Conic Optimal Energy Flow of Integrated Electricity and Natural Gas Systems

Rufeng Zhang, Member, IEEE, Tao Jiang, Senior Member, IEEE, Fangxing Li, Fellow, IEEE, Guoqing Li, Xue Li, Member, IEEE, and Houhe Chen, Member, IEEE

Abstract—In this letter, we propose a market-based bi-level conic optimal energy flow (OEF) model of integrated electricity and natural gas systems (IENGSs). Conic alternating current optimal power flow (ACOPF) is formulated in the upper-level model, and the generation cost of natural gas fired generation units (NGFGUs) is calculated based on natural gas locational marginal prices (NG-LMPs). The market clearing process of natural gas system is modeled in the lower-level model. The bilevel model is then transferred into a mixed-integer second-order cone programming (MISOCP) problem. Simulation results demonstrate the effectiveness of the proposed conic OEF model.

Index Terms—Optimal energy flow (OEF), bi-level model, second-order cone programming (SOCP), integrated energy system.

I. INTRODUCTION

THE optimal energy flow (OEF) is of great significance to integrated electricity and natural gas systems (IENGSs) [1]. The majority of existing works model the optimal operation or OEF problems of IENGS as a combination of traditional direct current optimal power flow (DCOPF) and additional constraints due to the feasibility of natural gas network flows. Thus, this is a single-level optimization model with electricity systems as the main consideration for optimization. However, the models can be enhanced in two aspects. First, critical factors may be modeled by nonlinear alternating current (AC) power flow equations [2], due to discoveries in recent works proving that a second-order cone AC power flow model can be effectively solved [3]. Second, we may consider modeling a natural gas market instead of modeling gas systems only as the constraints in an IENGS, which means that the suppliers with different prices can supply gas loads, and the generation costs of natural gas fired

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generation units (NGFGUs) relate to gas market clearing prices.

With the above motivations, the market-based OEF problem of IENGS is modeled in this letter as a bi-level conic optimization problem. The upper-level model is for the AC optimal power flow (ACOPF) problem of the electricity system, while the lower-level model is for the market-based natural gas OEF problem. Figure 1 illustrates the proposed OEF model, which is then converted for a mixed-integer second-order cone programming (MISOCP) problem, where NG-LMP is the natural gas locational marginal price. Numerical results verify the effectiveness of the conic model.



Fig. 1. Bi-level conic OEF model.

II. MARKET-BASED OEF MODEL

In IENGS, NGFGUs are the main coupling elements, which purchase gas to generate electricity. In the proposed OEF model, the generation cost of an NGFGU is equal to its gas purchasing cost measured by NG-LMPs. NG-LMPs can be calculated via the market clearing process of a natural gas system, which corresponds to the lower-level model. The ACOPF of an electricity system is the upper-level model, and the market-based bi-level OEF problem can be formulated as the following optimization model:

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R. Zhang, T. Jiang (corresponding author), G. Li, X. Li, and H. Chen are with the Department of Electrical Engineering, Northeast Electric Power University, Jilin 132012, China (e-mail: zhangrufeng@neepu.edu.cn; tjiang@neepu.edu.cn; lgq@neepu.edu.cn; xli@neepu.edu.cn; chenhouhe@neepu.edu.cn).

F. Li is with the Department of Electrical Engineering and Computer Science, The University of Tennessee, Knoxville, TN 37996, USA (e-mail: fli6@utk.edu). DOI: 10.35833/MPCE.2020.000244

$$\min \sum_{g_1 \in A} c_{g_1} p_{g_1} + \sum_{g_2 \in B} c_{ng,g_2} p_{g_2}$$
(1)

s.t.

$$p_{gl,i} + p_{g2,i} - p_{d,i} = \sum_{l \in \mathcal{Q}_l} M_{i,l} P_{e,l} + \sum_{l \in \mathcal{Q}_l} M_{loss,i,l} P_{loss,l} \quad \forall i \in \Phi$$
(2)

$$q_{g1,i} + q_{g2,i} - q_{d,i} = \sum_{l \in \mathcal{Q}_l} M_{i,l} Q_{e,l} + \sum_{l \in \mathcal{Q}_l} M_{loss,i,l} Q_{loss,l} \quad \forall i \in \Phi$$
(3)

$$M_{i,l}^{\mathrm{T}} u_{e,i} = 2R_{l}P_{e,l} + 2X_{l}Q_{e,l} + R_{l}P_{loss,l} + X_{l}Q_{loss,l} \quad \forall i \in \Phi, l \in \Omega_{l}(4)$$

$$P_{loss,l} = \frac{P_{e,l}^2 + Q_{e,l}^2}{u_{e,i}} R_l \quad l \in \Omega_l$$
(5)

$$Q_{loss,l} = \frac{P_{e,l}^2 + Q_{e,l}^2}{u_{e,l}} X_l \quad l \in \Omega_l$$
(6)

$$l_{e,l} = \frac{P_{e,l}^2 + Q_{e,l}^2}{u_{e,l}} \quad l \in \Omega_l$$
(7)

$$-p_l^{\max} \le P_{e,l} \le p_l^{\max} \quad \forall l \in \Omega_l \tag{8}$$

$$p_{gl,i}^{\min} \le p_{gl,i} \le p_{gl,i}^{\max} \quad gl \in A, \forall i \in \Phi$$
(9)

$$q_{gl,i}^{\min} \le q_{gl,i} \le q_{gl,i}^{\max} \quad gl \in A, \forall i \in \Phi$$

$$\tag{10}$$

 $p_{g2,i}^{\min} \le p_{g2,i} \le p_{g2,i}^{\max} \quad g2 \in B, \forall i \in \Phi$ $\tag{11}$

$$q_{g2,i}^{\min} \leq q_{g2,i} \leq q_{g2,i}^{\max} \quad g2 \in B, \forall i \in \Phi$$

$$(v_i^{\min})^2 \leq u_i \leq (v_i^{\max})^2 \quad i \in \Phi$$
(12)
(13)

$$\|2P_{e,l} \quad 2Q_{e,l} \quad l_{e,l} - u_{e,l}\| \le l_{e,l} + u_{e,l}$$
 (14)

$$CX_l P_{e,l} - CR_l Q_{e,l} = 0 \quad l \in \Omega_l$$
(15)

$$betta_{gf} \cdot y_G = p_{gf} \quad gf \in \Lambda(G) \tag{16}$$

$$c_{ng,gf} p_{g2} = \alpha_G y_G \quad gf \in \Lambda(G) \tag{17}$$

$$\alpha_G \in \arg\min \sum_{w \in Y} \mathcal{Q}_w y_w \tag{18}$$

s.t.

$$\sum_{w \in \theta_w(m)} y_w - \sum_{d_g \in \theta_d(m)} y_{d_g} - \sum_{G \in \theta_G(m)} y_G - \sum_{\{\cdot\} \in \theta_{\{:out}(m)} y_{\{\cdot\}}^{out} + \sum_{\{\cdot\} \in \theta_{i}, (m)} y_{\{\cdot\}}^{in} = 0 \quad \{\cdot\} = \{c, pl\}, m \in \Psi$$
(19)

$$\left\| \frac{2y_{pl,mn}}{\phi_{pl,mn}} \right\| \le U_m - U_n + 1 \quad pl \in \Gamma$$

$$(20)$$

$$y_{ct} - (1 - \kappa_c) y_{cf} = 0 \quad c \in I$$

$$\tag{21}$$

$$U_{ct} \le \gamma_c^2 U_{cf} \quad c \in I \tag{22}$$

$$y_{w}^{\min} \leq y_{w} \leq y_{w}^{\max} \quad w \in \Upsilon$$
(23)

$$(u_m^{\min})^2 \le U_m \le (u_m^{\max})^2 \quad m \in \Psi$$
(24)

where c_{g1} and $c_{ng,g2}$ are the cost coefficients of non-NGFGU g1 and NGFGU g2, respectively; p_{g1} and p_{g2} are the power outputs of non-NGFGU g1 and NGFGU g2, respectively; A and B are the sets of non-NGFGUs and NGFGUs, respectively; Φ and Q_1 are the sets of buses and transmission lines in the electricity system, respectively; $M_{i,l}$ is the element of incidence matrix associated with power injection of bus *i*

and power transmission of line l; $M_{loss,i,l}$ is the element of incidence matrix associated with power injection of bus *i* and loss of line l; $l_{e,l}$ is the square of line current; C is the element of branch-path incidence matrix; u_i is the square voltage of bus *i*; v_i^{max} and v_i^{min} are the upper and lower limits of the voltage of bus *i*, respectively; $u_{e,l}$ is the square voltage of the receiving end of line l; $p_{gl,i}$ and $q_{gl,i}$ are the active and reactive power generation of non-NGFGU g1 at bus i, respectively; $p_{g2,i}$ and $q_{g2,i}$ are the active and reactive power outputs of NGFGU g2, respectively; $P_{loss,l}$ and $Q_{loss,l}$ are the active and reactive losses of line l, respectively; p_l^{max} is the power transmission limit of line *l*; p_{g1}^{max} , p_{g1}^{min} , p_{g2}^{max} , and p_{g2}^{min} are the active power limits of non-NGFGU g1 and NGFGU g2, respectively; q_{g1}^{max} , q_{g1}^{min} , q_{g2}^{max} , and q_{g2}^{min} are the reactive power limits of g1 and g2, respectively; $p_{d,i}$ and $q_{d,i}$ are the active and reactive power loads, respectively; R_l and X_l are the resistance and reactance, respectively; $P_{e,l}$ and $Q_{e,l}$ are the active and reactive power flows, respectively; bettagf is the efficiency factor of NGFGUs; gf and p_{of} are the index and power output of NGFGUs, respectively; $\Lambda(G)$ is the set of NGF-GUs connected to node G in gas network; $c_{ng,gf}$ is the cost coefficient of NGFGUs; α_G is the NG-LMPs of node G with NGFGUs, and can be calculated by the dual variables of the gas balance equation (19) in the lower-level OEF problem; v_w and Q_w are the supply and the price of gas well w; c and *pl* are the indices of compressors and pipelines, respectively; $y_{pl,mn}$ is the pipeline flows of passive pipeline connecting nodes m and n; y_{ct} and y_{cf} are the injected and outlet flows of active pipelines, respectively; y_G is the gas load of NGF-GUs; y_{d_x} is the other gas load; d_g is the load node; $\theta_w(m)$, $\theta_d(m)$, and $\theta_G(m)$ are the sets of gas supply, load, and NGF-GUs connecting to node *m*, respectively; $y_{\{\}}^{out}$ and $y_{\{\}}^{in}$ are the pipeline flows into and out of node *m*, respectively; $\theta_{\text{sin}}(m)$ and $\theta_{\alpha_{i}}(m)$ are the sets of pipelines flowing into and out of node *m*, respectively; Ψ is the set of nodes in natural gas networks; Γ and I are the sets of passive and active pipelines, respectively; y_w^{max} and y_w^{min} are the limits of gas supplies of gas well w; U_m and U_n are the squares of nodal pressure; u_m^{max} and u_m^{min} are the nodal pressure limits; $\phi_{pl,mn}$ is the Weymouth equation coefficient; κ_c and γ_c are the fuel consumption coefficient and the compression factor of the compressors, respectively [4]; U_{ct} and U_{cf} are the squares of pressure of inlet and outlet nodes of compressor, respectively; and Yis the set of gas wells.

Equations (1)-(17) represent the upper-level ACOPF model for an electricity system with the formulation of secondorder cone programming (SOCP) [3], where (1) is the objective function minimizing the power generation costs. The power balance equations with definitions of real and reactive power flows as well as branch currents are given in (2)-(7).

Equations (8)-(13) denote the limits on active and reactive power of units and bus voltage, and (14) denotes the second-order cone (SOC) constraint, where $\|\cdot\|$ is the 2-norm.

Equation (15) is added to make the SOCP formulation for ACOPF more accurate [3]. Equation (16) denotes the coupling constraints of the two energy systems, and (17) denotes that the generation cost of NGFGUs is equal to the

gas purchasing cost.

s.t.

Equations (18)-(24) represent the lower-level OEF problem of the natural gas system, where (18) is the objective function minimizing the gas purchasing cost at gas wells. Equation (19) denotes the gas balance, while (20) denotes the passive Weymouth pipeline flow, which is relaxed and expressed as SOC inequalities. Moreover, (21) and (22) denote the active pipeline flow, and (23) and (24) denote the limits on gas supply of gas wells and nodal pressure, respectively.

III. SOLUTION METHOD

For the proposed conic OEF model, for a given upper decision y_G , the lower-level model is an SOCP problem. Hence, the lower-level model can be replaced by its primaldual counterpart [2]. Then, the bi-level OEF model can be transferred into a single-level model with the bilinear term $c_{ng,g2}p_{g2}$. In (17), the bilinear term $c_{ng,g2}p_{g2}$ is equal to the bilinear term $\alpha_G y_G$, where α denotes the dual variables corresponding to the NG-LMPs of the natural gas system. The bilinear term $\alpha_G y_G$ can be linearized through strong dual theory [5]. Then, the bilinear term $c_{ng,g2}p_{g2}$ is linearized and (17) is removed. The compact form of the resulting single-level model is as follows:

$$\min f(x,y)$$

$$vx = b \quad \omega y + Dx \ge g \tag{26}$$

$$\|Nx\| \le hx \tag{27}$$

(25)

$$v\tau + \zeta \eta + N\varphi + h\lambda = k \tag{28}$$

$$\|\varphi\| \leq \lambda \quad \lambda, \eta \geq 0 \tag{29}$$

$$\boldsymbol{Q}_{w}\boldsymbol{y}_{w} \leq \partial(1 - \boldsymbol{\omega}\boldsymbol{y}) + \boldsymbol{d}\boldsymbol{\eta}$$

$$\tag{30}$$

where y and x are the vectors of variables of the upper- and lower-level model, respectively; g and k are constant vectors; the constant matrices $(v, \zeta, b, \omega, D, N, h, d)$ are associated with the corresponding variables; and $(\eta, \lambda, \varphi, \tau)$ are the vectors of dual variables for the constraints of the lowerlevel model. Equations (26) and (27) are the primal constraints, (28) and (29) are the dual constraints, and (30) is the primal-dual constraint used to ensure the strong duality, which is analyzed in detail in [2]. In (30), the bilinear term can be linearized by the McCormick method [2]. To make the SOC relaxation tight, a penalty is added in (25) [6]. Then, the proposed bi-level conic OEF model of IENGS can be formulated as an MISOCP problem.

IV. NUMERICAL RESULTS

In this section, the IEEE 9-bus power system with 7-node gas system and the IEEE 118-bus power system with 14node gas system are utilized to illustrate the performance of the proposed market-based conic OEF model. The data of natural gas systems are given in [7]. In the IEEE 9-bus power system, unit 2 is assumed to be an NGFGU connecting node 1 in the 7-node gas system, and 12 NGFGUs are considered in the IEEE 118-bus power system. All models are implemented in MATLAB and solved by the Mosek solver on a PC with Intel Core i7 3.00 GHz CPU and 8 GB RAM.

A. IEEE 9-bus Power System with 7-node Gas System

The topology of the IEEE 9-bus power system with 7node gas system is shown in Fig. 2, where GU, P, and PLare the indices of generators, buses and power loads, respectively; and GW, N, C, and GL are the indices of gas wells, nodes, compressors and gas loads, respectively. In this case with modified residential gas load data, the results of generation costs and outputs of the NGFGU are compared with various electricity and residential gas load levels, the results of which are shown in Fig. 3.



Fig. 2. Topology of IEEE 9-bus power system with 7-node gas system.



Fig. 3. Generation costs and outputs of NGFGU with various load levels. (a) Generation costs. (b) Outputs of NGFGU.

As can be seen from Fig. 3, the generation costs increase with electricity load levels. Meanwhile, when the residential gas load level increases, the generation cost increases and the output of the NGFGU decreases for the same electricity load level. The reason is that NG-LMPs increase with gas load levels, and hence, the generation cost coefficient of the NGFGU increases. When the residential gas load level is 100%, the NGFGU is the cheapest unit and maintains the maximum output. In contrast, when the residential gas load level is high enough, the NGFGU is expensive and the gas consumption of the NGFGU is also constrained by the natural gas network. Hence, the output of the NGFGU increases with the electricity load level, but is less than the values when the gas load level is 100%. The results demonstrate that the load levels of one system could affect the operation of both energy systems.

B. IEEE 118-bus Power System with 14-node Gas System

To further demonstrate the validity of the bi-level conic OEF model, the IEEE 118-bus power system with a 14-node gas system with modified nodal pressure limits is applied, the topology of which is shown in Fig. 4.



Fig. 4. Topology of IEEE 118-bus power system with 14-node gas system.

We compare the proposed ACOPF-based bi-level OEF model with the DCOPF-based OEF model. For the DCOPFbased OEF model, linear programming DCOPF model is at the upper level. The main difference between the ACOPFand DCOPF-based OEF models is that voltage constraints, reactive power constraints, and electricity network losses are considered in the ACOPF-based model. The ACOPF-based bi-level OEF model is more accurate than the DCOPF-based model. A comparison of the outputs of the NGFGUs from the two models is shown in Fig. 5(a). It can be seen that the outputs of NGFGUs 2 and 10 are different in the two different models. The difference is primarily caused by voltage and reactive power constraints and electricity network losses. The comparison of the nodal pressures of the natural gas system is shown in Fig. 5(b). The pressures of nodes 6, 7, and 12-14 are different for the two cases, which verifies that different gas loads corresponding to different outputs of NGF-GUs would result in different operation statuses of the natural gas system. Hence, the errors in the DCOPF model will be proportionally transferred to the OEF model, and affect the operation of the natural gas system. For instance, for a power system with uncertainties, fast-regulated NGFGUs would be dispatched to match the power imbalances, and the errors of DCOPF may misestimate the available regulation capacities of the NGFGUs. The results demonstrate the effectiveness of considering ACOPF in OEF of IENGS.



Fig. 5. Comparison of results for ACOPF- and DCOPF-based bi-level OEF models. (a) Comparison of outputs of NGFGUs. (b) Comparison of nodal pressures.

C. Analysis of Computation Efficiency

The approximate error results, i.e., SOC relaxation gap, of Weymouth equations of the natural gas system are depicted in Fig. 6. It can be seen from Fig. 6 that the SOC relaxation gaps of all branches are relatively small. The largest gap is less than 0.00001, which is acceptable.



Fig. 6. Approximate error results of SOC relaxation for natural gas system.

To further verify the validity of the proposed OEF model on large systems, the IENGS composed of the IEEE 1354bus power system integrated with 4×48 -node gas systems is utilized, the topology of which is shown in Fig. 7.



Fig. 7. Topology of IEEE 1354-bus power system with 4×48-node gas systems.

The parameters of the 48-node natural gas system are given in [8]. The results of computation time of the three systems are shown in Table I. The computation time of the larger IEEE 1354-bus power system with 4×48 -node gas systems is 88.92 s, which is relatively short. The above results demonstrate that the proposed model can be solved with high computation efficiency.

TABLE I COMPUTATION TIME RESULTS

IENGS	Computation time (s)
9-bus (7-node)	1.32
118-bus (14-node)	4.78
1354-bus (4×48-node)	88.92

V. CONCLUSION

In this letter, a market-based bi-level conic OEF model of IENGS is proposed. The ACOPF of a power system is considered as the upper-level model, and a natural gas market clearing process is modeled and considered as the lower-level model. The proposed model is transferred and formulated as an MISOCP problem. The results verify the validity of the proposed model, and demonstrate that the energy system load levels of would affect the operation of both energy systems. The effectiveness of considering ACOPF in OEF of IENGS is further verified.

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Rufeng Zhang received the B.S., M.S., and Ph.D. degrees in electrical engineering in Northeast Electric Power University, Jilin, China, in 2013, 2016, and 2019, respectively. He is currently a Lecture with the Department of Electrical Engineering, Northeast Electric Power University. His research interests include integrated energy systems, power system operation and optimization, and renewable energy integration.

Tao Jiang received the B.S. and M.S. degrees in electrical engineering from Northeast Electric Power University, Jilin, China, in 2006 and 2011, respectively, and the Ph.D. degree in electrical engineering from Tianjin University, Tianjin, China, in 2015. He is currently a Professor with the Department of Electrical Engineering, Northeast Electric Power University. From 2014 to 2015, he was a Visiting Scholar with the Department of Electrical and Computer Engineering, North Carolina State University, Raleigh, USA. From October 2018 to October 2019, he was a Visiting Scholar with the Department of Electrical Engineering and Computer Science, The University of Tennessee, Knoxville, USA. His research interests include power system stability analysis and control, renewable energy integration, demand response, and smart grid.

Fangxing Li is also known as Fran Li. He received his B.S.E.E. and M.S.E.E. degrees from Southeast University, Nanjing, China, in 1994 and 1997, respectively, and his Ph.D. degree from Virginia Tech, Blacksburg, USA, in 2001. Currently, he is the James McConnell Professor at the University of Tennessee, Knoxville, USA. His research interests include renewable energy integration, demand response, electricity market, power system control, and power system computing.

Guoqing Li received the Ph.D. degree from Tianjin University, Tianjin, China, in 1998. He is a Professor with the Department of Electrical Engineering, Northeast Electric Power University, Jilin, China. His research interest includes power system security and stability, power system protection and optimization, and high-voltage direct current (HVDC).

Xue Li received the Ph.D. degree from the University of Tennessee, Knoxville, USA, in 2015. She is currently an Associate Professor with the Department of Electrical Engineering, Northeast Electric Power University, Jilin, China. Her research interest includes parallel computing applied in power system analysis.

Houhe Chen received the Ph.D. degree from North China Electric Power University, Beijing, China, in 2012. He is currently a Professor with the Department of Electrical Engineering, Northeast Electric Power University, Jilin, China. His research interests include power system operation and optimization, HVDC, and integrated energy systems.