoCoF target. The RoCoF target varies across power systems according to stability criteria and adequacy of flexible resources [25].

### C. Procedure

The procedure is based on a pool-based market where inertia provisions are auctioned. In this way, the inertia market determines the clearing price and allocation of inertia products among different generators. The mechanism is implemented as follows.

1) Initially, the regulator collects some basic parameters for the auction: ① the inertia constant and installed capacity of each generator; ② the minimum RoCoF and corresponding inertia requirement of the grid; and ③ the frequency of the auction, typically on an annual basis.

2) The generators submit a price-quantity pair per bid to the inertia auction. The quantity in the bid is the maximum inertia contribution that the generator can commit. The price represents the minimum fee that the generator is willing to accept for inertia provision.

3) The price of the last accepted bid determines the clearing price in the inertia market, as shown in Fig. 2. This clearing price (\$/s) is the per-unit fee that is paid to all accepted generators.

4) A generator with an accepted bid (s) accepts payment



(\$). All of this procedure applies once in each auction and the payment covers the following whole year.

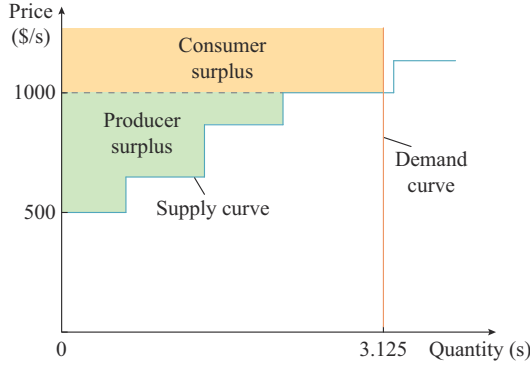


Fig. 2. Illustrative auction procedure and surplus of sellers and buyers.

### III. MATHEMATICAL FORMULATION OF PROPOSED STOCHASTIC GENERATION INVESTMENT EQUILIBRIUM MODEL

In this section, we form a bilevel model for each strategic producer. The upper-level problem maximizes the profit of a producer and determines the optimal investment decisions while anticipating the revenues from multiple markets. The lower-level problems represent the market clearing processes, taking the upper-level investment decisions as given parameters. Each producer has its bilevel model to maximize its profits while sharing the same lower-level constraints.

#### A. Upper-level: Producer's Profit-maximization Problem

##### 1) Objective Function

The objective function (1) aims to minimize the investment cost and maximize the profits from the energy, reserve, capacity, and inertia markets for a producer. We choose several representative scenarios to mimic the variability of renewable generation and demand. The energy and reserve markets are cleared hourly, resulting in an hourly price. The capacity market and inertia market are cleared annually and each of them results in a single annual price.

$$\min_{\substack{\bar{z}^U, \bar{z}^{EnR}, \bar{z}^C, \bar{z}^H}} \left\{ C^I - \sum_s \pi_s (f_s^E + f_s^R) - f^C - f^H \right\} \quad (1)$$

$$C^I = \sum_{g \in \Psi_G^S \cap \Omega_y} K_g^G x_g^G + \sum_{w \in \Psi_W^S \cap \Omega_y} K_w^W x_w^W + \sum_{b \in \Psi_B^S \cap \Omega_y} (K_b^P x_b^B + K_b^E e_b^{Smax}) \quad (2)$$

$$f_s^E = \sum_t \sigma_t \left[ \sum_{g \in \Psi_G^S \cap \Omega_y} p_{tgs}^{SG} (\lambda_{tgs}^E - C_g^{SG}) + \sum_{g \in \Psi_G^E \cap \Omega_y} p_{tgs}^{EG} (\lambda_{tgs}^E - C_g^{EG}) + \sum_{w \in \Psi_W^S \cap \Omega_y} p_{tws}^{SW} (\lambda_{tws}^E - C_w^{SW}) + \sum_{w \in \Psi_W^E \cap \Omega_y} p_{tws}^{EW} (\lambda_{tws}^E - C_w^{EW}) + \sum_{b \in \Psi_B^S \cap \Omega_y} p_{tbs}^{Sd} (\lambda_{tbs}^E - C_b^{Sd}) + \sum_{b \in \Psi_B^E \cap \Omega_y} p_{tbs}^{Ed} (\lambda_{tbs}^E - C_b^{Ed}) - \sum_{b \in \Psi_B^S \cap \Omega_y} p_{tbs}^{Sc} (\lambda_{tbs}^E + C_b^{Sc}) - \sum_{b \in \Psi_B^E \cap \Omega_y} p_{tbs}^{Ec} (\lambda_{tbs}^E + C_b^{Ec}) \right] \quad (3)$$

$$f_s^R = \sum_t \sigma_t \left( \sum_{g \in \Psi_G^S \cap \Omega_y} r_{tgs}^{SG} \lambda_{ts}^R + \sum_{g \in \Psi_G^E \cap \Omega_y} r_{tgs}^{EG} \lambda_{ts}^R + \sum_{b \in \Psi_B^S \cap \Omega_y} r_{tbs}^{SB} \lambda_{ts}^R + \sum_{b \in \Psi_B^E \cap \Omega_y} r_{tbs}^{EB} \lambda_{ts}^R \right) \quad (4)$$

$$f^C = \sum_{g \in \Psi_G^S \cap \Omega_y} c_g^{SG} \lambda^C + \sum_{g \in \Psi_G^E \cap \Omega_y} c_g^{EG} \lambda^C + \sum_{w \in \Psi_W^S \cap \Omega_y} c_w^{SW} \lambda^C + \sum_{w \in \Psi_W^E \cap \Omega_y} c_w^{EW} \lambda^C + \sum_{b \in \Psi_B^S \cap \Omega_y} c_b^{SB} \lambda^C + \sum_{b \in \Psi_B^E \cap \Omega_y} c_b^{EB} \lambda^C \quad (5)$$

$$f^H = \sum_{g \in \Psi_G^S \cap \Omega_y} h_g^{SG} \lambda^H + \sum_{g \in \Psi_G^E \cap \Omega_y} h_g^{EG} \lambda^H + \sum_{w \in \Psi_W^S \cap \Omega_y} h_w^{SW} \lambda^H + \sum_{w \in \Psi_W^E \cap \Omega_y} h_w^{EW} \lambda^H + \sum_{b \in \Psi_B^S \cap \Omega_y} h_b^{SB} \lambda^H + \sum_{b \in \Psi_B^E \cap \Omega_y} h_b^{EB} \lambda^H \quad (6)$$

Equation (2) represents the total investment cost. Note that the capital costs of storages consist of two parts: the cost associated with its rated power and the cost associated with its rated capacity. Equations (3) and (4) represent the producer's profits from the energy and reserve markets, respectively. Equations (5) and (6) represent the profits from the capacity and inertia markets, respectively.

##### 2) Upper-level Constraints

$$x_g^G, x_w^W, x_b^B \geq 0 \quad \forall g \in \Psi_G^S, \forall w \in \Psi_W^S, \forall b \in \Psi_B^S \quad (7)$$

$$e_b^{Smax} = \rho x_b^B \quad \forall b \in \Psi_B^S \quad (8)$$

$$o_{tgs}^{SG}, o_g^{SGCap}, o_w^{SWCap}, o_b^{SBCap}, o_g^{SGIne}, o_w^{SWIne}, o_b^{SBIne} \geq 0 \quad \forall g \in \Psi_G^S, \forall w \in \Psi_W^S, \forall b \in \Psi_B^S, \forall t \in T, \forall s \in S \quad (9)$$

$$o_{tgs}^{EG}, o_g^{EGCap}, o_w^{EWCap}, o_b^{EBCap}, o_g^{EGIne}, o_w^{EWIne}, o_b^{EBIne} \geq 0 \quad \forall g \in \Psi_G^E, \forall w \in \Psi_W^E, \forall b \in \Psi_B^E, \forall t \in T, \forall s \in S \quad (10)$$

$$\hat{h}_g^{SG} = H_g^{SG} x_g^G / \left( \sum_{g \in \Psi_G^S} x_g^G + \sum_{w \in \Psi_W^S} x_w^W + \sum_{b \in \Psi_B^S} x_b^B + \sum_{g \in \Psi_G^E} P_g^{Gmax} + \sum_{w \in \Psi_W^E} P_w^{Wmax} + \sum_{b \in \Psi_B^E} P_b^{Bmax} \right) \quad \forall g \in \Psi_G^S \quad (11)$$

$$\hat{h}_w^{SW} = H_w^{SW} x_w^W / \left( \sum_{g \in \Psi_G^S} x_g^G + \sum_{w \in \Psi_W^S} x_w^W + \sum_{b \in \Psi_B^S} x_b^B + \sum_{g \in \Psi_G^E} P_g^{Gmax} + \sum_{w \in \Psi_W^E} P_w^{Wmax} + \sum_{b \in \Psi_B^E} P_b^{Bmax} \right) \quad \forall w \in \Psi_W^S \quad (12)$$

$$\hat{h}_b^{SB} = H_b^{SB} x_b^B / \left( \sum_{g \in \Psi_G^S} x_g^G + \sum_{w \in \Psi_W^S} x_w^W + \sum_{b \in \Psi_B^S} x_b^B + \sum_{g \in \Psi_G^E} P_g^{Gmax} + \sum_{w \in \Psi_W^E} P_w^{Wmax} + \sum_{b \in \Psi_B^E} P_b^{Bmax} \right) \quad \forall b \in \Psi_B^S \quad (13)$$

$$\hat{h}_g^{EG} = H_g^{EG} P_g^{Gmax} / \left( \sum_{g \in \Psi_G^S} x_g^G + \sum_{w \in \Psi_W^S} x_w^W + \sum_{b \in \Psi_B^S} x_b^B + \sum_{g \in \Psi_G^E} P_g^{Gmax} + \sum_{w \in \Psi_W^E} P_w^{Wmax} + \sum_{b \in \Psi_B^E} P_b^{Bmax} \right) \quad \forall g \in \Psi_G^E \quad (14)$$

$$\hat{h}_w^{\text{EW}} = H_w^{\text{EW}} P_w^{\text{Wmax}} / \left( \sum_{g \in \Psi_G^S} x_g^G + \sum_{w \in \Psi_W^S} x_w^W + \sum_{b \in \Psi_B^S} x_b^B + \sum_{g \in \Psi_G^E} P_g^{\text{Gmax}} + \sum_{w \in \Psi_W^E} P_w^{\text{Wmax}} + \sum_{b \in \Psi_B^E} P_b^{\text{Bmax}} \right) \quad \forall w \in \Psi_W^E \quad (15)$$

$$\hat{h}_b^{\text{EB}} = H_b^{\text{EB}} P_b^{\text{Bmax}} / \left( \sum_{g \in \Psi_G^S} x_g^G + \sum_{w \in \Psi_W^S} x_w^W + \sum_{b \in \Psi_B^S} x_b^B + \sum_{g \in \Psi_G^E} P_g^{\text{Gmax}} + \sum_{w \in \Psi_W^E} P_w^{\text{Wmax}} + \sum_{b \in \Psi_B^E} P_b^{\text{Bmax}} \right) \quad \forall b \in \Psi_B^E \quad (16)$$

Constraint (7) represents that the capacity investments for thermal units, wind resources, and storages are non-negative. Constraint (8) sets the energy-to-power ratio of storages. Constraints (9) and (10) represent that the offers of the candidate and existing units in all markets are non-negative, respectively. Equations (11)-(16) represent the inertia provision of all candidate/existing units.

This paper also considers tie-breaking constraints for energy, reserve, capacity, and inertia markets when different producers offer the same price. For simplification, we incorporate tie-breaking constraints in the upper-level constraints to guarantee the linearity of the lower-level problems, as shown in [19].

The variable sets of the upper-level problem consist of the upper-level variable set  $\Xi^U$  and lower-level variable sets  $\Xi^{\text{EnR}}$ ,  $\Xi^C$ , and  $\Xi^H$ . The upper-level variables are those in the set  $\Xi^U = \{x_g^G, x_w^W, x_b^B, e_b^{\text{Smax}}, o_{\text{tgs}}^{\text{SG}}, o_{\text{tgs}}^{\text{SGCap}}, o_w^{\text{SWCap}}, o_b^{\text{SBCap}}, o_{\text{tgs}}^{\text{SGIne}}, o_w^{\text{SWIne}}, o_b^{\text{SBIne}}, o_{\text{tgs}}^{\text{EG}}, o_{\text{tgs}}^{\text{EGCap}}, o_w^{\text{EWCap}}, o_b^{\text{EBCap}}, o_{\text{tgs}}^{\text{EGIne}}, o_w^{\text{EWIne}}, o_b^{\text{EBIne}}, \hat{h}_g^{\text{SG}}, \hat{h}_w^{\text{SW}}, \hat{h}_b^{\text{SB}}, \hat{h}_g^{\text{EG}}, \hat{h}_w^{\text{EW}}, \hat{h}_b^{\text{EB}}\}$ . The lower-level variables related to the energy and reserve markets are those in the set  $\Xi^{\text{EnR}} = \{\lambda_{\text{tms}}^{\text{E}}, \lambda_{\text{tms}}^{\text{R}}, p_{\text{tgs}}^{\text{SG}}, p_{\text{tgs}}^{\text{EG}}, p_{\text{tws}}^{\text{SW}}, p_{\text{tws}}^{\text{EW}}, p_{\text{tbs}}^{\text{Sd}}, p_{\text{tbs}}^{\text{Ed}}, p_{\text{tbs}}^{\text{Sc}}, p_{\text{tbs}}^{\text{Ec}}, r_{\text{tgs}}^{\text{SG}}, r_{\text{tgs}}^{\text{EG}}, r_{\text{tbs}}^{\text{SB}}, r_{\text{tbs}}^{\text{EB}}, l_{\text{tms}}^{\text{shed}}, \theta_{\text{tms}}, e_{\text{tbs}}^{\text{S}}, e_{\text{tbs}}^{\text{E}}\}$ . The lower-level variables related to the capacity market are those in the set  $\Xi^C = \{\lambda^C, c_g^{\text{SG}}, c_g^{\text{EG}}, c_w^{\text{SW}}, c_w^{\text{EW}}, c_b^{\text{SB}}, c_b^{\text{EB}}, d_i^C\}$ . The lower-level variables related to the inertia market are those in the set  $\Xi^H = \{\lambda^H, h_g^{\text{SG}}, h_g^{\text{EG}}, h_w^{\text{SW}}, h_w^{\text{EW}}, h_b^{\text{SB}}, h_b^{\text{EB}}\}$ .

## B. Lower-level: Multiple Market-clearing Problems

### 1) Energy and Reserve Markets

The detailed formulations of energy and reserve markets are given in Appendix A. Note that this paper regards thermal generators, wind generators, and storages as price-makers.

### 2) Capacity Market

The detailed formulations of the capacity market are given in Appendix A as well. Note that we evaluate the capacity provision of different generation technologies by their firm capacity.

### 3) Inertia Market

$$(h_g^{\text{SG}}, h_g^{\text{EG}}, h_w^{\text{SW}}, h_w^{\text{EW}}, h_b^{\text{SB}}, h_b^{\text{EB}}, \lambda^H) \in \arg \{(18)-(25)\} \quad (17)$$

$$\min_{\Xi^C} \left\{ \sum_{g \in \Psi_G^S} o_g^{\text{SGIne}} h_g^{\text{SG}} + \sum_{g \in \Psi_G^E} o_g^{\text{EGIne}} h_g^{\text{EG}} + \sum_{w \in \Psi_W^S} o_w^{\text{SWIne}} h_w^{\text{SW}} + \sum_{w \in \Psi_W^E} o_w^{\text{EWIne}} h_w^{\text{EW}} + \sum_{b \in \Psi_B^S} o_b^{\text{SBIne}} h_b^{\text{SB}} + \sum_{b \in \Psi_B^E} o_b^{\text{EBIne}} h_b^{\text{EB}} \right\} \quad (18)$$

$$\sum_{g \in \Psi_G^S} h_g^{\text{SG}} + \sum_{g \in \Psi_G^E} h_g^{\text{EG}} + \sum_{w \in \Psi_W^S} h_w^{\text{SW}} + \sum_{w \in \Psi_W^E} h_w^{\text{EW}} + \sum_{b \in \Psi_B^S} h_b^{\text{SB}} + \sum_{b \in \Psi_B^E} h_b^{\text{EB}} \geq H^{\text{req}} (\lambda^H) \quad (19)$$

$$0 \leq h_g^{\text{SG}} \leq \hat{h}_g^{\text{SG}} \quad \forall g \in \Psi_G^S (\tau_g^{\text{SGmin}}, \tau_g^{\text{SG}}) \quad (20)$$

$$0 \leq h_g^{\text{EG}} \leq \hat{h}_g^{\text{EG}} \quad \forall g \in \Psi_G^E (\tau_g^{\text{EGmin}}, \tau_g^{\text{EG}}) \quad (21)$$

$$0 \leq h_w^{\text{SW}} \leq \hat{h}_w^{\text{SW}} \quad \forall w \in \Psi_W^S (\tau_w^{\text{SWmin}}, \tau_w^{\text{SW}}) \quad (22)$$

$$0 \leq h_w^{\text{EW}} \leq \hat{h}_w^{\text{EW}} \quad \forall w \in \Psi_W^E (\tau_w^{\text{EWmin}}, \tau_w^{\text{EW}}) \quad (23)$$

$$0 \leq h_b^{\text{SB}} \leq \hat{h}_b^{\text{SB}} \quad \forall b \in \Psi_B^S (\tau_b^{\text{SBmin}}, \tau_b^{\text{SB}}) \quad (24)$$

$$0 \leq h_b^{\text{EB}} \leq \hat{h}_b^{\text{EB}} \quad \forall b \in \Psi_B^E (\tau_b^{\text{EBmin}}, \tau_b^{\text{EB}}) \quad (25)$$

Formula (18) aims to maximize the social welfare of the inertia market. Formula (19) gives the inertia requirement of the power system. Constraints (20)-(25) represent the upper and lower limits of inertia provision of different generators. Dual variables are given in the bracket of the corresponding formulas. Renewable generators have zero inertia unless they are equipped with improved virtual inertia control facilities.

## IV. SOLUTION TECHNIQUES

In this section, we give solution techniques for the proposed generation investment equilibrium model. Each producer acts given other producers' investment decisions. An equilibrium among multiple bilevel problems is achieved if no producer alters its decision [38].

First, we transform each bilevel model into a single-level problem by replacing the lower-level problems with their primal-dual optimality conditions [39]. The resulting single-level model is a mathematical problem with equilibrium constraints (MPEC) problem [40]-[43]. Second, we use a diagonalization approach [44], [45] to find a Nash equilibrium among multiple producers. This approach solves each producer's MPEC model iteratively with decisions of other producers known from the previous iteration [19], [46]. This iteration terminates when all producers do not change actions.

Notably, it is possible that there is no equilibrium to the problem and that the proposed algorithm cannot converge. Facing multiple markets, producers can make profits from the energy market, reserve market, capacity market, and inertia market. This complexity of income structure increases the nonlinearity of the problem, which can lead to nonexistence of equilibrium and nonconvergence. With numerical observation, the solution might oscillate in some cases. Although this does not sound elegant in theory, it somehow reflects the reality. In many electricity markets in the world, the generation mix and overall generation capacity usually oscillate around the ideal point. In some years, generation capacities are too adequate, and suppliers decide to curtail generation capacity, leading to underinvestment. Consequently, in the next years, generation capacities are inadequate, and suppliers decide to increase generation capacity, leading to overinvestment in future. If the solution oscillates, the stopping criterion can be terminated at a certain number of iterations [7] such as 100 iterations. The average generation mix within a

cycle of oscillation can be used to examine the impact of the market.

Here, we introduce the method for choosing an initial value of the diagonalization approach. Different initial values will lead to different solutions. To obtain an appropriate and reasonable solution, we first find the least-cost capacity mix from the aspect of central planning. Then, taking this solution as an initial value, we use the proposed model to obtain the equilibrium.

The problem may also have multiple equilibria. Among different equilibria, we can set different objective functions to pick one equilibrium, such as one that maximizes social welfares or maximizes profits of all producers [45]. Since we choose the least-cost solution as the initial value, we probably will end up with an equilibrium with the least cost.

## V. NUMERICAL RESULTS

In this section, we run numerical experiments of six scenarios: ① equilibrium model with inertia market but without virtual inertia; ② equilibrium model with inertia market and with virtual inertia; ③ equilibrium model without inertia market; ④ centralized model with inertia requirements but without virtual inertia; ⑤ centralized model with inertia requirements and with virtual inertia; and ⑥ centralized model without inertia requirements. We use an illustrative example based on a 5-bus system and a large system example with the IEEE 118-bus system. All examples have been implemented and solved using GAMS with default GAMS options. We use dedicated MPEC solvers embedded in GAMS, NLPEC, which can give locally optimal solutions to MPEC problems. The simulations are carried out on an Intel Core i5 CPU with 3.20 GHz and 8 GB of RAM.

The model time horizon is one year, and the investment costs are annualized to match the time horizon. We choose four representative days ( $\pi_s = 1/4$ ,  $\sigma_t = 365$ ) to account for the variability of renewable generation and demand. Each representative day consists of 24 hours and represents one season. To highlight the impacts of the inertia market, the numerical experiment omits transmission constraints and demand response. And we only consider spinning reserves and ignore other reserve types.

### A. Illustrative Example: 5-bus System

The topology of the 5-bus system is shown in Fig. 3 and the parameters of generation technologies are listed in Table II, where  $CT$  represents the carbon tax, which is a part of the operation cost of thermal generators. This paper set the carbon tax as 30 \$/MWh. The investment cost for virtual inertia is 10000 \$/(MW·year<sup>-1</sup>). After the installation of the virtual inertia technology, the inertia constants of wind resources and storages will be 4 s.

Three existing thermal generators are located in bus 1, with installed capacities at 80 MW, 80 MW, and 120 MW, respectively. The planning margin reserve for the capacity of the system is 13.75%. The inertia requirement of the system is no less than 3.125 s. The price cap for the energy market is 400 \$/MWh and the bid of capacity demand is 8600 \$/MWh [47].

The sum of these two prices is 9000 \$/MWh, representing the value of lost load (VOLL) [19]. In the energy market, all units are assumed to be offered at their operation costs. Reserve offers of all units are their opportunity costs in the energy market. Capacity offers of candidate small thermal generators are their investment costs while candidate large thermal generators offer at 20% of their investment costs. Capacity offers of three existing thermal generators are 0 \$/(MW·year<sup>-1</sup>) (assuming that profits of these generators have covered their investment costs). Capacity offers of wind generation and storages are 0 \$/(MW·year<sup>-1</sup>). Under the cases with virtual inertia provision, inertia offers of thermal generators are 0 \$/(s·year<sup>-1</sup>). Inertia offers of wind generation and storages are their investment costs for virtual inertia provision. Under the cases without virtual inertia provision, inertia offers of candidate thermal generators are investment costs for virtual inertia installations to indicate the value of inertia provision. Detailed parameters can be found in [48].

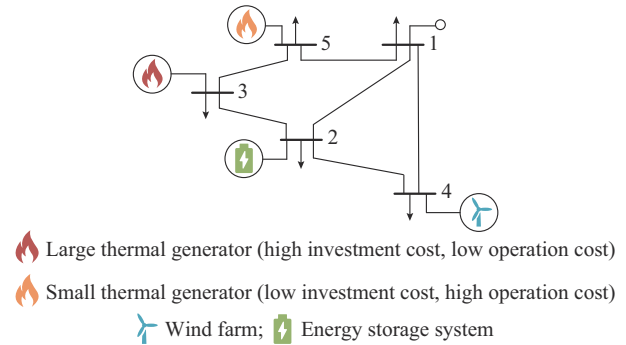


Fig. 3. Topology of 5-bus system.

TABLE II  
PARAMETERS OF GENERATION TECHNOLOGIES

| Technology         | Investment cost (\$/(MW·year <sup>-1</sup> )) | Operation cost (\$/MWh) | Ramp rate (p.u.) | Inertia constant (s) |
|--------------------|-----------------------------------------------|-------------------------|------------------|----------------------|
| Large thermal      | 34267                                         | 4+CT                    | 1/12             | 5.0                  |
| Small thermal      | 18933                                         | 7+CT                    | 2/15             | 4.3                  |
| Wind               | 35034                                         | 4.9                     |                  | 0                    |
| Storage            | 18730 (power),<br>2000 (energy)               | 1                       | 2/15             | 0                    |
| Existing thermal 1 |                                               | 4.8+CT                  | 1/12             | 4.3                  |
| Existing thermal 2 |                                               | 5+CT                    | 1/12             | 4.5                  |
| Existing thermal 3 |                                               | 5.1+CT                  | 1/12             | 4.8                  |

The rationale behind the assumed capacity offer costs is explained as follows.

1) Candidate small thermal generators are those peak units. Their operation costs decide the marginal price of the energy market during non-scarcity hours. During scarcity hours, the electricity price will be the price cap, i.e., 400 \$/MWh, which is higher than their operation cost, leading to profits. However, in a whole year, the total scarcity hours are few, usually less than 20 hours, due to the reliability requirement of the power system. Therefore, candidate small

thermal generators can hardly have profits in the energy market. They rely on the capacity market to recover nearly all their investment cost. As a result, their offers in the capacity market are their investment costs.

2) Candidate large thermal generators can recover part of their investment costs by profits from the energy market. The proportion of cost recovery from the energy market is assumed to be 80% [19]. Therefore, they only need 20% of their investment costs to be obtained in the capacity market.

3) Existing thermal generators have been working for many years. We assume that they have recovered all their investment costs. Therefore, they offer zero in the capacity market.

4) Wind generators offer zero in the capacity market. Wind generators can recover nearly all their investment costs in the energy market due to their cheap operation costs. Moreover, due to the variability and intermittence of wind generation, ISO usually does not regard them as capacity providers or take their firm capacity to be the capacity provision.

sion.

5) Storages offer zero in the capacity market. The generation capacity of storage depends on their stored energy, i.e., state of charge (SOC). Analogous to wind generators, ISO usually does not regard them as capacity providers or take their firm capacity to be the capacity provision.

### B. Comparative Results of Investment Decisions

Per Fig. 4(a) and Fig. 4(c) or Fig. 4(d) and Fig. 4(f), considering the inertia constraints, the share of wind generation decreases while that of thermal generation increases. In Fig. 4, centralized planning model is a least-cost model that incorporates energy, reserve, capacity, and inertia constraints instead of running markets to procure these products. Wind resources feature low operation costs and low inertia. Thus, it is a prior choice if the inertia constraints are not considered. When the inertia constraints are considered, however, thermal units, with high inertia constants, will replace wind generators to improve the overall system inertia.

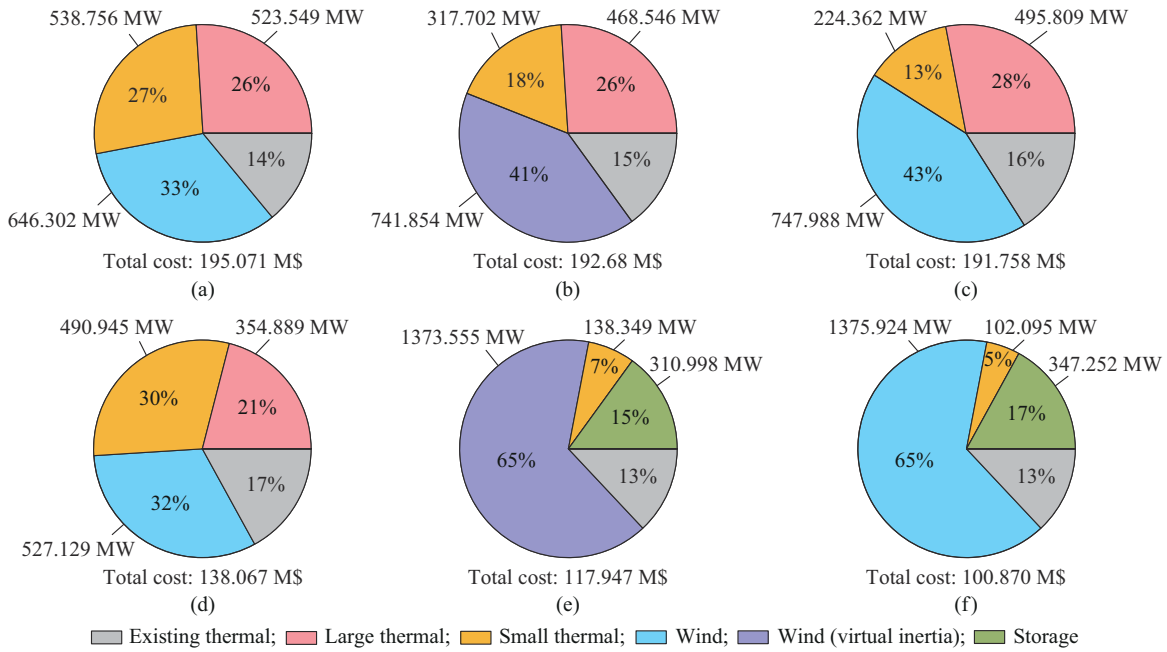


Fig. 4. Generation mix for 5-bus system of equilibrium model and centralized planning model. (a) Equilibrium model with inertia market but without virtual inertia. (b) Equilibrium model with inertia market and virtual inertia. (c) Equilibrium model without inertia market. (d) Centralized planning model with inertia requirements but without virtual inertia. (e) Centralized planning model with inertia requirements and virtual inertia. (f) Centralized planning model without inertia requirements.

Per Fig. 4(b) and Fig. 4(c) or Fig. 4(e) and Fig. 4(f), if the wind generators are equipped with virtual inertia devices, the introduction of inertia constraints does not significantly reduce wind generation shares but boosts virtual inertia facility penetration.

We compare the total cost, including the investment cost and operation cost, across six cases. We use those cases without inertia constraints as benchmarks. The total costs increase when considering inertia constraints with virtual inertia provision. The reason is that inertia provision, similar to reserve provision, is an additional requirement for the power system with high penetration of renewable generation. The total costs increase further when considering inertia con-

straints but without virtual inertia. The reason is that, with carbon taxes, it is more cost-competitive to invest wind resources with virtual inertia facilities than to substitute wind generators with thermal generators.

Compared with the centralized planning model, the market-based model also yields higher shares of thermal generation. This is because the reserve, capacity, and inertia markets favor thermal generators due to their firm ramping ability, reliable capacity, and high inertia constants. Some researchers criticize that these markets discriminate against renewable resources and may hinder decarbonization. However, we argue that reserve service, reliable firm capacity, and adequate inertia are crucial to the long-term reliability of the



electricity supply. To achieve the low-carbon targets, renewable generations are encouraged to participate in these markets such as by equipping virtual inertia devices.

It is also noted that storages only show up in Fig. 4(e) and Fig. 4(f), which are both the results of the central planning model. This is because storages can hardly make profits in the market-oriented model. First, their outputs depend on the SOC. When they see a scarcity price, they probably are with low energy stored and thus cannot discharge. Second, for storages, it is hard to forecast the highest and lowest prices to decide when to discharge and charge, respectively. Therefore, in the market-oriented model, producers dislike storages due to the low profits brought by them. Moreover, storages neither show up in Fig. 4(d). Because in that case, storages cannot provide inertia while the inertia requirements must be met. In that case, thermal generators are favored by the ISO to provide inertia support.

### C. Comparative Results of System Inertia Level

The overall system inertia of the six cases is shown in Fig. 5. It is necessary to incorporate inertia requirements both in the equilibrium model and in the centralized model. When the inertia requirements are omitted, the overall system inertia cannot meet the minimum inertia requirement of the power system (3.125 s). Moreover, with virtual inertia provision, the overall system inertia is higher than that without virtual inertia provision.

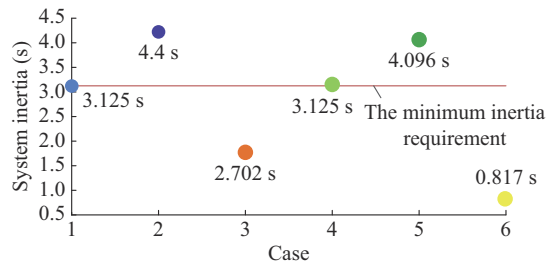


Fig. 5. Overall system inertia of six cases.

### D. Economic Results

#### 1) Prices for Different Products

The energy and reserve prices for cases 1, 2, and 3 are shown in Fig. 6. The capacity prices for cases 1, 2, and 3 are 6.853 k\$/(MW·year<sup>-1</sup>). The inertia prices for cases 1 and 2 are 10 k\$/(s·year<sup>-1</sup>).

As observed from Fig. 6(a), case 2 has the lower energy price than case 1. This is because compared to case 1, case

2 increases wind generation by 15%. More wind generation leads to lower energy prices due to their cheap operation costs. Moreover, case 3 has the highest energy prices. This is because smaller thermal generators (high operation costs) are more dispatched in case 3. Note that, although case 3 has the highest percentage of wind generation, its wind generation is only more than that of case 2 by 0.8%. Besides, due to the absence of inertia requirements, case 3 has the lowest installed capacity leading to the highest utilization level of small thermal generators.

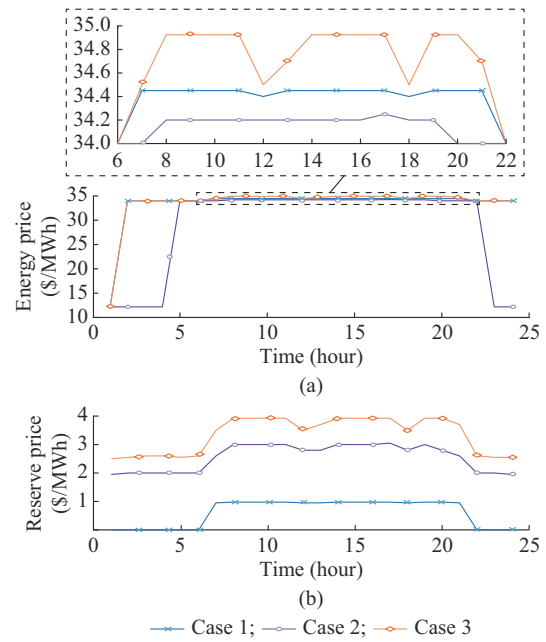


Fig. 6. Energy prices and reserve prices for cases 1-3. (a) Energy prices. (b) Reserve prices.

As observed from Fig. 6(b), case 1 has the most capacities of small thermal generators and the lowest reserve prices, while case 3 has the fewest capacities of small thermal generators and the highest prices. This is because small thermal generators have the highest operation costs leading to the lowest opportunity costs for energy. In an energy-and-reserve joint market, these lowest opportunity costs for energy lead to the lowest reserve prices. Therefore, the more the capacities of small thermal generators are, the lower the reserve prices are.

#### 2) Consumers' Payment

The annual payment and the average payment are listed in Table III.

TABLE III  
CONSUMERS' PAYMENTS

| Case | Annual payment (\$) |                     |                     |                     |                     | Average payment (\$/MWh) |         |          |         |        |
|------|---------------------|---------------------|---------------------|---------------------|---------------------|--------------------------|---------|----------|---------|--------|
|      | Energy              | Reserve             | Capacity            | Inertia             | Total               | Energy                   | Reserve | Capacity | Inertia | Total  |
| 1    | $1.466 \times 10^8$ | $4.193 \times 10^5$ | $4.992 \times 10^6$ | $3.125 \times 10^4$ | $1.520 \times 10^8$ | 33.668                   | 0.096   | 1.147    | 0.007   | 34.918 |
| 2    | $1.303 \times 10^8$ | $4.604 \times 10^5$ | $4.992 \times 10^6$ | $3.125 \times 10^4$ | $1.358 \times 10^8$ | 29.936                   | 0.106   | 1.147    | 0.007   | 31.196 |
| 3    | $1.476 \times 10^8$ | $1.961 \times 10^6$ | $4.992 \times 10^6$ |                     | $1.546 \times 10^8$ | 33.910                   | 0.450   | 1.147    |         | 35.507 |

Consistent with the trends of prices for different products, case 3 has the highest energy payment while case 2 has the lowest one. For reserve payment, case 3 has the highest payment while case 1 has the lowest one.

For the composition of average payment, energy fee is the majority, accounting for around 96% of the total payment. Capacity fee accounts for around 3.4% of the total payment. The sum of reserve and inertia fees hold slight proportions, with around 0.6%.

#### E. Large System Example: IEEE 118-bus System

We conduct numerical experiments on the IEEE 118-bus system to verify the scalability of the proposed model. The

initial parameters are from MATPOWER 7.1. Other settings of parameters keep the same as those in the 5-bus system. All parameters can be found in [48].

The generation mix of different cases for the IEEE 118-bus system is shown in Fig. 7. It is shown that the results of the IEEE 118-bus system have analogous conclusions with those of the 5-bus system. Note that the percentage of each generation technology keeps the same as that of the 5-bus system. This is because the proposed model ignores the constraints of networks. Moreover, the main differences in parameters between the IEEE 118-bus and 5-bus systems are the demand levels and existing generators, which are not marginal units, thus not deciding the clearing prices.

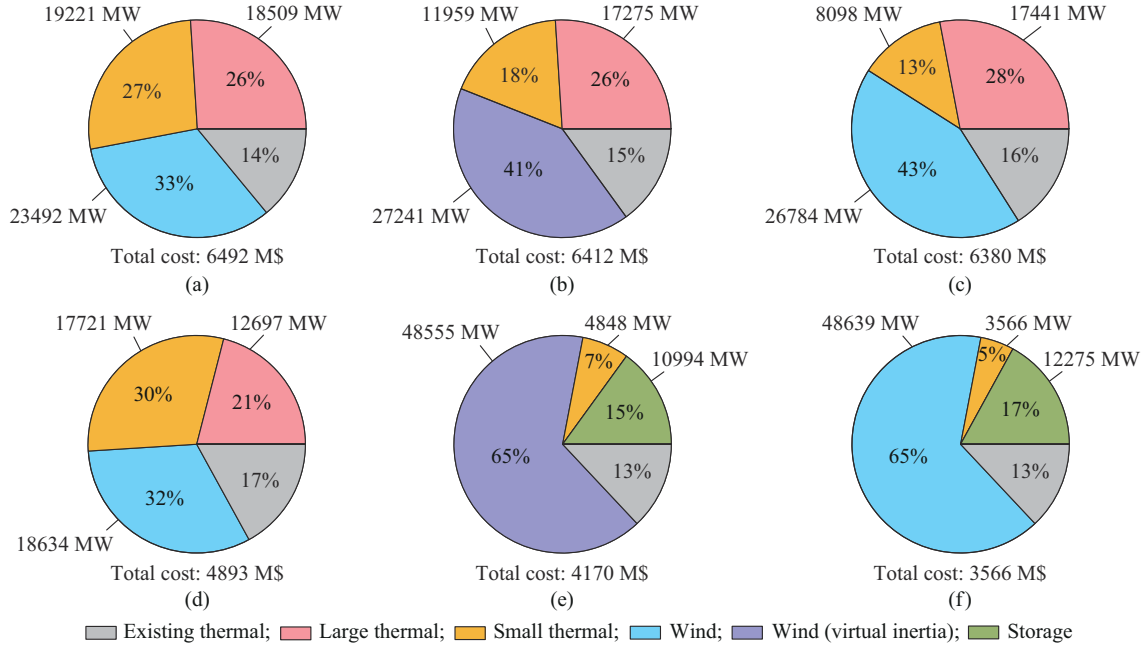


Fig. 7. Generation mix for IEEE 118-bus system of equilibrium model and centralized planning model. (a) Equilibrium model with inertia market but without virtual inertia. (b) Equilibrium model with inertia market and virtual inertia. (c) Equilibrium model without inertia market. (d) Centralized planning model with inertia requirements but without virtual inertia. (e) Centralized planning model with inertia requirements and virtual inertia. (f) Centralized planning model without inertia requirements.

## VI. CONCLUSION

In this paper, we propose an auction-based inertia market mechanism and present a generation investment equilibrium model to examine the impacts of the inertia incentives on the generation mix. Our findings are as follows. First, it is necessary to incorporate inertia constraints and design an inertia remuneration mechanism to ensure inertia adequacy with high shares of renewable generation. Second, with inertia requirements but without virtual inertia provision, a sizable share of wind generators has to be replaced by thermal generators to ensure inertia adequacy. Third, with inertia requirements and virtual inertia provision, thermal generators only replace 0%-2% shares of wind generation. Fourth, considering carbon taxes, it is more cost-competitive to invest wind resources with virtual inertia facilities than to substitute wind generators with thermal generators. The findings imply that, by increasing virtual inertia facility penetration, a power system with adequate inertia is possible and cost-ef-

fective with high shares of renewable generation.

## APPENDIX A

In the appendix, we give the detailed formulations of problems described in Section III-B, including energy and reserve markets and the capacity market.

### 1) Energy and Reserve Markets

$$(\lambda_{ts}^E, \lambda_{ts}^R, p_{tgs}^{SG}, p_{tgs}^{EG}, p_{tws}^{SW}, p_{tws}^{EW}, p_{tbs}^{Sd}, p_{tbs}^{Ed}, p_{tbs}^{Sc}, p_{tbs}^{Ec}, r_{tgs}^{SG}, r_{tgs}^{EG}, r_{tbs}^{SB}, r_{tbs}^{EB}) \in \arg \{(A2)-(A30)\} \quad \forall t \in T, \forall s \in S \quad (A1)$$

$$\min_{\Xi^{E\&R}} \left\{ \sum_{g \in \Psi_G^S} o_{tgs}^{SG} p_{tgs}^{SG} + \sum_{g \in \Psi_G^E} o_{tgs}^{EG} p_{tgs}^{EG} + \sum_{w \in \Psi_W^S} C_w^{SW} p_{tws}^{SW} + \sum_{w \in \Psi_W^E} C_w^{EW} p_{tws}^{EW} + \sum_{g \in \Psi_G^S} C_g^{SGR} r_{tgs}^{SG} + \sum_{g \in \Psi_G^E} C_g^{EGR} r_{tgs}^{EG} + \sum_{b \in \Psi_B^S} C_b^{SBR} r_{tbs}^{SB} + \sum_{b \in \Psi_B^E} C_b^{EBR} r_{tbs}^{EB} + \sum_{n \in N} P_n^{E} I_{tms}^{shed} \right\} \quad (A2)$$

$$\sum_{g \in \Psi_{G(n)}^S} p_{tgs}^{SG} + \sum_{g \in \Psi_{G(n)}^{EG}} p_{tgs}^{EG} + \sum_{w \in \Psi_{W(n)}^{SW}} p_{tws}^{SW} + \sum_{w \in \Psi_{W(n)}^{EW}} p_{tws}^{EW} + \sum_{b \in \Psi_{B(n)}^{Sd}} (p_{tbs}^{Sd} - p_{tbs}^{Sc}) + \sum_{b \in \Psi_{B(n)}^{Ed}} (p_{tbs}^{Ed} - p_{tbs}^{Ec}) - \sum_{m \in \Phi_n} B_{nm} (\theta_{tms} - \theta_{tms}) + l_{tms}^{shed} = D_{tms} \quad \forall n \in \Psi_N \quad (\lambda_{tms}^E) \quad (A3)$$

$$0 \leq p_{tgs}^{SG} \leq x_g^G \quad \forall g \in \Psi_G^S \quad (\mu_{tgs}^{SGmin}, \mu_{tgs}^{SG}) \quad (A4)$$

$$0 \leq p_{tgs}^{EG} \leq P_g^{Gmax} \quad \forall g \in \Psi_G^{EG} \quad (\mu_{tgs}^{EGmin}, \mu_{tgs}^{EG}) \quad (A5)$$

$$0 \leq p_{tws}^{SW} \leq P_{tws}^{SW} x_w^W \quad \forall w \in \Psi_W^S \quad (\mu_{tws}^{SWmin}, \mu_{tws}^{SW}) \quad (A6)$$

$$0 \leq p_{tws}^{EW} \leq P_{tws}^{EW} P_w^{Wmax} \quad \forall w \in \Psi_W^{EW} \quad (\mu_{tws}^{EWmin}, \mu_{tws}^{EW}) \quad (A7)$$

$$0 \leq p_{tbs}^{Sc} \leq x_b^B \quad \forall b \in \Psi_B^S \quad (\mu_{tbs}^{Scmin}, \mu_{tbs}^{Sc}) \quad (A8)$$

$$0 \leq p_{tbs}^{Sd} \leq x_b^B \quad \forall b \in \Psi_B^S \quad (\mu_{tbs}^{Sdmin}, \mu_{tbs}^{Sd}) \quad (A9)$$

$$0 \leq p_{tbs}^{Ec} \leq P_b^{Bmax} \quad \forall b \in \Psi_B^{EG} \quad (\mu_{tbs}^{Ecmin}, \mu_{tbs}^{Ec}) \quad (A10)$$

$$0 \leq p_{tbs}^{Ed} \leq P_b^{Bmax} \quad \forall b \in \Psi_B^{EG} \quad (\mu_{tbs}^{Edmin}, \mu_{tbs}^{Ed}) \quad (A11)$$

$$0 \leq l_{tms}^{shed} \leq D_{tms} \quad \forall n \in \Psi_N \quad (\mu_{tms}^{lsmin}, \mu_{tms}^{ls}) \quad (A12)$$

$$e_{tbs}^S = e_{(t-1)bs}^S + (p_{tbs}^{Sc} l_b^{Sc} - p_{tbs}^{Sd} / \eta_b^{Sd}) \Delta t \quad \forall b \in \Psi_B^S \quad (\phi_{tbs}^S) \quad (A13)$$

$$e_{tbs}^S = 50\% \cdot e_b^{Smax} \quad t=0, T, \forall b \in \Psi_B^S \quad (\phi_{tbs}^{S0}, \phi_{tbs}^{ST}) \quad (A14)$$

$$e_{tbs}^E = e_{(t-1)bs}^E + (p_{tbs}^{Ec} l_b^{Ec} - p_{tbs}^{Ed} / \eta_b^{Ed}) \Delta t \quad \forall b \in \Psi_B^{EG} \quad (\phi_{tbs}^E) \quad (A15)$$

$$e_{tbs}^E = 50\% \cdot e_b^{Emax} \quad t=0, T, \forall b \in \Psi_B^{EG} \quad (\phi_{tbs}^{E0}, \phi_{tbs}^{ET}) \quad (A16)$$

$$0 \leq e_{tbs}^S \leq e_b^{Smax} \quad \forall b \in \Psi_B^S \quad (\phi_{tbs}^{Smin}, \phi_{tbs}^{Smax}) \quad (A17)$$

$$0 \leq e_{tbs}^E \leq e_b^{Emax} \quad \forall b \in \Psi_B^{EG} \quad (\phi_{tbs}^{Emin}, \phi_{tbs}^{Emax}) \quad (A18)$$

$$0 \leq r_{tgs}^{SG} \leq R_g^{S10} x_g^G \quad \forall g \in \Psi_G^S \quad (\lambda_{tgs}^{SGmin}, \lambda_{tgs}^{SGmax}) \quad (A19)$$

$$0 \leq r_{tgs}^{EG} \leq R_g^{E10} P_g^{Gmax} \quad \forall g \in \Psi_G^{EG} \quad (\lambda_{tgs}^{EGmin}, \lambda_{tgs}^{EGmax}) \quad (A20)$$

$$p_{tgs}^{SG} + r_{tgs}^{SG} \leq x_g^G \quad \forall g \in \Psi_G^S \quad (\lambda_{tgs}^{SG}) \quad (A21)$$

$$p_{tgs}^{EG} + r_{tgs}^{EG} \leq P_g^{Gmax} \quad \forall g \in \Psi_G^{EG} \quad (\lambda_{tgs}^{EG}) \quad (A22)$$

$$0 \leq r_{tbs}^{SB} \leq R_b^{S10} x_b^B \quad \forall b \in \Psi_B^S \quad (\lambda_{tbs}^{SBmin}, \lambda_{tbs}^{SBmax}) \quad (A23)$$

$$0 \leq r_{tbs}^{EB} \leq R_b^{E10} P_b^{Bmax} \quad \forall b \in \Psi_B^{EG} \quad (\lambda_{tbs}^{EBmin}, \lambda_{tbs}^{EBmax}) \quad (A24)$$

$$p_{tbs}^{Sd} + r_{tbs}^{SB} \leq x_b^B \quad \forall b \in \Psi_B^S \quad (\lambda_{tbs}^{SB}) \quad (A25)$$

$$p_{tbs}^{Ed} + r_{tbs}^{EB} \leq P_b^{Bmax} \quad \forall b \in \Psi_B^{EG} \quad (\lambda_{tbs}^{EB}) \quad (A26)$$

$$\sum_{g \in \Psi_G^S} r_{tgs}^{SG} + \sum_{g \in \Psi_G^{EG}} r_{tgs}^{EG} + \sum_{b \in \Psi_B^S} r_{tbs}^{SB} + \sum_{b \in \Psi_B^{EG}} r_{tbs}^{EB} \geq R_t^{req} \quad (\lambda_{ts}^R) \quad (A27)$$

$$-F_{nm}^{max} \leq B_{nm} (\theta_{tms} - \theta_{tms}) \leq F_{nm}^{max} \quad \forall n \in \Psi_N, \forall m \in \Phi_n \quad (v_{tms}^{min}, v_{tms}) \quad (A28)$$

$$-\pi \leq \theta_{tms} \leq \pi \quad \forall n \in \Psi_N \quad (\zeta_{tms}^{min}, \zeta_{tms}) \quad (A29)$$

$$\theta_{tms} = 0 \quad n=1 \quad (\zeta_{ts}^1) \quad (A30)$$

Formula (A2) aims to maximize the social welfare of the energy and reserve markets. Formula (A3) ensures nodal power balance. Constraints (A4), (A5), and (A19) - (A22) represent the technical limits of thermal units. Constraints (A6) and (A7) represent the technical limits for wind farms. Constraints (A8)-(A11), (A17), (A18), and (A23)-(A26) represent the technical limits for candidate and existing storages. Constraint (A12) limits the ranges of load shedding. Constraints (A13) and (A15) depict the dynamics of the SOC of storages. Con-

straints (A14) and (A16) set the initial and final SOC of storages to be its half-rated capacity, respectively. Constraint (A27) gives the reserve requirement. Constraints (A28) - (A30) restrict the power flow and voltage phase angles. Dual variables are given in the bracket of the corresponding formulas.

## 2) Capacity Market

$$(c_g^{SG}, c_g^{EG}, c_w^{SW}, c_w^{EW}, c_b^{SB}, c_b^{EB}, \lambda^C) \in \arg \{(32)-(40)\} \quad (A31)$$

$$\min_{\lambda^C} \left\{ \sum_{g \in \Psi_G^S} o_g^{SGCap} c_g^{SG} + \sum_{g \in \Psi_G^{EG}} o_g^{EGCap} c_g^{EG} + \sum_{w \in \Psi_W^S} o_w^{SWCap} c_w^{SW} + \sum_{w \in \Psi_W^{EW}} o_w^{EWCap} c_w^{EW} + \sum_{b \in \Psi_B^S} o_b^{SBCap} c_b^{SB} + \sum_{b \in \Psi_B^{EG}} o_b^{EBCap} c_b^{EB} - \sum_{i \in I} B_i^{Cap} d_i^C \right\} \quad (A32)$$

$$\sum_{g \in \Psi_G^S} c_g^{SG} + \sum_{g \in \Psi_G^{EG}} c_g^{EG} + \sum_{w \in \Psi_W^S} c_w^{SW} + \sum_{w \in \Psi_W^{EW}} c_w^{EW} + \sum_{b \in \Psi_B^S} c_b^{SB} + \sum_{b \in \Psi_B^{EG}} c_b^{EB} = \sum_{i \in I} d_i^C \quad (\lambda^C) \quad (A33)$$

$$0 \leq c_g^{SG} \leq x_g^G \quad \forall g \in \Psi_G^S \quad (\lambda_g^{SGmin}, \lambda_g^{SG}) \quad (A34)$$

$$0 \leq c_g^{EG} \leq P_g^{Gmax} \quad \forall g \in \Psi_G^{EG} \quad (\lambda_g^{EGmin}, \lambda_g^{EG}) \quad (A35)$$

$$0 \leq c_w^{SW} \leq \gamma_w^{SW} x_w^W \quad \forall w \in \Psi_W^S \quad (\lambda_w^{SWmin}, \lambda_w^{SW}) \quad (A36)$$

$$0 \leq c_w^{EW} \leq \gamma_w^{EW} P_w^{Wmax} \quad \forall w \in \Psi_W^{EW} \quad (\lambda_w^{EWmin}, \lambda_w^{EW}) \quad (A37)$$

$$0 \leq c_b^{SB} \leq \gamma_b^{SB} x_b^B \quad \forall b \in \Psi_B^S \quad (\lambda_b^{SBmin}, \lambda_b^{SB}) \quad (A38)$$

$$0 \leq c_b^{EB} \leq \gamma_b^{EB} P_b^{Bmax} \quad \forall b \in \Psi_B^{EG} \quad (\lambda_b^{EBmin}, \lambda_b^{EB}) \quad (A39)$$

$$0 \leq d_i^C \leq D_i^C \quad \forall i \in I \quad (\lambda_i^{dmin}, \lambda_i^d) \quad (A40)$$

Formula (A32) aims to maximize the social welfare of the capacity market. Constraint (A33) represents that the total firm capacity is provided by thermal units, wind farms, and storages. Constraints (A34) and (A35) give the technical limits for the capacity supply of thermal units. Constraints (A36) and (A37) give the technical limits for the firm capacity supply of wind generation. Constraints (A38) and (A39) give the technical limits of the firm capacity supply of storages. The maximum firm capacity that wind resources or storages can provide is the rated power scaled by a capacity factor. The scaling factor is the ratio of its scheduled power (during the scarcity period) to its rated power in the last year. Constraint (A40) represents the upper and lower limits of the capacity demand for the  $i^{th}$  segment of the demand curve.

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