Distributionally Robust Transmission Expansion Planning Considering Uncertainty of Contingency Probability

Weilun Wang, Mingqiang Wang, Xueshan Han, Ming Yang, Qiuwei Wu, and Ran Li

***Abstract*—The outage of power system equipment is one of the most important factors that affect the reliability and economy of power system. It is crucial to consider the influence of contingencies elaborately in planning problem. In this paper, a distributionally robust transmission expansion planning model is proposed in which the uncertainty of contingency probability is considered. The uncertainty of contingency probability is described by uncertainty interval based on the outage rate of single equipment. An epigraph reformulation and Benders decomposition are applied to solve the proposed model. Finally, the feasibility and effectiveness of the proposed model are illustrated on the IEEE RTS system and the IEEE 118-bus system.**

***Index Terms*—Benders decomposition, distributionally robust, equipment outage rate, transmission expansion planning.**

I. Introduction

POWER transmission system is mainly used to deliver the electricity from generators to distribution systems and further to consumers [1]. With the restructure of power industry and the integration of large-scale renewable energy source (RES), transmission expansion planning (TEP) has become one of the most important strategic decisions in power systems [1]. TEP aims to determine when and where to construct new transmission lines [2], [3]. During the last decade, a great share of research related to TEP has focused on addressing various uncertainties. The research on TEP considering uncertainties can be categorized as follows.

1) In terms of time scale of uncertainties, the uncertainties can be generally categorized into long- and short-term uncertainties [4]. The long-term uncertainties correspond to those which will be realized in the long run, e. g., load growth [5], future share of RES in power systems, future fuel cost, and policy regulations. In contrast, the short-term uncertainties correspond to those which will be realized in the operation stage, e. g., power generation of RES and consumption of loads. Reference [4] proposes a TEP model considering both long-term (the future installed capacity of units and future peak demand) and short-term uncertainties (daily variation of demand and power production). Reference [6] proposes a coordinated investment for transmission and storage systems considering both long- and short-term uncertainties.

2) For the short-term uncertainties concerned, in terms of uncertainty sources, the uncertainties are mainly caused by load, RES output, and contingencies caused by equipment outage events. References [7] and [8] propose TEP models considering the uncertainty of wind power production. Reference [9] proposes a TEP model considering the uncertainty of load. References [4], [10] - [14] propose more complex TEP models which consider several uncertainties together, and [15] considers the correlation between different uncertainties.

3) In terms of the methods for addressing the uncertainties, the uncertainties can be described by probability distribution functions (PDFs) or discretized scenarios, uncertainty intervals, and set of PDFs, etc. Then, stochastic optimization (SO) [16], robust optimization (RO), and distributionally robust optimization (DRO) can be applied in TEP models [17]. References [11], [15], [18], and [19] consider the uncertainties of RES output and load, and build three- or two-stage RO models with different ambiguity sets such as the uncertainty interval set and the ellipsoidal uncertainty set. References [10], [20], and [21] describe the uncertainties of load, wind power, and natural gas demand by scenarios and establish SO TEP models. Reference [22] establishes a DRO model for TEP which considers both long- and short-term uncertainties of RES output and load.

It can be found that when the uncertainties are concerned in TEP, the existing research mainly concentrates on analyzing the effect of uncertainties caused by RES output and load. Although the uncertainty of contingencies caused by equipment outage events is also an important source of uncertainty, its effect is not explicitly considered.

In TEP models, the uncertainty of contingencies caused by equipment outage events can be considered either in a deterministic approach or in a probabilistic approach [23]. For the deterministic approach concerned, usually the $N-k$ security criterion is applied [24], [25]. This approach is easy to understand, but the contingency probability is ignored, and

Manuscript received: October 28, 2020; revised: February 28, 2021; accepted: May 17, 2021. Date of CrossCheck: May 17, 2021. Date of online publication: July 14, 2021.

This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (http://creativecommons.org/licenses/by/4.0/).

W. Wang, M. Wang (corresponding author), X. Han, M. Yang, and R. Li are with the Key Laboratory of Power System Intelligent Dispatch and Control, Ministry of Education, Shandong University, Jinan 250002, China (e-mail: wangwl_sdu@163.com; wang0367@sdu.edu.cn; xshan@sdu.edu.cn; myang@sdu.edu.cn; 13791057205@163.com).

Q. Wu is with Department of Electrical Engineering, Technical University of Denmark, Region Hovedstaden, Denmark (e-mail: qw@elektro.dtu.dk).

DOI: 10.35833/MPCE.2020.000768

the result may be conservative or radical. The probabilistic approach determines the optimal planning results by balancing the investment cost, pre-contingency, and post-contingency cost simultaneously [26], and it can strike a good balance between the security and economics. However, the solution of the probabilistic approach is closely related to the contingency probability which can be analytically expressed as a function of equipment outage rate. Usually, a fixed equipment outage rate obtained from historical statistics and/or operation experience is usually applied [27]. But it is difficult to obtain the exact value of equipment outage rate due to the insufficient quality and quantity of historical data [28], [29], and the law of large numbers is not available. Therefore, an estimated equipment outage rate is usually used. However, the estimated equipment outage rate would be significantly deviated from the real value, which would cause suboptimal solution of TEP problem.

Various models have been proposed to model the uncertainties of contingencies caused by equipment outage events. Reference [28] proposes a transmission system hardening model and constructs an ambiguity set for $N-k$ contingency probability distribution in which the contingency probability is described by interval. Then, the reliability assessment is implemented for each given harden plan considering the elaborately-selected contingency events. However, the uncertainty of contingency probability is only used in the post-processing for each given harden plan, and it is not involved in the optimization model. Reference [30] models the contingency probability by ambiguity set, and a DRO approach is proposed for the unit commitment problem considering contingency. The ambiguity set of contingency probability proposed in [30] is extended in a TEP problem in [31] and a distributionally robust TEP model is proposed. However, the ambiguity set of contingency probability is not well analyzed. Instead, there are many research works on the equipment outage rate.

Compared with the distributionally robust TEP model in [31], the ambiguity set of contingency probability is further analyzed in this paper. Considering the analytical relationship between the contingency probability and equipment outage rate, the distributionally robust TEP model considering the ambiguity set of equipment outage rate is proposed in this paper, and the existing research works on equipment outage rate can be applied in the TEP model. For the calculation of the uncertainty of equipment outage rate, the interval probability approach is most widely applied. For example, [32] establishes the relationship between the equipment outage rate and operation conditions based on the imprecise Dirichlet model, and the interval of the outage rate is finally obtained. When the ambiguity set of equipment outage rate is boiled down to an interval, the proposed distributionally robust TEP model can be recast as a robust stochastic optimization (RSO) model [33], which is easier to be solved compared with the original DRO model. The proposed model can be transformed into a multi-level optimization problem and can be solved by the mixed-integer linear programming (MILP) commercial solver based on the dual theory, epigraph reformulation [34], and Benders decomposition. Finally, the effectiveness and validity of the proposed model are illustrated using the IEEE RTS system and the IEEE 118-bus system.

The major contributions of this paper are as follows.

1) A novel distributionally robust TEP model is proposed considering the uncertainty of equipment outage rate. An interval equipment outage rate is applied and the TEP model is transformed into an RSO model.

2) Combining the techniques of dual theory, epigraph reformulation, and Benders decomposition, the proposed model can be efficiently solved by MILP commercial solver.

The remainder of this paper is organized as follows. Section II presents the proposed the mathematical formulation of the proposed model. Section III describes the uncertainty set of contingency probability. Section IV gives the solution methodology. In Section V, numerical results are provided and analyzed, and the relevant conclusions are drawn in Section VI.

II. Mathematical Formulation of Proposed Model

The proposed model is explicitly formulated as:

$$\min\left[\sum_{l\in N_L^+} u_l C_l + \sum_{g\in N_G}\lambda^{normal} C_g + \sup_{\lambda_s\in N_\Lambda}\sum_{s\in N_S}\left(\min\left(\lambda_s\cdot VOLL\cdot\sum_{i\in N_I} lol_{s,i}\right)\right)\right] \tag{1}$$

s.t.

$$\sum_{i=B(g)} P_g - d_i = \sum_{\substack{i=B_{start}(l)\\ l\in N_L^+\cup N_L}} f_l - \sum_{\substack{i=B_{end}(l)\\ l\in N_L^+\cup N_L}} f_l \quad \forall i \tag{2}$$

$$f_l X_l = \sum_{i=B_{start}(l)}\theta_i - \sum_{i=B_{end}(l)}\theta_i \quad \forall l\in N_L \tag{3}$$

$$f_l X_l = u_l\left(\sum_{i=B_{start}(l)}\theta_i - \sum_{i=B_{end}(l)}\theta_i\right) \quad \forall l\in N_L^+ \tag{4}$$

$$\theta_i = 0 \quad i\in I_{ref} \tag{5}$$

$$-T_l^{\max}\le f_l\le T_l^{\max} \quad \forall l\in N_L \tag{6}$$

$$-u_l T_l^{\max}\le f_l\le u_l T_l^{\max} \quad \forall l\in N_L^+ \tag{7}$$

$$P_g^{\min}\le P_g\le P_g^{\max} \quad \forall g \tag{8}$$

$$\sum_{i=B(g)} P_{s,g} - \left(d_i - lol_{s,i}\right) = \sum_{\substack{i=B_{start}(l)\\ l\in N_L^+\cup N_L}} f_{s,l} - \sum_{\substack{i=B_{end}(l)\\ l\in N_L^+\cup N_L}} f_{s,l} \quad \forall i,\forall s \tag{9}$$

$$f_{s,l} X_l = \sum_{i=B_{start}(l)}\theta_{s,i} - \sum_{i=B_{end}(l)}\theta_{s,i} \quad \forall l\in N_L,\forall s \tag{10}$$

$$f_{s,l} X_l = u_l\left(\sum_{i=B_{start}(l)}\theta_{s,i} - \sum_{i=B_{start}(l)}\theta_{s,i}\right) \quad \forall l\in N_L^+,\forall s \tag{11}$$

$$\theta_{s,i} = 0 \quad \forall s, i\in I_{ref} \tag{12}$$

$$-T_l^{\max}\le f_{s,l}\le T_l^{\max} \quad \forall l\in N_L,\forall s \tag{13}$$

$$-u_l T_l^{\max}\le f_{s,l}\le u_l T_l^{\max} \quad \forall l\in N_L^+,\forall s \tag{14}$$

$$P_g^{\min}\le P_{s,g}\le P_g^{\max} \quad \forall g,\forall s \tag{15}$$

$$0\le lol_{s,i}\le d_i \quad \forall i,\forall s \tag{16}$$

where g, i, and s are the indexes of units, buses, and contingencies, respectively; N_G, N_I, and N_S are the sets of units, buses, and contingencies, respectively; l is the index of transmission lines, including existing lines and candidate lines; N_L and N_L^+ are the sets of the existing lines and candidate lines, respectively; u_l is a binary variable which represents whether transmission line l is constructed (1) or not (0); C_l is the investment cost of candidate transmission line l; λ^{normal} is the probability of normal operation condition; C_g is the operation cost of unit g; λ_s is the random contingency probability of contingency s; N_Λ is the set of the distribution functions of λ_s; $VOLL$ is the value of lost load; $B(g)$, $B_{start}(l)$, and $B_{end}(l)$ are the bus on which unit g is located, the start bus of line l, and the end bus of line l, respectively; $lol_{s,i}$ is the loss of load on bus i under contingency s; P_g and $P_{s,g}$ are the power outputs of unit g under normal operation condition and contingency s, respectively; d_i is the forecasted load on bus i; f_l and $f_{s,l}$ are the power flows of transmission line l under normal operation condition and contingency s, respectively; X_l is the reactance of transmission line l; θ_i and $\theta_{s,i}$ are the voltage phase angles on bus i under normal operation condition and contingency s, respectively; $T_l^{\max}$ is the capacity of transmission line l; I_{ref} is the set of reference buses; and $P_g^{\min}$ and $P_g^{\max}$ are the minimum and maximum output power of unit g, respectively.

The objective function (1) minimizes the total cost, which includes the investment cost, operation cost, and expected load-shedding cost. Constraints (2)-(8) correspond to the normal operation condition. Formula (2) describes the nodal power balance constraint. Formulas (3) and (4) calculate the power flows on the existing and candidate transmission lines, respectively. Formula (5) imposes the phase angle requirement on the reference bus. Formulas (6) and (7) give the line flow limits on the existing and candidate transmission lines, respectively. Formula (8) gives the limits of unit output power.

Formulas (9)-(16) are the constraints corresponding to the contingency scenarios considered. In this paper, the outages of units and transmission lines are considered. Both the single-equipment outage and multi-equipment outage are considered. The output power of outage units and the power flow on outage transmission lines are set to be zero. These constraints have similar meaning compared with those corresponding to the normal operation condition, while (16) limits the loss of load under contingency s on each bus.

Since λ_s is a random parameter, the model cannot be solved directly and it needs to be transformed into deterministic form by constructing the uncertainty set.

III. Uncertainty Set of Contingency Probability

The probability of the system-wide contingency can be analytically expressed by the probability of equipment outage rate. For a contingency s which corresponds to m equipment that malfunction simultaneously and n equipment that operate normally, the contingency probability can be expressed as [35]:

$$\lambda_s = \prod_{i=1}^{m}\gamma_i \prod_{j=1}^{n}\left(1-\gamma_j\right) \tag{17}$$

where γ_i and γ_j are the outage rates of equipment i and j, respectively. Since γ_j is usually very small, $\prod_{j=1}^{n}\left(1-\gamma_j\right)$ is approximately equal to 1 and can be ignored. Then the contingency probability can be expressed as:

$$\lambda_s \approx \prod_{i=1}^{m}\gamma_i \tag{18}$$

When γ_i varies within an interval $[\gamma_i^{\min}, \gamma_i^{\max}]$ [36], the contingency probability also varies within an interval, and the upper and lower limits of the interval $[\lambda_s^{\min}, \lambda_s^{\max}]$ can be expressed as:

$$\lambda_s^{\max} = \prod_{i=1}^{m}\gamma_i^{\max} \tag{19}$$

$$\lambda_s^{\min} = \prod_{i=1}^{m}\gamma_i^{\min} \tag{20}$$

Similar to that addressed in the traditional robust model, the random contingency probability can be equivalently expressed as a deterministic form with the introduction of auxiliary variable α_s.

$$\lambda_s = \bar{\lambda}_s + \alpha_s \hat{\lambda}_s \tag{21}$$

where α_s is an auxiliary variable which describes the contingency probability deviating from midpoint and $-1 \le \alpha_s \le 1$; and $\bar{\lambda}_s$ and $\hat{\lambda}_s$ are the midpoint and radius of the uncertainty interval of contingency probability, respectively. $\bar{\lambda}_s$ and $\hat{\lambda}_s$ can be expressed as:

$$\bar{\lambda}_s = \left(\lambda_s^{\max} + \lambda_s^{\min}\right)/2 \tag{22}$$

$$\hat{\lambda}_s = \left(\lambda_s^{\max} - \lambda_s^{\min}\right)/2 \tag{23}$$

Meanwhile, the sum of all contingency probabilities concerned should be equal to 1, i.e.,

$$\sum_{s \in N_s}\lambda_s = \sum_{s \in N_s}\left(\bar{\lambda}_s + \alpha_s \hat{\lambda}_s\right) = 1 \tag{24}$$

In order to limit the conservativeness, the number of scenarios that reach the worst case at the same time is constrained by the corresponding budget constraint, which is expressed as:

$$\sum_{s \in N_s}\left|\alpha_s\right| \le \Gamma \tag{25}$$

where Γ is the conservativeness parameter, which represents the number of contingency scenarios that reach the worst case. Then, the entire ambiguity set of contingency probability can be written as:

$$\mathbb{Q} = \left\{\alpha_s \,\middle|\, \sum_{s \in N_s}\left(\bar{\lambda}_s + \alpha_s \hat{\lambda}_s\right) = 1, \sum_{s \in N_s}\left|\alpha_s\right| \le \Gamma, -1 \le \alpha_s \le 1; \forall s \in N_s\right\} \tag{26}$$

IV. Solution Methodology

After the random contingency probability is expressed in the deterministic form, the entire optimization will be transformed into an RSO [37] TEP model and can be expressed as a min-max-min optimal problem. The original objective function can be transformed as:

$$\min_{u_l,P_g}\left\{\sum_{l\in N_L}u_lC_l+\sum_{g\in N_G}\lambda^{normal}C_g+\max_{\alpha_s\in\mathbb{Q}}\sum_{s\in N_S}\left[\left(\bar{\lambda}_s+\alpha_s\hat{\lambda}_s\right)\min_{lol_{s,i}}\left(VOLL\cdot\sum_{i\in N_I}lol_{s,i}\right)\right]\right\} \tag{27}$$

The objective function (27) minimizes the investment cost and operation cost under normal operation condition, and the load shedding cost against the worst case of contingency probability.

A. Linearization of Nonlinear Terms

In this paper, a quadratic operation cost function which can be easily piecewise linearized is applied. The products of binary variables and continuous variables exist in constraints (4), (7), and (11), and can be linearized by the big M method [21]. The constraints (4) and (11) can be linearized as:

$$-M\left(1-u_l\right)\le f_l-\frac{1}{X_l}\left(\sum_{i=B_{start}(l)}\theta_i-\sum_{i=B_{end}(l)}\theta_i\right)\le M\left(1-u_l\right)\quad\forall l\in N_L^+ \tag{28}$$

$$-M\left(1-u_l\right)\le f_{s,l}-\frac{1}{X_l}\left(\sum_{i=B_{start}(l)}\theta_{s,i}-\sum_{i=B_{end}(l)}\theta_{s,i}\right)\le M\left(1-u_l\right)\quad\forall s,\forall l\in N_L^+ \tag{29}$$

Another nonlinear term is caused by the absolute sign in (26). $\left|\alpha_s\right|$ can be linearized as:

$$\left|\alpha_s\right|=\alpha_s^++\alpha_s^-\quad\forall s \tag{30}$$

$$\alpha_s=\alpha_s^+-\alpha_s^-\quad\forall s \tag{31}$$

where α_s^+ and α_s^- are the auxiliary non-negative variables.

B. Model Simplification and Duality

For the contingency probability concerned, the uncertain range is significant compared with the forecasted value. However, the interval width of the contingency probability under the normal operation condition is relatively small compared with the forecasted value, thus the contingency probability under normal operation condition can be simplified as the fixed forecasted value.

In order to transform the three-level optimization problem of (27) into a single-level model, firstly, the optimal value of the loss of load, i. e., the recourse function, is reformulated as:

$$Q(s)=\min\left(VOLL\cdot\sum_{i\in N_I}lol_{s,i}\right) \tag{32}$$

Then, the three-level objective function possesses a form of bi-level optimization, i.e.,

$$\min\left[\sum_{l\in N_L}u_lC_l+\sum_{g\in N_G}\lambda^{normal}C_g+VOLL\cdot\sum_{s\in N_S}\sum_{i\in N_I}\bar{\lambda}_s\cdot lol_{s,i}+\max_{\alpha_s^+,\alpha_s^-}\sum_{s\in N_S}\left(\alpha_s^+-\alpha_s^-\right)\hat{\lambda}_sQ(s)\right] \tag{33}$$

Considering the inner maximization problem can be equivalently expressed as a minimization problem based on the duality theory, the bi-level optimization problem can be recast as a single-level optimization problem. The inner maximization problem can be expressed as:

$$\begin{cases}\max\limits_{\alpha_s^+,\alpha_s^-}\sum\limits_{s\in N_S}\left(\alpha_s^+-\alpha_s^-\right)\hat{\lambda}_sQ(s)\\ \text{s.t. }\sum\limits_{s\in N_s}\left(\alpha_s^+-\alpha_s^-\right)\hat{\lambda}_s=1-\sum\limits_{s\in N_s}\bar{\lambda}_s & (r^1)\\ \sum\limits_{s\in N_s}\left(-\alpha_s^+-\alpha_s^-\right)\ge\Gamma & (r^2)\\ \alpha_s^+-\alpha_s^-\ge-1\quad\forall s & (r_s^3)\\ -\alpha_s^++\alpha_s^-\ge1\quad\forall s & (r_s^4)\\ \alpha_s^+\ge0\\ \alpha_s^-\ge0\end{cases} \tag{34}$$

where r^1, r^2, r_s^3, and r_s^4 are the dual variables of corresponding constraints. The dual model can be written as:

$$\min_{r^1,r^2,r_s^3,r_s^4}\left[\left(1-\sum_{s\in N_s}\bar{\lambda}_s\right)r^1+\Gamma r^2+\sum_{s\in N_S}\left(r_s^3+r_s^4\right)\right] \tag{35}$$

s.t.

$$\hat{\lambda}_sr^1+r^2+r_s^3\ge\hat{\lambda}_sQ(s)\quad\forall s \tag{36}$$

$$-\hat{\lambda}_sr^1+r^2+r_s^4\ge-\hat{\lambda}_sQ(s)\quad\forall s \tag{37}$$

Based on the epigraph reformulation [34], the optimal value of (35) can be directly expressed as:

$$\left(1-\sum_{s\in N_s}\bar{\lambda}_s\right)r^1+\Gamma r^2+\sum_{s\in N_S}\left(\max\left(-\hat{\lambda}_sr^1-r^2+\hat{\lambda}_sQ(s),0\right)+\max\left(\hat{\lambda}_sr^1-r^2-\hat{\lambda}_sQ(s),0\right)\right) \tag{38}$$

Through the above operation, the variables r_s^3 and r_s^4 and constraints (36) and (37) are eliminated, and the computational burden can be significantly reduced. By substituting (37) into (33), (33) can be expressed as:

$$\min\left[\sum_{l\in N_L^+}u_lC_l+\sum_{g\in N_G}\lambda^{normal}C_g+\left(1-\sum_{s\in N_s}\bar{\lambda}_s\right)r^1+\Gamma r^2+\sum_{s\in N_S}\bar{\lambda}_sQ(s)+\sum_{s\in N_S}\left(\max\left(-\hat{\lambda}_sr^1-r^2+\hat{\lambda}_sQ(s),0\right)+\max\left(\hat{\lambda}_sr^1-r^2-\hat{\lambda}_sQ(s),0\right)\right)\right] \tag{39}$$

The objective function (39) can be further transformed into (40) based on the epigraph reformulation [34] by introducing auxiliary variables z_s^1, z_s^2, and z_s^3, which is also subjected to constraints (2)-(10), (12)-(16), (28), (29), and (41)-(44).

$$\min\left[\sum_{l\in N_L}u_lC_l+\sum_{g\in N_G}\lambda^{normal}C_g+\left(1-\sum_{s\in N_s}\bar{\lambda}_s\right)r^1+\Gamma r^2+\sum_{s\in N_S}\left(z_s^1+z_s^2+z_s^3\right)\right] \tag{40}$$

s.t.

$$z_s^1\ge VOLL\cdot\bar{\lambda}_sQ(s)\quad\forall s \tag{41}$$

$$z_s^2 \geq -\hat{\lambda}_s r^1 - r^2 + VOLL \cdot \hat{\lambda}_s Q(s) \quad \forall s \tag{42}$$

$$z_s^3 \geq \hat{\lambda}_s r^2 - r^2 - VOLL \cdot \hat{\lambda}_s Q(s) \quad \forall s \tag{43}$$

$$\begin{cases} z_s^1 \geq 0 \\ z_s^2 \geq 0 \\ z_s^3 \geq 0 \end{cases} \tag{44}$$

This model cannot be solved directly due to $Q(s)$ in constraints (41)-(43). When the inner maximization problem (32) is regarded as a sub-problem, the same configuration of Benders decomposition can be shared by the bi-level maximization problem. Especially, considering the inner problem is dependent with the contingency and the contingency events are independent with each other, the Benders decomposition is applied to iteratively solve the bi-level optimization problem [33]. For the sub-problem, $Q(s)$ is the objective function, and (9), (10), (12)-(16), and (29) are the corresponding constraints with some variables given in the master problem.

C. Benders Decomposition

When Benders decomposition is applied, the corresponding model has to be divided into master problem and sub-problems. The master problem and sub-problems involve the constraints related to the normal condition and contingencies, respectively. Meanwhile, the sub-problems are transformed to dual sub-problems. The procedure of the Benders decomposition is shown in Fig. 1 [38]-[40], where *UB* and *LB* are the upper bound and lower bound of the convergence gap of Benders decomposition, respectively.

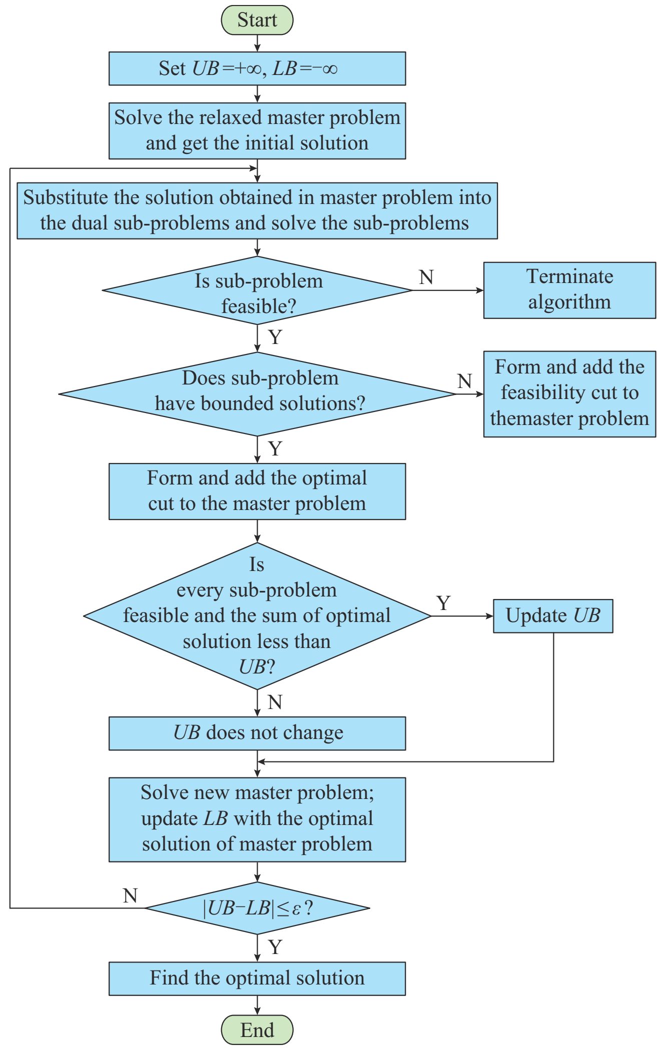

Fig. 1. Flowchart of Benders decomposition.

V. Case Studies

The proposed model is tested on the IEEE RTS system and IEEE 118-bus system. The data of equipment outage rate are taken from [41] and [42]. In the cases presented in this paper, the simultaneous outage of three or more equipment is not considered. The convergence tolerance of the Bender decomposition is set to be 0.01%. The proposed model is coded on the GAMS platform and is solved by the commercial solver CPLEX [20]. The optimization is implemented on a computer with Win7 system, Intel Core i7-4790 processors at 3.6 GHz, and 4 GB of RAM.

A. Case 1: IEEE RTS System

The IEEE RTS system consists of 24 buses, 26 units, and 38 existing transmission lines, and the corresponding data can be found in [42]. The load is 2850 MW. Similar to [1], two transmission lines are allowed to be constructed in each transmission corridor. The maximum capacity of each transmission line is reduced to half of the original value [42]. The investment cost of each candidate transmission line is also given as in [42]. *VOLL* is set to be 5000 \$/MWh and the conservativeness parameter is set to be 30. A width parameter (WP) is introduced to quantify the interval width and the relationship is expressed as:

$$\left[\gamma^{\min}, \gamma^{\max}\right] = \left[\bar{\gamma}/WP, \bar{\gamma} \cdot WP\right] \tag{45}$$

where $\bar{\gamma}$ is the average outage rate of a single equipment based on the historical data.

Three scenarios are considered in this case. In scenario 1, a fixed estimated contingency probability is applied. In scenario 2, the contingency probability is described by an interval and the WP is set to be 10. In scenario 3, the performance of the construction plan in scenario 1 is tested with the uncertain contingency probability. The costs of the three scenarios are shown in Table I and the construction plans of scenarios 1 and 2 are shown in Table II.

TABLE I
COSTS OF THREE SCENARIOS IN CASE 1

Scenario	Total cost (M\$)	Investment cost (M\$)	Operation cost (M\$)	Expected load-shedding cost (M\$)
1	533.247	166.150	366.850	0.248
2	552.241	173.440	366.167	12.635
3	552.889	166.150	366.850	19.889

TABLE II
CONSTRUCTION PLAN OF SCENARIOS 1 AND 2 IN CASE 1

Scenario	Newly constructed transmission line
1	6, 10, 16, 23, 28
2	6, 10, 16, 23, 28, 30

In Table I, it can be observed from scenarios 1 and 2 that when considering the uncertainty of contingency probability, the total cost, investment cost, and expected load-shedding cost increase while the operation cost almost does not change. The increase of investment cost means that new transmission lines are constructed when the contingency probability is considered, which illustrates that the planning of transmission lines is affected by the contingency probability. Moreover, the expected load-shedding cost increases tens of times when the contingency probability is considered. Meanwhile, it can be observed from the costs of scenarios 3 that the construction plan in scenario 1 will lead to a higher expected load-shedding cost, so the construction plan needs to be changed in order to minimize the total cost. The comparison reveals that using a fixed estimated contingency probability would over-estimate the system reliability level and the results may be over-optimistic. From Table II, it can be found that when considering the uncertainty of contingency probability, a new transmission line which is located in the corridor of transmission line 30 is constructed.

The total cost and expected load-shedding cost with different WPs and *VOLL* are given in Tables III and IV.

TABLE III
TOTAL COST WITH DIFFERENT WPS AND *VOLL* IN CASE 1

VOLL ($/MWh)	Total cost with different WPs ($)					
	5	6	7	8	9	10
5000	538.0	540.2	542.8	545.7	549.1	552.2
6000	539.0	541.6	544.7	548.3	551.9	554.8
7000	540.0	543.0	546.7	550.9	554.0	557.3
8000	541.0	544.5	548.6	552.6	556.0	559.8
9000	542.0	545.9	550.8	554.2	558.0	562.4
10000	543.0	547.8	552.0	555.8	560.1	564.9

TABLE IV
EXPECTED LOAD-SHEDDING COST WITH DIFFERENT WPS AND *VOLL* IN CASE 1

VOLL ($/MWh)	Expected load-shedding cost with different WPs ($)					
	5	6	7	8	9	10
5000	4.97	7.16	9.75	12.73	16.12	12.64
6000	5.97	8.59	11.70	15.29	12.28	15.16
7000	6.96	10.03	13.65	11.32	14.33	17.69
8000	7.96	11.46	15.61	12.94	16.38	26.82
9000	8.95	12.89	11.14	14.56	18.42	22.77
10000	9.95	8.22	12.38	16.17	20.49	25.30

From Tables III and IV, it can be observed that the total cost and expected load-shedding cost increase when the *VOLL* or WP increases.

The construction plans with different WPs and *VOLL* in case 1 are given in Table V, where the symbol √ means that a new transmission line is constructed in the corridor of transmission line 30. In Table V, it can be found that new transmission lines are intended to be constructed when the *VOLL* or WP becomes larger. The construction plan of new transmission lines depends on the balance between the expected load-shedding cost and line construction cost and operation cost.

TABLE V
CONSTRUCTION PLANS WITH DIFFERENT WPS AND *VOLL* IN CASE 1

VOLL ($/MWh)	Construction plan with different WPs					
	5	6	7	8	9	10
5000						√
6000					√	√
7000				√	√	√
8000			√	√	√	
9000			√	√	√	√
10000		√	√	√	√	√

The relationship between the total cost and conservativeness parameter Γ in case 1 is shown in Fig. 2. The *VOLL* is set to be 5000 $/MWh, and the WP is set to be 10.

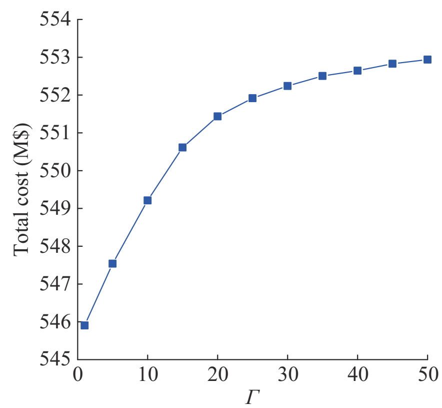

Fig. 2. Relationship between total cost and Γ in Case 1.

From Fig. 2, it can be found that when Γ increases, the total cost also increases and finally becomes saturate. This is because the more serious situation with less occurring probability is considered.

B. Case 2: IEEE 118-bus System

The IEEE 118-bus system consists of 118 nodes, 186 transmission lines, and 54 units. All data of the system can be found in [43]. Two transmission lines are allowed to be constructed in each transmission corridor. The construction price for each candidate transmission line is 0.4 M$/(km·MW). The load is 6350 MW. The *VOLL* and Γ are set to be 5000 $/MWh and 30, respectively. Table VI shows the costs with different WPs in case 2.

Similar to that in case 1, it can be found from Table VI that the total cost, investment cost, and expected load-shedding cost increase when the WP becomes larger. Table VII shows the construction plans with different WPs in case 2.

Table VIII shows the costs with different *VOLL* in case 2. The influence of *VOLL* is also similar to that in case 1. From Table VIII, it can be found that the total cost, investment cost, and expected load-shedding increase when *VOLL* becomes larger. The operation cost is almost unchanged. Table IX shows the construction plans with different *VOLL* in case 2. It can be found that the construction plan changes with different *VOLL*.

TABLE VI
COSTS WITH DIFFERENT WPS IN CASE 2

WP	Total cost (M$)	Investment cost (M$)	Operation cost (M$)	Expected load-shedding cost (M$)
1	878.03	112.0	763.83	2.20
3	887.42	112.0	763.83	11.58
5	907.76	112.0	763.83	31.93
7	932.25	123.2	763.83	45.22
10	972.96	140.0	763.35	69.62

TABLE VII
CONSTRUCTION PLANS WITH DIFFERENT WPS IN CASE 2

WP	Constructed transmission line	Number of new transmission lines
1	37, 42, 49, 75, 106, 121, 159, 184	8
3	37, 42, 49, 75, 106, 121, 159, 184	8
5	37, 42, 49, 75, 106, 121, 159, 184	8
7	4, 24, 29, 37, 42, 75, 106, 109, 121, 159, 184	11
10	4, 23, 24, 29, 37, 42, 49, 75, 106, 109, 121, 155, 159, 184	14

TABLE VIII
COSTS WITH DIFFERENT *VOLL* IN CASE 2

VOLL ($/MWh)	Total cost (M$)	Investment cost (M$)	Operation cost (M$)	Expected load-shedding cost (M$)
5000	973.0	140.0	763.4	69.6
6000	985.6	140.0	763.4	82.3
7000	998.5	147.9	763.3	87.2
8000	1012.3	147.9	763.3	101.1
9000	1025.0	147.9	763.3	113.7
10000	1037.6	147.9	763.3	126.4

TABLE IX
CONSTRUCTION PLANS WITH DIFFERENT *VOLL* IN CASE 2

VOLL ($/MWh)	Constructed transmission line
5000	4, 23, 24, 29, 37, 42, 49, 75, 106, 109, 121, 155, 159, 184
6000	4, 23, 24, 29, 37, 42, 49, 75, 106, 109, 121, 155, 159, 184
7000	4, 29, 37, 42, 49, 75, 106, 109, 121, 153, 159, 184
8000	4, 29, 37, 42, 49, 75, 106, 109, 121, 153, 159, 184
9000	4, 29, 37, 42, 49, 75, 106, 109, 121, 153, 159, 184
10000	4, 29, 37, 42, 49, 75, 106, 109, 121, 153, 159, 184

The computation time with different WPs in case 2 is given in Table X. It can be observed that when considering the uncertainty of contingency probability, the computation time increases nearly 10 times. Although a longer computation time is required, it does not affect the feasibility of the proposed model since the computation time is not vital in the planning problem. Besides, a lot of techniques [44] such as the critical constraint screening and parallel computation can be applied to enhance the computation efficiency.

TABLE X
COMPUTATION TIME WITH DIFFERENT WPS IN CASE 2

WP	Iteration times	Computation time (min)
1	1	131.0
10	5	1342.5

VI. CONCLUSION

In this paper, an DRO TEP model considering the uncertainty of contingency probability is proposed. The uncertainty of contingency probability can be expressed by the uncertainty of the outage of equipment. The proposed model involves random parameters and can be reformulated into a tri-level optimization problem. It is finally recast as a bi-level model by using the epigraph reformulation and dual theory, which can be solved by the Benders decomposition. The case studies show that the cost and construction plan are significantly influenced by the uncertainty of contingency probability. Using a fixed contingency probability would cause a less reliable construction plan and the result would be over-optimistic.

REFERENCES

[1] S. Lumbreras and A. Ramos, "The new challenges to transmission expansion planning: survey of recent practice and literature review," *Electric Power Systems Research*, vol. 134, pp. 19-29, May 2016.

[2] N. G. Ude, H. Yskandar, and R. C. Graham, "A comprehensive state-of-the-art survey on the transmission network expansion planning optimization algorithms," *IEEE Access*, vol. 7, pp. 123158-123181, Aug. 2019.

[3] M. Chen, C. Gao, Z. Li *et al*., "Aggregated model of data network for the provision of demand response in generation and transmission expansion planning," *IEEE Transactions on Smart Grid*, vol. 12, no. 1, pp. 512-523, Jan. 2021.

[4] X. Zhang and A. J. Conejo, "Robust transmission expansion planning representing long- and short-term uncertainty," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1329-1338, Mar. 2018.

[5] M. Alraddadi, A. J. Conejo, and R. M. Lima, "Expansion planning for renewable integration in power system of regions with very high solar irradiation," *Journal of Modern Power Systems and Clean Energy*, vol. 9, no. 3, pp. 485-494, May 2021.

[6] A. J. Conejo, Y. Cheng, N. Zhang *et al*., "Long-term coordination of transmission and storage to integrate wind power," *CSEE Journal of Power and Energy Systems*, vol. 3, no. 1, pp. 36-43, Mar. 2017.

[7] M. Majidi-Qadikolai and R. Baldick, "A generalized decomposition framework for large-scale transmission expansion planning," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1635-1649, Mar. 2018.

[8] W. Wu, Z. Hu, Y. Song *et al*., "Transmission network expansion planning based on chronological evaluation considering wind power uncertainties," *IEEE Transactions on Power Systems*, vol. 33, no. 5, pp. 4787-4796, Sept. 2018.

[9] A. Moreira, G. Strbac, R. Moreno *et al*., "A five-level milp model for flexible transmission network planning under uncertainty: a min-max regret approach," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 486-501, Jan. 2018.

[10] O. Ziaee, O. Alizadeh-Mousavi, and F. F. Choobineh, "Co-optimization of transmission expansion planning and TCSC placement considering the correlation between wind and demand scenarios," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 206-215, Jan. 2018.

[11] J. Li, Z. Li, F. Liu *et al*., "Robust coordinated transmission and generation expansion planning considering ramping requirements and construction periods," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 268-280, Jan. 2018.

[12] S. Dehghan, N. Amjady, and A. J. Conejo, "A multistage robust transmission expansion planning model based on mixed binary linear decision rules–Part I," *IEEE Transactions on Power Systems*, vol. 33, no. 5, pp. 5341-5350, Sept. 2018.

[13] L. Baringo and A. Baringo, "A stochastic adaptive robust optimization approach for the generation and transmission expansion planning," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 792-802, Jan. 2018.

[14] J. P. Bukenberger and M. D. Webster, "Approximate latent factor algorithm for scenario selection and weighting in transmission expansion planning," *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1099-1108, Mar. 2020.

[15] X. Zhu, Z. Yu, and X. Liu, "Security constrained unit commitment with extreme wind scenarios," *Journal of Modern Power Systems and Clean Energy*, vol. 8, no. 3, pp. 464-472, May 2020.

[16] C. Roldán, R. Mínguez, R. García-Bertrand *et al*., "Robust transmission network expansion planning under correlated uncertainty," *IEEE Transactions on Power Systems*, vol. 34, no. 3, pp. 2071-2082, May 2019.

[17] A. J. Conejo, L. Baringo, S. J. Kazempour *et al*., "Transmission expansion planning," in *Investment in Electricity Generation and Transmission: Decision Making Under Uncertainty*. New York: Springer Publishing Company, 2016.

[18] F. Verástegui, Á. Lorca, D. E. Olivares *et al*., "An adaptive robust optimization model for power systems planning with operational uncertainty," *IEEE Transactions on Power Systems*, vol. 34, no. 6, pp. 4606-4616, Nov. 2019.

[19] R. Mínguez, R. García-Bertrand, J. M. Arroyo *et al*., "On the solution of large-scale robust transmission network expansion planning under uncertain demand and generation capacity," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 1242-1251, Mar. 2018.

[20] B. Zhao, A. J. Conejo, and R. Sioshansi, "Coordinated expansion planning of natural gas and electric power systems," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 3064-3075, May 2018.

[21] Z. Zhuo, E. Du, N. Zhang *et al*., "Incorporating massive scenarios in transmission expansion planning with high renewable energy penetration," *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1061-1074, Mar. 2020.

[22] A. Velloso, D. Pozo, and A. Street, "Distributionally robust transmission expansion planning: a multi-scale uncertainty approach," *IEEE Transactions on Power Systems*, vol. 35, no. 5, pp. 3353-3365, Sept. 2020.

[23] J. Choi, T. Mount, R. J. Thomas *et al*., "Probabilistic reliability criterion for planning transmission system expansions," *IET Proceedings: Generation, Transmission, and Distribution*, vol. 153, no. 6, pp. 719-727, Dec. 2006.

[24] A. Arabali, M. Ghofrani, M. Etezadi-Amoli *et al*., "A multi-objective transmission expansion planning framework in deregulated power systems with wind generation," *IEEE Transactions on Power Systems*, vol. 29, no. 6, pp. 3003-3011, Nov. 2014.

[25] J. Wang, H. Zhong, Q. Xia *et al*., "Transmission network expansion planning with embedded constraints of short circuit currents and *N*–1 security," *Journal of Modern Power Systems and Clean Energy*, vol. 3, no. 3, pp. 312-320, Sept. 2015.

[26] J. Qiu, H. Yang, Z. Dong *et al*., "A linear programming approach to expansion co-planning in gas and electricity markets," *IEEE Transactions on Power Systems*, vol. 31, no. 5, pp. 3594-3606, Sept. 2016.

[27] F. Bouffard, F. D. Galiana, and A. J. Conejo, "Market-clearing with stochastic security-part I: formulation," *IEEE Transactions on Power Systems*, vol. 20, no. 4, pp. 1818-1826, Nov. 2005.

[28] A. Bagheri and C. Zhao, "Distributionally robust reliability assessment for transmission system hardening plan under *N*–*k* security criterion," *IEEE Transactions on Reliability*, vol. 68, no. 2, pp. 653-662, Jun. 2019.

[29] R. Billinton and R. N. Allan. *Reliability Evaluation of Power Systems*. New York: Plenum Press, 1984.

[30] C. Zhao and R. Jiang, "Distributionally robust contingency-constrained unit commitment," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 94-102, Jan. 2018.

[31] D. Alvarado, A. Moreira, R. Moreno *et al*., "Transmission network investment with distributed energy resources and distributionally robust security," *IEEE Transactions on Power Systems*, vol. 34, no. 6, pp. 5157-5168, Nov. 2019.

[32] M. Yang, J. Wang, H. Diao *et al*., "Interval estimation for conditional failure rates of transmission lines with limited samples," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2752-2763, Jul. 2018.

[33] T. Aurélie, "Robust stochastic programming with uncertain probabilities," *IMA Journal of Management Mathematics*, vol. 19, no. 3, pp. 289-321, Jul. 2008.

[34] S. Boyd, L. Vandenberghe, and L. Faybusovich., "Convex optimization," *IEEE Transactions on Automatic Control*, vol. 51, no. 11, pp. 1859-1859, Nov. 2006.

[35] M. Wang, H. B. Gooi, and S. Chen, "Optimizing probabilistic spinning reserve using an analytical expected-energy-not-supplied formulation, " *IET Generation, Transmission & Distribution*, vol. 5, no. 7, pp. 772-780, Jul. 2011.

[36] M. Yang, J. Wang, H. Diao *et al*., "Interval estimation for conditional failure rates of transmission lines with limited samples," *IEEE Transactions on Smart Grid*, vol. 9, no. 4, pp. 2752-2763, Jul. 2018.

[37] C. Yang, Z. Xiang, G. Ma *et al*, "Multi-drive overhead-line failure rate model," *Power System Protection and Control*, vol. 46, no. 12, pp. 9-15, Jun. 2018.

[38] J. F. Benders, "Partitioning procedures for solving mixed-variables programming problems," *Numerische Mathematik*, vol. 4, pp. 238-252, Dec. 1962.

[39] L. C. da Costa, F. S. Thomé, J. D. Garcia *et al*., "Reliability-constrained power system expansion planning: a stochastic risk-averse optimization approach," *IEEE Transactions on Power Systems*, vol. 36, no. 1, pp. 97-106, Jan. 2021.

[40] M. Esmaili, M. Ghamsari-Yazdel, N. Amjady *et al*., "Convex model for controlled islanding in transmission expansion planning to improve frequency stability," *IEEE Transactions on Power Systems*, vol. 36, no. 1, pp. 58-67, Jan. 2021.

[41] J. Aghaei, N. Amjady, A. Baharvandi *et al*., "Generation and transmission expansion planning: MILP-based probabilistic model," *IEEE Transactions on Power Systems*, vol. 29, no. 4, pp. 1592-1601, Jul. 2014.

[42] C. Grigg, P. Wong, P. Albrecht *et al*., "The IEEE reliability test system-1996: a report prepared by the reliability test system task force of the application of probability methods subcommittee," *IEEE Transactions on Power Systems*, vol. 14, no. 3, pp. 1010-1020, Aug. 1999.

[43] IIT. (2008, Apr.). IEEE 118-bus system. [Online]. Available: http://motor.ece.iit.edu/data/

[44] J. P. Bukenberger and M. D. Webster, "Approximate latent factor algorithm for scenario selection and weighting in transmission expansion planning," *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1099-1108, Mar. 2020.

Weilun Wang received the B.E. degree in electrical engineering from Shandong University, Jinan, China, in 2018. He is currently pursuing the M.S. degree in electrical engineering at Shandong University. His research interests include optimization, planning, and unit commitment.

Mingqiang Wang received the Ph.D. degree from Nanyang Technological University, Singapore, in 2012. He is currently an Associate Professor with the School of Electrical Engineering, Shandong University, Jinan, China. His research interests include power system economic operation and microgrids.

Xueshan Han received the Ph.D. degree in electrical engineering from the Harbin Institute of Technology, Harbin, China, in 1994. He is currently a Full Professor of electrical engineering with Shandong University, Jinan, China. His research interests include power system operation and control, power system reliability assessment, and power economics.

Ming Yang received the Ph.D. degrees in electrical engineering from Shandong University, Jinan, China, in 2009. He is currently an Associate Professor with Shandong University. His research interests include power system optimal operation and control.

Qiuwei Wu received the Ph.D. degree in power system engineering from Nanyang Technological University, Singapore, in 2009. He is currently working at Technical University of Denmark, Copenhagen, Denmark. His research interests include operation and control of power systems with high penetration of renewables.

Ran Li received the B.E. degree in electrical engineering and its automation from Northeast Normal University, Changchun, China, in 2018. She is currently pursuing the M.S. degree with the School of Electrical Engineering, Shandong University, Jinan, China. Her research interests include power system optimal operation and control.