Data-driven Probabilistic Static Security Assessment for Power System Operation Using High-order Moments

Guanzhong Wang, Zhiyi Li, Feng Zhang, Ping Ju, Hao Wu, and Changsen Feng

***Abstract*—In this letter, a new formulation of Lebesgue integration is used to evaluate the probabilistic static security of power system operation with uncertain renewable energy generation. The risk of power flow solutions violating any pre-defined operation security limits is obtained by integrating a semi-algebraic set composed of polynomials. With the high-order moments of historical data of renewable energy generation, the integration is reformulated as a generalized moment problem which is then relaxed to a semi-definite program (SDP). Finally, the effectiveness of the proposed method is verified by numerical examples.**

***Index Terms*—Data-driven analysis, probabilistic static security assessment, power system operation, distributionally robust approach.**

I. Introduction

WITH rapid adoption of intermittent renewable energy sources, e.g., wind power and solar generation, uncertainties in renewable energy generation have been threatening the security of power system operation. Probabilistic static security assessment (PSSA) is a well-established way to characterize the influence of those uncertainties on power system operation [1]. The static security assessment is widely used to check the overvoltage/undervoltage condition of buses and the overload status of lines and transformers in the present condition of power system or some contingency operation points due to component outages or fluctuations of renewable energy generation in the near future. In this letter, we focus on the fluctuation and uncertainty of the renewable energy generation.

In common practices of power engineering, only partial statistical properties, e. g., the mean or first-order moment, the variance or second-order moment and high-order moment, are available for modeling stochastic renewable energy generation [2]. Thus, the probability distributions (PDs) of power system operation states are uncertain where it is of practical significance to evaluate the worst-case probability of overload or overvoltage/undervoltage condition among all the admissible PDs. To this end, the moment-based distributionally robust optimization methods are proposed in the literature [3]. However, existing moment-based methods are only applicable to either linear power flow equations or PDs with the first two moments, eventually compromising the trustworthiness of PSSA.

This letter aims to perform high-order (more than 2) moment-based distributionally robust probabilistic static security assessment (DR-PSSA) based on nonlinear power flow equations (NPFEs) with uncertain input parameters. The DR-PSSA is formulated as a generalized moment problem [4], which is then relaxed to a semi-definite program (SDP).

II. Problem Formulation

This section gives a brief introduction of the DR-PSSA problem in terms of a single continuous random parameter $q \in \Delta \subseteq \mathbb{R}$, where Δ is the range or set of random parameters.

Mathematically, when uncertain parameters are incorporated into NPFEs, PSSA is aimed at evaluating the probability of normal power flow solutions violating any pre-defined operation security limits. $K(q)$ and $K_{bad}(q)$ are defined as the feasible solutions of power flow equations and the insecure solutions, i.e., at least one security limit is violated, pertinent to random variable q whose PD is P_q, respectively. Accordingly, PSSA is equivalent to the integration (Lebesgue integration) of q over $K_{bad}(q)$:

$$Pr = \int_{K_{bad}(q)} \mathrm{d}P_q \tag{1}$$

where Pr is the calculated probability that achieves a value between 0 and 1. Assuming that NPFEs are solvable for any $q \in \Delta$, the integration of q over $K(q)$ always equals 1.

As NPFEs are usually formulated in rectangle coordinate in the existing literature [1], and they can be expanded into a set of equivalent second polynomial inequalities with a doubled size, $K(q)$ and $K_{bad}(q)$ both can be represented as a semi-algebraic set of polynomials [4]. Suppose that

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$$K(q)=\{q\in\Delta, g_i(\boldsymbol{v},q)\geq 0, i=1,2,\ldots\} \tag{2}$$

where $\{g_i(\boldsymbol{v},q)\geq 0, i=1,2,\ldots\}$ is a unified form of NPFEs; and $\boldsymbol{v}=[\boldsymbol{e},\boldsymbol{f}]^{\mathrm{T}}$ is the state vector consisting of real and imaginary parts of each bus voltage, i.e., $\boldsymbol{e}$ and $\boldsymbol{f}$. The probability of any branch flow or bus voltage exceeding any security limit can be evaluated through representing $K_{bad}(q)$ as a set of inequalities in state variables. For example, to evaluate the undervoltage probability of each bus by comparing with its limit V_{i0}, $K_{bad}(q)$ can be stated as:

$$K_{bad}(q)=\{g_0(\boldsymbol{v})\geq 0\}\cap K(q)=\{q\in\Delta, g_i(\boldsymbol{v},q)\geq 0, i=0,1,\ldots\} \tag{3}$$

where $g_0(\boldsymbol{v})=V_{i0}^2-e_i^2-f_i^2$, and e_i and f_i are the i^{th} elements of $\boldsymbol{e}$ and $\boldsymbol{f}$, respectively.

Next, we focus on how to calculate $\overline{Pr}$, i.e., the upper probability over all the admissible PDs with given moments of power flow solutions in $K_{bad}(q)$. The operation risk can be evaluated by $\overline{Pr}$. $\overline{Pr}$ can be regarded as the optimum in the following semi-infinite linear optimization problem [5]:

$$\overline{Pr}=\sup_{P_{\boldsymbol{v},q}}\int_{K_{bad}(q)}\mathrm{d}P_{\boldsymbol{v},q} \tag{4}$$

s.t.

$$\int_{K(q)}\mathrm{d}P_{\boldsymbol{v},q}=1 \tag{5}$$

$$\int_{K(q)}q^i\mathrm{d}P_{\boldsymbol{v},q}=m_i \quad i=1,2,\ldots \tag{6}$$

where $P_{\boldsymbol{v},q}$ is the joint PD of $\boldsymbol{v}$ and q; and m_i is the provided i^{th} moment of q. The semi-infinite linear optimization problem (4)-(6) is referred to as the generalized moment problem [4].

Hence, DR-PSSA maximizes the integration on $K_{bad}(q)$, i.e., (4), by finding $P_{\boldsymbol{v},q}$ in the worst case supported on $K(q)$, i.e., (5), while respecting the moment constraint, i.e., (6). Moreover, if multiple violations are encountered, the violations can be directly added into the set $K_{bad}(q)$, which means that the proposed method can deal with the intersection set of multiple violations in the set $K_{bad}(q)$.

III. Computationally Tractable SDP Relaxations

A. Reformulation of DR-PSSA

First, define $\boldsymbol{x}=[\boldsymbol{v}^{\mathrm{T}},q]^{\mathrm{T}}=[x_1,x_2,\ldots,x_n]^{\mathrm{T}}$. DR-PSSA can be reformulated as an equivalent generalized moment problem through introducing an auxiliary measure P of $\boldsymbol{x}$ based on Theorem 3.1 in [6], where PD is regarded as a special instance of measures. The proposed DR-PSSA problem, i.e., (4)-(6), is then reformulated as (7)-(9), where (8) is the compact form of (5) and (6) when $m_0=1$.

$$\overline{Pr}=\sup_{P,P_{\boldsymbol{x}}}\int_{K_{bad}(q)}\mathrm{d}P \tag{7}$$

s.t.

$$\int_{K(q)}q^i\mathrm{d}P_{\boldsymbol{x}}=m_i \quad i=0,1,\ldots \tag{8}$$

$$\begin{cases}P\leq P_{\boldsymbol{x}}\\ P\in M(K_{bad}(q))\\ P_{\boldsymbol{x}}\in M(K(q))\end{cases} \tag{9}$$

where $P_{\boldsymbol{x}}$ is the PD of random variable q whose supporting set is $K(q)$; and $M(K(q))$ is the set of all possible PDs of q. $P\leq P_{\boldsymbol{x}}$ means $\int_{\boldsymbol{K}_s}\mathrm{d}P\leq\int_{\boldsymbol{K}_s}\mathrm{d}P_{\boldsymbol{x}}$ for any $\boldsymbol{K}_s\subseteq\mathbb{R}^n$. $P_{\boldsymbol{x}}\in M(K(q))$ means $\int_{K_c}\mathrm{d}P_{\boldsymbol{x}}=0$ for any $K_c\cap K(q)=\varnothing$, $K_c\subseteq\mathbb{R}^n$. $P\in M(K_{bad}(q))$ is defined in the same way as $P_{\boldsymbol{x}}\in M(K(q))$.

B. SDP Relaxations

DR-PSSA can be approximated as $\overline{Pr_d}$ by performing d-order SDP relaxations of $y_{\boldsymbol{x}}=\{y_{\boldsymbol{x\alpha}}\}$ and $y=\{y_{\boldsymbol{\alpha}}\}$, respectively, of which moment sequences correspond to $P_{\boldsymbol{x}}$ and P [7], as shown in (10)-(14). $y_{\boldsymbol{x\alpha}}$ is the approximated moment with an order up to $2d$ for $P_{\boldsymbol{x}}$, i.e., $y_{\boldsymbol{x\alpha}}=\int_{K(q)}\boldsymbol{x}^{\boldsymbol{\alpha}}\mathrm{d}P_{\boldsymbol{x}}$, and $\boldsymbol{x}^{\boldsymbol{\alpha}}=x_1^{\alpha_1}x_2^{\alpha_2}\cdot\ldots\cdot x_n^{\alpha_n}$ with $\sum_{i=1}^{n}\alpha_i\leq d$, where α_i is an integer and d is also an integer larger than the degree of polynomials in $K(q)$ and $K_{bad}(q)$. $y_{\boldsymbol{\alpha}}$ is the approximated moment with an order up to $2d$ for P, i.e., $y_{\boldsymbol{\alpha}}=\int_{K_{bad}(q)}\boldsymbol{x}^{\boldsymbol{\alpha}}\mathrm{d}P$.

$$\overline{Pr_d}=\sup(y_0) \tag{10}$$

s.t.

$$\begin{cases}y_{\boldsymbol{x}0}=1\\ y_{\boldsymbol{x}\alpha_i}=m_i \quad i=1,2,\ldots\end{cases} \tag{11}$$

$$\boldsymbol{M}_d(y_{\boldsymbol{x}}-y)\geq\boldsymbol{0} \tag{12}$$

$$\begin{cases}\boldsymbol{M}_d(y)\geq\boldsymbol{0}\\ \boldsymbol{M}_{d-r_j}(y;g_j)\geq\boldsymbol{0} \quad j=0,1,\ldots\end{cases} \tag{13}$$

$$\begin{cases}\boldsymbol{M}_d(y_{\boldsymbol{x}})\geq\boldsymbol{0}\\ \boldsymbol{M}_{d-r_j}(y_{\boldsymbol{x}};g_j)\geq\boldsymbol{0} \quad j=1,2,\ldots\end{cases} \tag{14}$$

where y_0 is one of the decision variables, which denotes the probability or the security risk; $2r_j$ is the degree of polynomial g_j in $K(q)$ or $K_{bad}(q)$; and $\boldsymbol{M}_d$ and $\boldsymbol{M}_{d-r_j}$ are the moment matrix and localizing matrix [4], [7], respectively, whose entries are deterministic linear combinations of available moment sequences. In addition, a larger d generally leads to a more accurate approximation of DR-PSSA while the order of given moments $m_i\ (i=1,2,\ldots)$ is set to be smaller than $2d$ [6].

The processes of constructing SDP relaxations (10)-(14) for the generalized moment problem (7)-(9) are detailed as follows. The objective function (10) and linear constraints (11) are derived by substituting moment sequences y and $y_{\boldsymbol{x}}$ into (7) and (8), respectively. The semi-definite constraints (12)-(14) are sufficient conditions for satisfying (9) according to Lemma 2.4 and Theorem 2.2 in [6].

The proposed SDP relaxations lead to computationally tractable convex optimization problems. For any $\boldsymbol{x}\in\mathbb{R}^n$, $\boldsymbol{M}_d$ is a $k\times k$ positive semi-definite matrix in d-order SDP relaxations, where $k=(2n+d)!/((2n)!d!)$. Although the proposed method would suffer the curse of dimensionality, we can further take advantage of the inherent sparsity of NPFE to facilitate its implementation in large-scale applications [7]. Note that the sparsity-based method for optimal power flow problem proposed in [7] cannot be directly invoked to tackle the curse of dimensionality of the proposed method, because uncertain parameters in this paper bring more complexity than those in [7]. Due to limited space, we will introduce the solution to tackle the curse of dimensionality of the proposed method in our future work.

IV. Numerical Examples

Numerical experiments are carried out based on the revised 4-bus system in [8] (corresponding to case4gs in the MATPOWER library), where bus 1 is the slack bus, all the loads in the original system are tripled, and the active power output of the generator at bus 4 is doubled. An uncertain active power generation source is added at bus 2, which features a continuous uniform distribution on the interval of [−5 p.u., 5 p.u.]. The degree of NPFEs is set to be 2 [7].

In this section, we intend to validate the effectiveness of the proposed method through setting the Monte Carlo simulation as a benchmark. To be specific, we calculate the probability of voltage magnitude at bus 2 (denoted as V_2 in Fig. 1) being lower than a pre-defined limit V_{20} by the two methods. The solid and dashed lines in Fig. 1 denote the results pertinent to DR-PSSA with two- and four-order moments of the aforementioned uniform distribution, respectively, which indicate that the results will be less conservative if the order of moments becomes higher. Moreover, the dotted line in Fig. 1 depicts the simulation results obtained from 2000 scenarios of the uniform distributions, which is always a lower bound of DR-PSSA with various order moments. Thus, the accuracy of the results of the DR-PSSA is also confirmed.

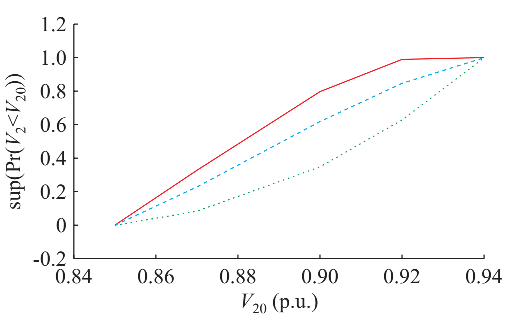

Fig. 1. Probabilities with 2-order SDP relaxations and a uniform distribution.

In addition, all of the three probabilities stay at the same value when the voltage magnitude is lower than 0.85 p.u. or higher than 0.94 p.u., which means the operation interval of voltage magnitudes pertinent to the pre-defined active power variations, i.e., [−5 p.u., 5 p.u.], is [0.85 p.u., 0.94 p.u.].

V. Conclusion

A DR-PSSA method for static security assessment of power system operation based on the partial moments of uncertain parameters is proposed in this letter. The proposed method improves the ability of moment-based distributionally robust optimization in dealing with nonlinear models and high-order moment, making the static security assessment of power system operation more robust to uncertainties. Numerical examples show that distribution uncertainties in power system operation are effectively incorporated in DR-PSSA. As the follow-up study, we will explore more structural information, e.g., symmetry and unimodality, embedded in the PDs of uncertain parameters to improve the computation performance of the proposed method.

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