Equilibria in Interdependent Natural-gas and Electric Power Markets: an Analytical Approach

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***Abstract*—Natural-gas and electric power systems and their corresponding markets have evolved over time independently. However, both systems are increasingly interdependent since combined cycle gas turbines that use natural gas to produce electricity increasingly couple them together. Therefore, suitable analysis techniques are most needed to comprehend the consequences on market outcomes of an increasing level of integration of both systems. There is a vast literature on integrated natural-gas and electric power markets assuming that the two markets are operated centrally by a single operator. This assumption is often untrue in the real world, which necessitates developing models for these interdependent yet independent markets. In this vein, this paper addresses the gap in the literature and provides analytical Nash-Cournot equilibrium models to represent the joint operation of natural-gas and electric power markets with the assumption that the market participants in each market make their own decisions independently seeking the maximum profits, as often is the case in the real world. We develop an analytical equilibrium model and apply the Karush-Kuhn-Tucker (KKT) approach to obtain Nash-Cournot equilibria for the interdependent natural-gas and electric power markets. We use a double-duopoly case to study the interaction of both markets and to derive insightful analytical results. Moreover, we derive closed-form analytical expressions for spot-market equilibria in both natural-gas and electric power markets, which are relevant and of practical significance for decision makers. We complement the double-duopoly study with a detailed sensitivity analysis.**

***Index Terms*—Electric power market, natural-gas market, Nash-Cournot equilibria, analytical equilibrium model.**

I. Introduction

THE natural-gas systems and electric power systems were created independently and have evolved over time as two separate infrastructures with limited or no coordination [1], [2]. The increasing availability of natural gas and its competitive price [2], the high efficiency of natural-gas-fired power plants, twice the efficiency of coal-fired power plants [3], and the reduced carbon dioxide emissions of natural gas compared with the other fossil fuels [4], [5] increase the use of natural gas for electricity generation and thus the interdependency between natural-gas systems and electric power systems. On the other hand, the rise of the climate change awareness and public concerns about depletion of fossil-based sources persuaded governments to invest heavily on replacing fossil fuels with renewable energy sources such as wind and solar. The intermittent nature of renewables, however, requires a backup energy source that could be utilized to respond to demand fluctuations quickly, particularly during the peak hours. Natural gas has been found as an environmentally-friendly source in accommodating the intermittent nature of renewables [6], which elevates the interdependency of the natural-gas systems and electric power systems. This strong interdependency has reached to the markets of the two systems as well, which necessitates coordinated analysis to maximize potential economic and environmental benefits and ensure efficient operations of the two systems.

The literature on the analysis of natural-gas and electric power markets individually is diverse. However, the literature on the coordinated analysis of the integrated natural-gas and electric power markets as well as the tools to comprehend their interdependency is still limited [2], [6], [7]. The available literature is divided into two main clusters. The first cluster assumed that the two markets were operated centrally by a single operator whereas the second cluster assumed that these markets, albeit being interdependent, functioned individually.

The approach of the first cluster, which can be found in the works of [8]-[23], may be considered unrealistic given the existing structure of natural-gas and electric power markets in most places. As a case in point, [8] assumed that the power system operator had full knowledge of the natural-gas network and the natural-gas demand forecast and developed a trilevel optimization program for the unit commitment problem to avoid potential economic and reliability risks in the coupled markets. A similar assumption was made in [9], where a unit commitment model was developed for the power system considering the line pack of pipelines to enhance the flexibility provided by natural-gas systems to electric power systems. Moreover, a non-deterministic model was developed in [10] to study the fully coordinated natural-gas

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and electric power markets for day-ahead clearing and pricing considering wind power uncertainty. Similarly, [11] assumed a central entity to manage the demand response of integrated natural-gas and electric power systems, developed a distributionally robust scheduling model, and studied the effect of such demand response model on locational marginal price of natural gas and electricity in the coupled markets. Reference [20] developed a data-driven stochastic co-optimization and settlement of a power gas coupled system for the day-ahead markets, and obtained better outcomes in terms of the tractability and accuracy of algorithms compared with other proposed distributionally robust models. Reference [21] studied a synergistic operations of the coupled natural-gas and electric power markets and used an alternating direction method of multipliers to solve the optimal power flow in coupled natural-gas and electric power markets by minimizing the power generation cost and natural-gas production cost. Reference [22] integrated two optimization models, one for maximizing the revenue of electric power markets and one for minimizing the operation cost of natural-gas market, to find the optimal decision in interdependent markets within the European regulatory framework. This model was based on the assumption that the two markets exchange the information and therefore the accuracy of the proposed model depends highly on the level of the information exchanged between the participants of the two markets. Likewise, the operational equilibrium of natural-gas and electric power systems with limited information exchange was studied in [23]. Reference [23] further explored the impact of different levels of access to the information on the system decisions, prices, and operational costs in the two markets. They used second-order cone relaxation methods to obtain the operational equilibrium modeled as a mixed-integer second-order conic problem, which raises the concern of the obtained equilibrium being infeasible. Therefore, the assumptions such as the central entity for demand response of integrated markets, fully coordinated or synergistic decision making in integrated natural-gas and electric power markets with full or limited information exchange among the participants of the two markets do not seem realistic and are often not the cases in the real world.

In this paper, we adopt the the approach of the second cluster, which can be found in [24]-[27]. For example, [24] designed a framework for the coupled natural-gas and electric power markets assuming the individuality of the two markets in their decision making. To overcome the computational difficulty of a large-scale second-order cone programming method, a best-response decomposition algorithm was presented to identify the equilibrium in the coupled market. However, the proposed algorithm showed convergence difficulty and the equilibrium existence was argued through intuitive discussions based on the price-demand curves. A similar assumption was made in [27] where deep reinforcement learning was employed to develop a dynamic energy conversion and management strategy for the integrated natural-gas and electric power markets. Such methods based on artificial intelligence (AI) are suitable to solve multi-agent decision making problems when sufficient historical data are available to train the AI-based algorithm. Expectedly, the implementation of the AI-based models is challenging and computationally demanding and the accuracy of the models is highly dependent on the quality and availability of historical data. Therefore, we adopt the same concept on the individuality of the two interdependent markets, i. e., each market is formed by individual market participants and decision makers, and propose a new model to alleviate the challenges mentioned above.

Equilibrium models have been used in the literature to analyze the non-integrated natural-gas [28]-[30] and electric power markets [31], [32]. For example, [28] analyzed the interplay of natural-gas markets and natural-gas transport in a multilevel equilibrium model considering the nonconvex physics of the natural-gas flow. On the electric power market side, we may highlight the analysis in [32] where the impact of information asymmetry on the equilibrium of electric power markets was investigated. A limited number of papers such as [33]-[35] used equilibrium problem with equilibrium constraints (EPEC) and mathematical programming with equilibrium constraints (MPEC) to study the coupled natural-gas and electric power markets. Aside from the computational challenges and complexity, these computational models allow describing specific cases in very detail, but generally do not allow the identification of general trends or interactions in both markets. In this paper, we use an analytical equilibrium approach for the interdependent natural-gas and electric power markets, and more specifically, propose a Nash-Cournot equilibrium model to analyze the interdependency and interactions of natural-gas market producers and electric power market producers, which helps comprehend the general behavior of the two interdependent markets. The Nash-Cournot equilibrium model has been used extensively to address problems in electric power markets, e.g., the works of [31], [36]-[41], as well as to study natural-gas problems, e. g., the works of [42]-[46]. We use the Nash-Cournot equilibrium model to model the interdependent natural-gas and electric power markets. This approach is not only less complicated and less computationally challenging but also allows us to derive closed-form analytical expressions for the spot-market equilibria in both natural-gas and electric power markets. Such expressions enable the system operators and other stakeholders of the two markets to evaluate their decisions and conduct comparative statistics or other analyses of market equilibria easily. Furthermore, it allows policymakers and regulators to examine the implications or benefits of the proposed policies and regulations on the operations of the interdependent natural-gas and electric power markets quickly without the usual complexity of computational models. To summarize, the main contributions of this paper are as follows.

1) We develop an analytical equilibrium model to analyze the interdependency of the natural-gas and electric power markets. Such analytical models based on classical optimization theory allow characterizing the optimality of the outcomes as well.

2) We use a Nash-Cournot equilibrium model to derive relevant equilibria between natural-gas and electric power mar-

kets when there is limited coordination or information exchange between the two markets, as often is the case in the real world.

3) Our approach allows us to derive closed-form analytical expressions for spot-market equilibria in both natural-gas and electric power markets, which allows policy-makers and regulators to examine the potential outcomes of the proposed policies and regulations quickly without the usual complexity of other models.

4) We use a double-duopoly case along with detailed sensitivity analyses to investigate the interaction of both markets and derive insightful analytical results.

The remainder of this paper is organized as follows. Section II presents the proposed market model and shows some of its structural properties. The model is demonstrated via stylized analysis and numerical examples in Section III. Section IV draws the conclusions.

II. Market Model

In this section, we present the proposed Nash-Cournot equilibrium model of the natural-gas and electric power markets.

A. Natural-gas Market Model

We model the natural-gas market as consisting of a set K of producing firms. The natural-gas producers compete in natural-gas spot market to serve two groups of natural-gas consumers, i.e., electric power firms who convert natural gas to electric power and other natural-gas consumers. The natural-gas producing firms are modeled as being quantity-setting competitors. This means that the firms simultaneously determine their production levels in the spot market, after which the market price is adjusted to clear the market, i.e., the demand is exactly equal to the supply. Let q_k^{GP} and q_k^{GG}, measured in cf, denote the production levels of natural-gas producing firm k being sold to the electric power sector and other natural-gas consumers, respectively, in the spot market. We assume that the firms have quadratic production costs. Thus, the cost to firm k of producing q_k^{GP} and q_k^{GG} in a spot market is:

$$\mathbb{C}_k^G(q_k^{GP},q_k^{GG})=a_k^G(q_k^{GG}+q_k^{GP})+\frac{1}{2}c_k^G(q_k^{GG}+q_k^{GP})^2 \tag{1}$$

where a_k^G and c_k^G are the non-negative coefficients of the quadratic cost function of natural-gas producing firm k. We assume that the demand in the spot market changes linearly with the price. Thus, the natural-gas spot price, which is measured in \$/Ccf, is given by:

$$P^G(q^G)=\gamma^G-\beta^G\sum_{k\in K}(q_k^{GG}+q_k^{GP}) \tag{2}$$

where γ^G and β^G are the non-negative coefficients of the linear price-demand function in natural-gas spot market; and $q^G=\sum_{k\in K}(q_k^{GG}+q_k^{GP})$ is the total natural-gas production of all producing firms in the natural-gas market. We assume hereafter that $P^G(q^G)$ is sufficiently large compared with $\mathbb{C}_k^G(q_k^{GP},q_k^{GG})$ for all firms to produce a strictly positive amount of energy in the spot market.

The natural-gas spot market consists of the firms determining their production levels to maximize their profits, meaning that the production level of firm k is derived from the following profit-maximization problem:

$$\max_{q_k^{GG},q_k^{GP}}\pi_k^G=P^G(q^G)(q_k^{GG}+q_k^{GP})-\mathbb{C}_k^G(q_k^{GP},q_k^{GG}) \tag{3}$$

s.t.

$$q_k^{GG}+q_k^{GP}\le q_k^{\max} \tag{4}$$

where π_k^G and $q_k^{\max}$ are the profit and the maximum production capacity of firm k, respectively. We also exclude the non-negativity constraint because we assume that the market price is sufficiently high for all firms to produce a strictly positive quantity.

We apply the Karush-Kuhn-Tucker (KKT) optimality conditions [47] to problem (3), (4) and derive the following KKT conditions, which are both necessary and sufficient for a global optimum [47]:

$$\begin{aligned}&-\left[\frac{\partial P^G(q^G)}{\partial q_k^{GG}}(q_k^{GG}+q_k^{GP})+P^G(q^G)\left(1+\frac{\partial q_k^{GP}}{\partial q_k^{GG}}\right)\right]+\\&a_k^G\left(1+\frac{\partial q_k^{GP}}{\partial q_k^{GG}}\right)+c_k^G(q_k^{GG}+q_k^{GP})\left(1+\frac{\partial q_k^{GP}}{\partial q_k^{GG}}\right)+\lambda_k^{GG}\left(1+\frac{\partial q_k^{GP}}{\partial q_k^{GG}}\right)=0\end{aligned} \tag{5}$$

$$\begin{aligned}&-\left[\frac{\partial P^G(q^G)}{\partial q_k^{GP}}(q_k^{GG}+q_k^{GP})+P^G(q^G)\left(1+\frac{\partial q_k^{GG}}{\partial q_k^{GP}}\right)\right]+\\&a_k^G\left(1+\frac{\partial q_k^{GG}}{\partial q_k^{GP}}\right)+c_k^G(q_k^{GG}+q_k^{GP})\left(1+\frac{\partial q_k^{GG}}{\partial q_k^{GP}}\right)+\lambda_k^{GG}\left(1+\frac{\partial q_k^{GG}}{\partial q_k^{GP}}\right)=0\end{aligned} \tag{6}$$

$$0\le\lambda_k^{GG}\perp[q_k^{\max}-(q_k^{GG}+q_k^{GP})]\ge 0 \tag{7}$$

where λ_k^{GG} is the Lagrange multiplier associated with the capacity constraint (4). To derive a more refined version of (5) and (6), we calculate equivalent terms for the partial derivatives in these equations. Considering (2), we can obtain:

$$\frac{\partial P^G(q^G)}{\partial q_k^{GG}}=-\beta^G\left(1+\frac{\partial q_k^{GP}}{\partial q_k^{GG}}\right) \tag{8}$$

$$\frac{\partial P^G(q^G)}{\partial q_k^{GP}}=-\beta^G\left(1+\frac{\partial q_k^{GG}}{\partial q_k^{GP}}\right) \tag{9}$$

Note that $\partial q_k^{GP}/\partial q_k^{GG}$ and $\partial q_k^{GG}/\partial q_k^{GP}$ are zero in general unless firm k has reached its maximum capacity, in which they take the value of -1. Therefore, we can obtain:

$$\frac{\partial q_k^{GP}}{\partial q_k^{GG}}=\begin{cases}-1 & k\in\Psi\\0 & \text{otherwise}\end{cases} \tag{10}$$

$$\frac{\partial q_k^{GG}}{\partial q_k^{GP}}=\begin{cases}-1 & k\in\Psi\\0 & \text{otherwise}\end{cases} \tag{11}$$

where Ψ is the set of natural-gas producing firms working at the maximum capacity in the spot market.

Replacing (10) and (11) in (8) and (9) renders:

$$\frac{\partial P^G(q^G)}{\partial q_k^{GG}}=\begin{cases}0 & k\in\Psi\\-\beta^G & \text{otherwise}\end{cases} \tag{12}$$

$$\frac{\partial P^G(q^G)}{\partial q_k^{GP}} = \begin{cases} 0 & k \in \Psi \\ -\beta^G & \text{otherwise} \end{cases} \tag{13}$$

Considering (12) and (13) together with the KKT conditions (5) - (7), we derive closed-form analytical expressions for the total production capacity of firm k in the spot market and the spot market clearing price in (14) and (15), respectively.

$$q_k^{GG*} + q_k^{GP*} = \begin{cases} q_k^{\max} & k \in \Psi \\ \dfrac{P^G(q^G) - a_k^G}{\beta^G + c_k^G} & \text{otherwise} \end{cases} \tag{14}$$

$$P^{G*}(q^G) = \frac{\gamma^G - \beta^G \left(\sum_{k' \in \Psi} q_{k'}^{\max} - \sum_{k'' \notin \Psi} \dfrac{a_{k''}^G}{\beta^G + c_{k''}^G} \right)}{1 + \beta^G \sum_{k'' \notin \Psi} \dfrac{1}{\beta^G + c_{k''}^G}} \tag{15}$$

where $*$ denotes the value of the variables at the obtained equilibrium.

B. Electric Power Market Model

Similar to the natural-gas market, we model the electric power market as consisting of a set I of electric power firms. The electric power firms compete in the spot market to serve the electricity consumers. The electric power firms are assumed to have two ways of generating electricity, i.e., converting natural gas to electricity and converting non-natural-gas energy sources to electricity. The electric power firms, modeled as quantity-setting competitors, simultaneously determine their generation levels in the spot market, after which the market price adjusts to clear the market.

Let q_i^{PG} denote the natural gas being converted to electricity by electric power firm i, which is measured in cf, at a conversion rate of α^{PG} and let q_i^P denote the generation level, which is measured in MW, of electric power generated by firm i from converting non-natural-gas energy sources to electricity. We assume that the firms have quadratic generation costs. Thus, the cost of firm i for generating q_i^P electricity from non-natural-gas energy sources and converting q_i^{PG} natural gas to electricity in a spot market is:

$$\mathbb{C}_i^P(q_i^P, q_i^{PG}) = a_i^P q_i^P + \frac{1}{2} c_i^P (q_i^P)^2 + P^{G*}(q^G) q_i^{PG} \tag{16}$$

where a_i^P and c_i^P are the non-negative coefficients of the quadratic cost function of electric power firm i. We assume that the electricity demand in the spot market changes linearly with the price. Thus, the spot electricity price, which is measured in \$/MW, is given by:

$$P^P(Q^P, Q^{PG}) = \gamma^P - \beta^P \sum_i (q_i^P + \alpha^{PG} q_i^{PG}) \tag{17}$$

where γ^P and β^P are the non-negative coefficients of the linear price-demand function in electric power spot market; and $Q^{PG} = \{q_i^{PG} | i \in I\}$ and $Q^P = \{q_i^P | i \in I\}$ are the sets of generation levels of all electric power firms from natural-gas and non-natural-gas energy sources, respectively.

We note that (16) and (17) couple together the natural-gas and electric power markets. Furthermore, the total natural gas consumed by the firms in the electric power market to generate electric power should match the total natural gas sold by the firms in the natural-gas market to the firms in the electric power market. Thus, we can obtain (18), which is used not only to check for the consistency of an equilibrium but also to differentiate the total natural gas sold to firms in the electric power market and other natural-gas consumers.

$$\sum_{i \in I} q_i^{PG} = \sum_{k \in K} q_k^{GP} \tag{18}$$

The electric power spot market consists of the firms determining their generation decisions to maximize their profits, meaning that the generation decision of firm i is derived from the following profit-maximization problem:

$$\max_{q_i^P, q_i^{PG}} \pi_i^P = P^P(Q^P, Q^{PG})(q_i^P + \alpha^{PG} q_i^{PG}) - \mathbb{C}_i^P(q_i^P, q_i^{PG}) \tag{19}$$

s.t.

$$q_i^P \le q_i^{P,\max} \tag{20}$$

$$q_i^{PG} \le q_i^{PG,\max} \tag{21}$$

where π_i^P, $q_i^{PG,\max}$, and $q_i^{P,\max}$ are the profit, the maximum natural-gas-based generating capacity, and the maximum non-natural-gas-based generating capacity of the firm i, respectively. We also exclude the non-negativity constraint because we assume that the market price is sufficiently high for all firms to produce a strictly positive quantity.

We apply the KKT optimality conditions to the problem (19) - (21) and derive the following KKT conditions, which are both necessary and sufficient for a global optimum [47]:

$$-\left[\frac{\partial P^P(Q^P, Q^{PG})}{\partial q_i^P} (q_i^P + \alpha^{PG} q_i^{PG}) + P^P(Q^P, Q^{PG}) \left(1 + \alpha^{PG} \frac{\partial q_i^{PG}}{\partial q_i^P} \right) \right] + \left[a_i^P + c_i^P q_i^P + P^{G*}(q^G) \frac{\partial q_i^{PG}}{\partial q_i^P} \right] + \lambda_i^{PG} \frac{\partial q_i^{PG}}{\partial q_i^P} + \lambda_i^P = 0 \tag{22}$$

$$-\left[\frac{\partial P^P(Q^P, Q^{PG})}{\partial q_i^{PG}} (q_i^P + \alpha^{PG} q_i^{PG}) + P^P(Q^P, Q^{PG}) \left(\alpha^{PG} + \frac{\partial q_i^P}{\partial q_i^{PG}} \right) \right] + \left(a_i^P \frac{\partial q_i^P}{\partial q_i^{PG}} + c_i^P q_i^P \frac{\partial q_i^P}{\partial q_i^{PG}} + P^{G*}(q^G) \right) + \lambda_i^P \frac{\partial q_i^P}{\partial q_i^{PG}} + \lambda_i^{PG} = 0 \tag{23}$$

$$0 \le (q_i^{P,\max} - q_i^P) \perp \lambda_i^P \ge 0 \tag{24}$$

$$0 \le (q_i^{PG,\max} - q_i^{PG}) \perp \lambda_i^{PG} \ge 0 \tag{25}$$

where λ_i^P and λ_i^{PG} are the Lagrange multipliers associated with the capacity constraints (20) and (21), respectively. To derive a more refined version of (22) and (23), we calculate equivalent terms for the partial derivative terms in these equations. Note that the two energy sources of generating electric power, i.e., natural-gas and non-natural-gas energy sources, are independent, we can obtain:

$$\frac{\partial q_i^P}{\partial q_i^{PG}} = \frac{\partial q_i^{PG}}{\partial q_i^P} = 0 \tag{26}$$

Considering (17) and (26), we can obtain:

$$\frac{\partial P^P(Q^P, Q^{PG})}{\partial q_i^P} = -\beta^P \tag{27}$$

$$\frac{\partial P^P(Q^P, Q^{PG})}{\partial q_i^{PG}} = -\beta^P \alpha^{PG} \tag{28}$$

Therefore, (22) and (23) can be rewritten as:

$$\beta^P(q_i^P+\alpha^{PG}q_i^{PG})-P^P(Q^P,Q^{PG})+a_i^P+c_i^P q_i^P+\lambda_i^P=0 \tag{29}$$

$$\beta^P\alpha^{PG}(q_i^P+\alpha^{PG}q_i^{PG})-\alpha^{PG}P^P(Q^P,Q^{PG})+P^{G*}(q^G)+\lambda_i^{PG}=0 \tag{30}$$

Formulae (24), (25), (29), and (30) provide the solution of the problem in the electric power market and yield analytical expressions for the generation levels of the firms in the electric power market. Considering (24) and (25) may or may not be binding for each electric power firm yields the following four cases to obtain the generation portfolio of each electric power firm as a function of the market clearing price. We further analyze these formulae considering the following four cases.

1) Case 1: $q_i^P=q_i^{P,\max}$ and $q_i^{PG}\neq q_i^{PG,\max}$. Considering (25) and (30), we can obtain the generation level of electric power from natural gas for firm i as:

$$\alpha^{PG}q_i^{PG}=\frac{\alpha^{PG}P^P(Q^P,Q^{PG})-P^{G*}(q^G)-\alpha^{PG}\beta^P q_i^{P,\max}}{\alpha^{PG}\beta^P} \tag{31}$$

Equation (31) accordingly results in the total electric power generation of firm i as:

$$q_i^{P*}+\alpha^{PG}q_i^{PG*}=q_i^{P,\max}+\frac{\alpha^{PG}P^P(Q^P,Q^{PG})-P^{G*}(q^G)-\alpha^{PG}\beta^P q_i^{P,\max}}{\alpha^{PG}\beta^P} \tag{32}$$

2) Case 2: $q_i^P\neq q_i^{P,\max}$ and $q_i^{PG}=q_i^{PG,\max}$. Considering (24) and (29), we can obtain the generation level of electric power from non-natural-gas energy sources for firm i as:

$$q_i^P=\frac{P^P(Q^P,Q^{PG})-a_i^P-\alpha^{PG}\beta^P q_i^{P,\max}}{\beta^P+c_i^P} \tag{33}$$

Equation (33) accordingly results in the total electric power generation of firm i as:

$$q_i^{P*}+\alpha^{PG}q_i^{PG*}=\frac{P^P(Q^P,Q^{PG})-a_i^P-\alpha^{PG}\beta^P q_i^{P,\max}}{\beta^P+c_i^P}+\alpha^{PG}q_i^{PG,\max} \tag{34}$$

3) Case 3: $q_i^P\neq q_i^{P,\max}$ and $q_i^{PG}\neq q_i^{PG,\max}$. Considering (24), (25), (29), and (30), we can obtain:

$$q_i^P=\frac{P^{G*}(q^G)-\alpha^{PG}a_i^P}{\alpha^{PG}c_i^P} \tag{35}$$

$$q_i^{PG}=\frac{P^P(Q^P,Q^{PG})-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_i^P}\right)P^{G*}(q^G)+\dfrac{\beta^P a_i^P}{c_i^P}}{\beta^P\alpha^{PG}} \tag{36}$$

Equations (35) and (36) accordingly result in the total electric power generation of firm i as:

$$q_i^{P*}+\alpha^{PG}q_i^{PG*}=\frac{P^{G*}(q^G)-\alpha^{PG}a_i^P}{\alpha^{PG}c_i^P}+\frac{P^P(Q^P,Q^{PG})-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_i^P}\right)P^{G*}(q^G)+\dfrac{\beta^P a_i^P}{c_i^P}}{\beta^P} \tag{37}$$

4) Case 4: $q_i^P=q_i^{P,\max}$ and $q_i^{PG}=q_i^{PG,\max}$. The total electric power generation of firm i can be obtained as:

$$q_i^{P*}+\alpha^{PG}q_i^{PG*}=q_i^{P,\max}+\alpha^{PG}q_i^{PG,\max} \tag{38}$$

To calculate the market clearing price, the total electric power generation of the firms derived from (32), (34), (37), (38) are plugged in (17). Note that each firm may fall into any of the four aforementioned cases in the spot market, which results in $|I|^4$ potential equilibria in the electric power market, where $|I|$ denotes the number of electric power firms. Similarly, the natural-gas market yields $|K|^2$ potential equilibria. Therefore, we may obtain $|I|^4|K|^2$ potential equilibria for the integrated natural-gas and electric power markets.

III. Numerical Analysis

We illustrate the proposed equilibrium model and solution methodology with a stylized case study. This analysis considers a duopoly in the natural-gas market and a duopoly in the electric power market with the values for the parameters of the model. This analysis is illustrative because we can present the interactions between the two markets and highlight how the equilibria are obtained.

A. Stylized Double-duopoly Case Study

In this subsection, we analyze a stylized double-duopoly case in which there are two producers in the natural-gas market and two electric power firms in the electric power market. The power equilibrium outcomes (14), (15), coupling condition (18), and natural-gas equilibrium outcomes (17), (32), (34), (37), and (38) are used. Table I summarizes the parameter data of the natural-gas market and Table II provides the parameter data of the electric power market. In a duopoly competition in the natural-gas market, four potential equilibria are expected in a spot market. Table III summarizes these four equilibria in terms of the production level of the two producers, their costs and profits, and the equilibrium consistency. Equilibria 1 and 3 are marked as consistent whereas Equilibria 2 and 4 are inconsistent due to violating the maximum production capacity of electric power firm 1.

TABLE I
PARAMETER DATA OF NATURAL-GAS MARKET FOR STYLIZED DOUBLE-DUOPOLY CASE

Parameter	Value	Parameter	Value
a_1^G (\$/cf)	0.005	$q_1^{\max}$ (cf)	2500
a_2^G (\$/cf)	0.006	$q_2^{\max}$ (cf)	3500
c_1^G (\$/cf^2)	10^{-7}	γ^G (\$/cf)	0.01
c_2^G (\$/cf^2)	10^{-7}	β^G (\$/cf^2)	5×10^{-7}

TABLE II
PARAMETER DATA OF ELECTRIC POWER MARKET FOR STYLIZED DOUBLE-DUOPOLY CASE

Parameter	Value	Parameter	Value
a_1^P (\$/kWh)	0.08	$q_2^{P,\max}$ (kWh)	300
a_2^P (\$/kWh)	0.085	$q_1^{PG,\max}$ (cf)	1500
c_1^P (\$/kWh^2)	0.00005	$q_2^{PG,\max}$ (cf)	1000
c_2^P (\$/kWh^2)	0.00003	γ^P (\$/kWh)	0.125
$q_1^{P,\max}$ (kWh)	200	β^P (\$/kWh^2)	0.00005

Given that there is at least one consistent equilibrium in the natural-gas market, we may examine the existence of consistent equilibria in the electric power market. Note that there exist 16 potential equilibria for each consistent equilibrium of the natural-gas market, i.e., 32 equilibria in total. Table IV provides all equilibria of electric power market for Equilibrium 1 of the natural-gas market and Table V provides the next 16 equilibria of electric power market for Equilibrium 3 of the natural-gas market.

TABLE III
EQUILIBRIA OF NATURAL-GAS MARKET

Equilibrium	P^{G*} ($/Ccf)	$q_1^{GG*}+q_1^{GP*}$ (kWh)	$q_2^{GG*}+q_2^{GP*}$ (kWh)	$\mathbb{C}_1^{G*}$ ($)	$\mathbb{C}_2^{G*}$ ($)	π_1^{G*} ($)	π_2^{G*} ($)	Consistency
1	0.0075	2500.00	2500.00	12.81	15.31	5.94	3.44	Yes
2	0.0068	2954.55	3500.00	15.21	21.61	4.80	2.09	No
3	0.0070	2500.00	3500.00	12.81	21.61	4.69	2.89	Yes
4	0.0072	3645.83	1979.17	18.89	12.07	7.31	2.15	No

TABLE IV
EQUILIBRIA OF ELECTRIC POWER MARKET FOR EQUILIBRIUM 1 OF NATURAL-GAS MARKET

Equilibrium	P^{P*} ($/MW)	q_1^{P*} (kWh)	q_2^{P*} (kWh)	q_1^{PG*} (cf)	q_2^{PG*} (cf)	$\mathbb{C}_1^{P*}$ ($)	$\mathbb{C}_2^{P*}$ ($)	π_1^{P*} ($)	π_2^{P*} ($)	Consistency
1	0.0917	200.00	300.00	1333.33	333.33	27.00	29.35	3.56	1.21	Yes
2	0.0957	200.00	71.43	2142.83	1000.00	33.07	13.65	6.58	2.76	No
3	0.0917	200.00	−333.33	1333.33	6666.67	27.00	23.33	3.56	7.22	No
4	0.0900	200.00	300.00	1000.00	1000.00	24.50	34.35	2.50	1.65	Yes
5	0.0945	70.00	300.00	1500.00	900.00	16.97	33.60	3.82	3.26	Yes
6	0.1000	125.00	125.00	1500.00	1000.00	21.65	18.36	5.86	4.14	Yes
7	0.0945	70.00	−333.33	1500.00	7233.33	16.97	27.58	3.82	9.27	No
8	0.0942	66.67	300.00	1500.00	1000.00	16.69	34.35	3.71	3.32	Yes
9	0.0917	−100.00	300.00	4333.33	333.33	24.75	29.35	5.81	1.21	No
10	0.0957	−100.00	71.43	5142.86	1000.00	30.82	13.65	8.83	2.76	No
11	0.0917	−100.00	−333.33	4333.33	6666.67	24.75	23.33	5.81	7.22	No
12	0.0900	−100.00	300.00	4000.00	1000.00	22.25	34.35	4.75	1.65	No
13	0.0913	200.00	300.00	1500.00	250.00	28.25	28.73	3.69	0.93	Yes
14	0.0977	200.00	96.15	1500.00	1000.00	28.25	15.81	5.94	3.35	Yes
15	0.0913	200.00	−333.33	1500.00	6583.33	28.25	28.25	3.69	3.69	No
16	0.0875	200.00	300.00	1500.00	1000.00	28.25	34.35	2.38	0.65	Yes

TABLE V
EQUILIBRIA OF ELECTRIC POWER MARKET FOR EQUILIBRIUM 3 OF NATURAL-GAS MARKET

Equilibrium	P^{P*} ($/MW)	q_1^{P*} (kWh)	q_2^{P*} (kWh)	q_1^{PG*} (cf)	q_2^{PG*} (cf)	$\mathbb{C}_1^{P*}$ ($)	$\mathbb{C}_2^{P*}$ ($)	π_1^{P*} ($)	π_2^{P*} ($)	Consistency
1	0.0883	200.00	300.00	1666.67	666.67	28.67	31.52	3.72	0.87	No
2	0.0938	200.00	47.62	3142.86	1000.00	39.00	11.08	9.24	2.77	No
3	0.0883	200.00	−500.00	1666.67	8666.67	28.67	21.92	3.72	10.47	No
4	0.0875	200.00	300.00	1500.00	1000.00	27.50	33.85	3.13	1.15	Yes
5	0.0925	50.00	300.00	1500.00	1500.00	14.56	37.35	3.94	4.28	No
6	0.1000	125.00	125.00	1500.00	1000.00	20.89	17.86	6.61	4.64	Yes
7	0.0925	50.00	−500.00	1500.00	9500.00	14.56	27.75	3.94	13.88	No
8	0.0942	66.67	300.00	1500.00	1000.00	15.94	33.85	4.46	3.82	Yes
9	0.0883	−200.00	300.00	5666.67	666.67	24.67	31.52	7.72	0.87	No
10	0.0938	−200.00	47.62	6761.90	1000.00	32.33	11.08	12.34	2.77	No
11	0.0883	−200.00	−500.00	5666.67	8666.67	24.67	21.92	7.72	10.47	No
12	0.0875	−200.00	300.00	5500.00	1000.00	23.50	33.85	7.13	1.15	No
13	0.0888	200.00	300.00	1500.00	750.00	27.50	32.10	3.56	1.18	Yes
14	0.0977	200.00	96.15	1500.00	1000.00	27.50	15.31	6.69	3.85	Yes
15	0.0888	200.00	−500.00	1500.00	8750.00	27.50	27.50	3.56	3.56	No
16	0.0875	200.00	300.00	1500.00	1000.00	27.50	33.85	3.13	1.15	Yes

As illustrated in Tables IV and V, there are 14 consistent equilibria for this case of the integrated natural-gas and electric power markets.

We conclude our analysis of the stylized double-duopoly case with a number of sensitivity analyses. Figure 1 depicts the equilibria of production level of producers (up to four) in the natural-gas market as a function of the slope of natural-gas price β^G. We use different colors in our figures to differentiate these equilibria from one another. Note that in some cases, the equilibria may result in identical values, but only one colored dot is used to represent them. The total natural-gas production of the producers tend to decrease with the increase in β^G unless the producers operate at the maximum capacity. For example, the solid-line and dotted-line arrows in Fig. 1 highlight the equilibria in which one of the natural-gas producers (producer 1) lowers its production level with an increase in β^G while the other producer (producer 2) operates at the maximum capacity. It is worth noting that there are more than one consistent equilibria for most values of β^G. Figure 2 exhibits the market clearing price of the equilibria (up to four) in the natural-gas market as a function of β^G. Figure 2 highlights that the market clearing price decreases as β^G increases. The solid-line and dotted-line arrows in Figs. 1 and 2 show that the producers, as firms seeking the maximum profit, tend to keep the prices high by producing less when they are not operating at the maximum capacity.

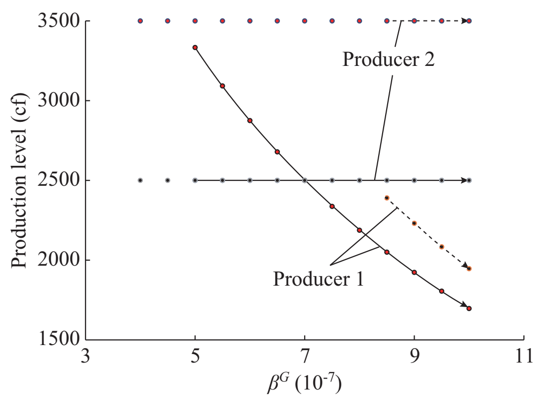

Fig. 1. Production level of producers in natural-gas market as a function of β^G.

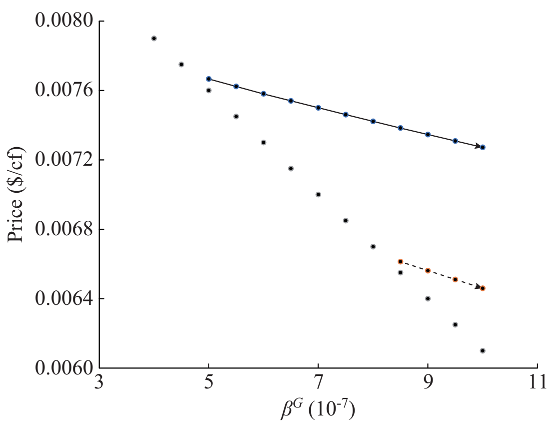

Fig. 2. Market clearing price in natural-gas market as a function of β^G.

Figures 3, 4, and 5 show the equilibria of market clearing price in the electric power market, the generation level of electric power firm 1, and the generation level of electric power firm 2, respectively, with respect to the changes in β^P.

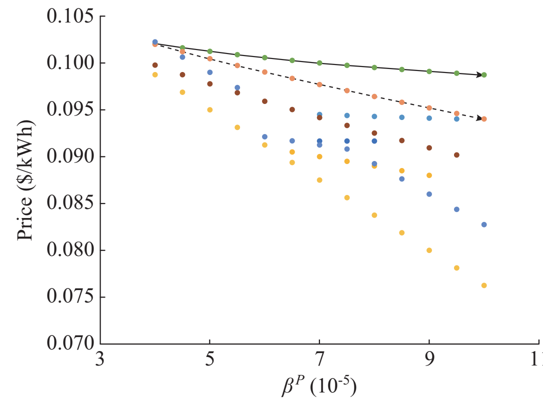

Fig. 3. Market clearing price in electric power market as a function of β^P.

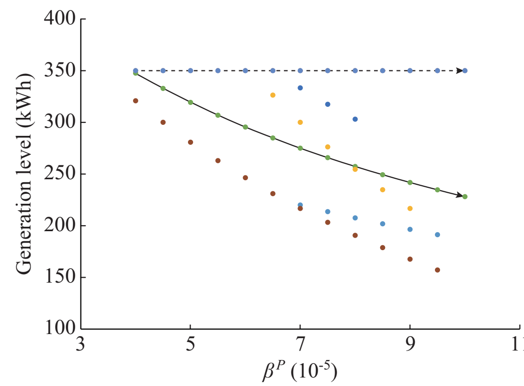

Fig. 4. Generation level of electric power firm 1 as a function of β^P.

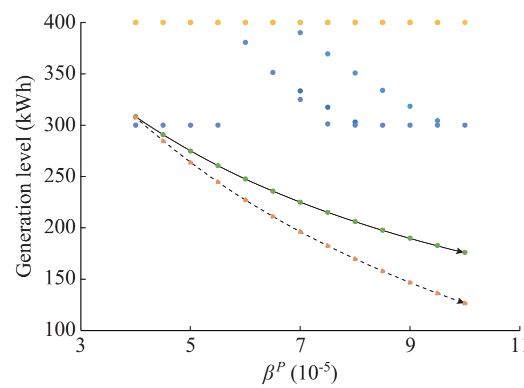

Fig. 5. Generation level of electric power firm 2 as a function of β^P.

In general, the increase in β^P causes a decrease in the market clearing price as well as the total generation level of the electric power firms. If a firm is not operating at the maximum capacity, it lowers its generation level as the market becomes less attractive with the increase of β^P. As a case in point, the dotted-line arrows in Figs. 3-5 illustrate the equilibria in which electric power firm 1 is operating at the maximum capacity, while electric power firm 2 lowers its generation level against the increase in β^P. The solid-line arrows in Figs. 3-5 illustrate the equilibria in which neither of the firms is operating at the maximum capacity, and both firms lower their generation levels against the increase in β^P, which naturally yields higher market clearing prices compared with the equilibria marked with the dotted-line arrows, i. e., when only one producer lowers its generation.

Next, we analyze how a change in the cost parameters of an electric power firm, namely a_1^P in this case study, influences the equilibria in electric power market. Figures 6 and 7 depict the generation levels of electric power firms 1 and 2 with respect to the changes in a_1^P, respectively.

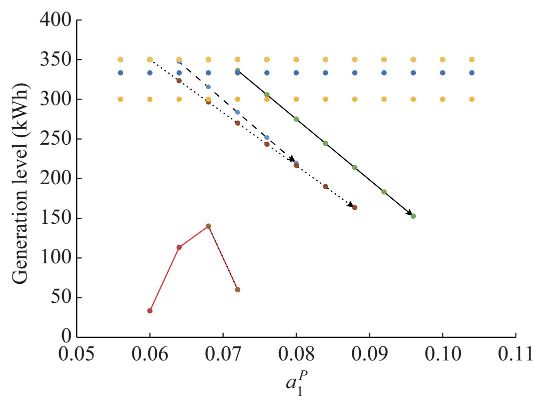

Fig. 6. Generation level of electric power firm 1 as a function of a_1^P.

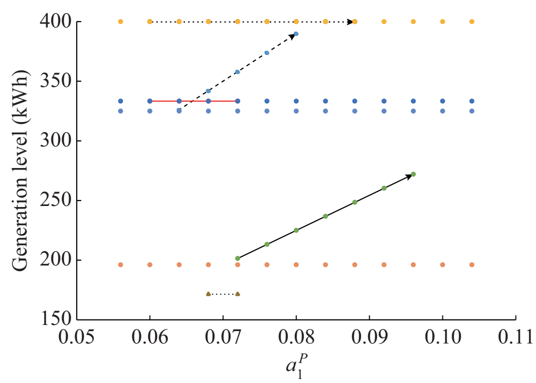

Fig. 7. Generation level of electric power firm 2 as a function of a_1^P.

Intuitively, one expects that an increase in a_1^P, i.e., an increase in the generation cost of electric power firm 1, causes a decrease in the generation level of electric power firm 1 and potentially an increase in the generation level of electric power firm 2, if electric power firm 2 is not operating at the maximum capacity. The arrows show a set of equilibria that are aligned with this intuition. We use different line formats for the arrows and lines to draw attention to some equilibria but use the same line format for the arrows and lines in related figures to illustrate the behavior of the equilibria with respect to the changes in the parameter values of the sensitivity analysis. The equilibria highlighted by the red line, however, counter this intuition. Albeit the available capacity, the generation level of electric power firm 2 does not change while the generation level of electric power firm 1 increases. Such equilibria and market behavior stem from the profit-maximization nature of these firms. A higher profit may be earned occasionally, as it is the case for electric power firm 1, if a lower price helps a firm sell more of its product. Such market complexities necessitate developing analytical models such as the one proposed in this paper to shed light on the behavior of the market participants, and more importantly assist the stakeholders and decision-makers of the interdependent natural-gas and electric power markets to make more informed decisions when pursuing new policies, regulations, market incentives, and others.

Finally, we conduct a sensitivity analysis to evaluate the impact of the natural gas to power conversion factor α^{PG} on the total generation level as well as the market clearing price of the two electric power firms. Figures 8 and 9 show the total generation levels of electric power firms 1 and 2 with different values of α^{PG}, respectively. Figure 10 depicts the corresponding market clearing price. Note that with different values of α^{PG}, there may be multiple consistent equilibria. Generally, the total generation level increases as the conversion factor increases, i. e., higher values of α^{PG}, and that drives the market clearing price down, as shown in Fig. 10. The higher generation level in the electric power market stems from the equilibria in which either both firms comparably increase their generation levels as indicated by the dashed-line arrows, or one firm dominantly increases its generation level while forcing the other one to lower its generation level, as the case shown by solid-line and dotted-line arrows. The former keeps the market clearing price at higher values while the latter drives down the price even further.

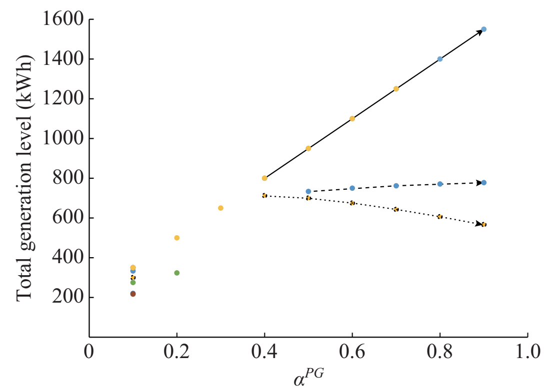

Fig. 8. Total generation level of electric power firm 1 as a function of α^{PG}.

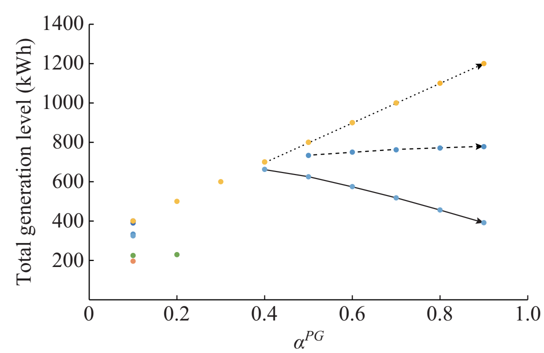

Fig. 9. Total generation level of electric power firm 2 as a function of α^{PG}.

A higher conversion factor makes the natural gas a more competitive energy source in the electric power market and enables the electric power firms to supply more electricity to their customers. Furthermore, Figs. 11 and 12 highlight the equilibria in which higher conversion factor enables the electric power firms to purchase less natural gas albeit supplying more electricity by converting natural gas more efficiently.

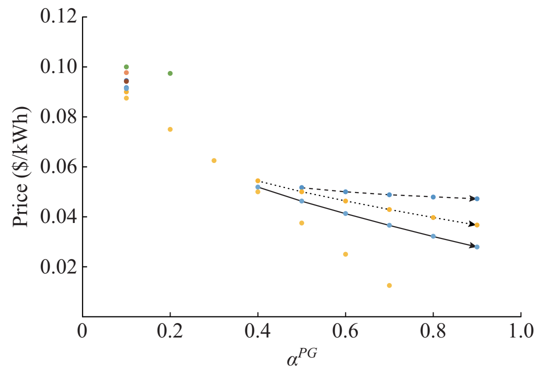

Fig. 10. Market clearing price in electric power market as a function of α^{PG}.

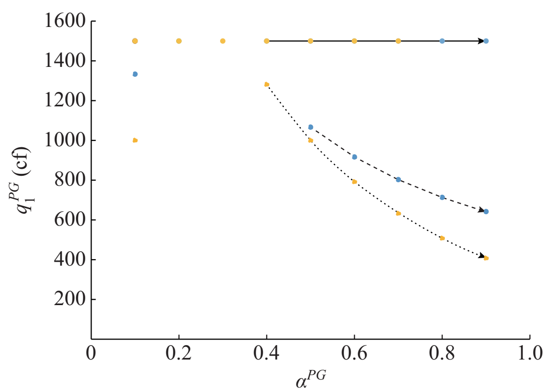

Fig. 11. Natural gas purchased by electric power firm 1 as a function of α^{PG}.

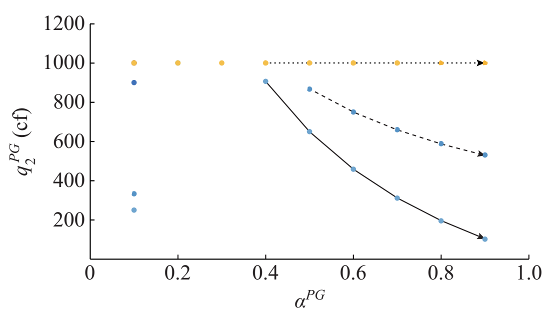

Fig. 12. Natural-gas purchased by electric power firm 2 as a function of α^{PG}.

B. Double-duopoly Case in Analytical Terms

The proposed model allows the derivation of analytical expressions that characterize the multi-firm equilibria in the interdependent natural-gas and electric power markets. Such analytical expressions may be utilized to provide insights on the outcomes and characteristics of operation decisions. Hence, we derive these analytical expressions for the double-duopoly case, as discussed in the previous section. Table VI shows the analytical expression of of the equilibria of the market clearing price and the generation levels of the electric power firms in a duopoly competition in the natural-gas market. Tables VII and VIII show the market clearing price and the generation levels of the electric power firms in analytical terms in a duopoly competition in the electric power market, respectively.

TABLE VI
ANALYTICAL EXPRESSION OF EQUILIBRIA IN NATURAL-GAS MARKET

Equilibrium	P^G	$q_1^{GG*}+q_1^{GP*}$	$q_2^{GG*}+q_2^{GP*}$
1	$\dfrac{\gamma^G-\beta^G q_1^{\max}+\beta^G \dfrac{a_2}{\beta^G+c_2^G}}{1+\beta^G \dfrac{1}{\beta^G+c_2^G}}$	$q_1^{\max}$	$\dfrac{P^{G*}-a_2}{\beta^G+c_2^G}$
2	$\dfrac{\gamma^G-\beta^G q_2^{\max}+\beta^G \dfrac{a_1}{\beta^G+c_1^G}}{1+\beta^G \dfrac{1}{\beta^G+c_1^G}}$	$\dfrac{P^{G*}-a_1}{\beta^G+c_1^G}$	$q_2^{\max}$
3	$\gamma^G-\beta^G(q_1^{\max}+q_2^{\max})$	$q_1^{\max}$	$q_2^{\max}$
4	$\dfrac{\gamma^G+\beta^G\left(\dfrac{a_1}{\beta^G+c_1^G}+\dfrac{a_2}{\beta^G+c_2^G}\right)}{1+\beta^G\left(\dfrac{1}{\beta^G+c_1^G}+\dfrac{1}{\beta^G+c_2^G}\right)}$	$\dfrac{P^{G*}-a_1}{\beta^G+c_1^G}$	$\dfrac{P^{G*}-a_2}{\beta^G+c_2^G}$

C. Discussion

As shown in the figures throughout the paper, we find the cases in which a set of parameter values may yield multiple equilibria. As is common in non-cooperative games, these equilibria introduce trade-offs in terms of which market agents, i.e., firms and consumers, benefit from one equilibrium relative to another. On the other hand, another set of parameter values may not yield consistent equilibria, which suggests that only mixed-strategy equilibria which are not found by the proposed model and which are beyond the scope of our work may exist. Therefore, there is no way to guarantee the existence of convergence to one consistent equilibrium, and there is no way to prove that a particular market equilibrium has been achieved until the spot markets clear in both markets.

Further, we discuss that one of the key advantages of the proposed model is that it allows identifying general trends and interactions in both natural-gas and electric power markets, which helps comprehend the general behavior of the two markets. As a case in point, any of the beneficiaries of the proposed model may need to investigate how to decrease the interdependency of the two markets and intuitively may expect that higher efficiency converters, i. e., higher values of α^{PG}, will increase the attractiveness of natural gas for electric power firms, drive up the consumption of natural gas in electric power market, and make the two markets more interdependent. In such scenarios, the proposed model can be utilized to test the hypothesis and provide insightful results quickly. In this specific scenario, the proposed model is utilized to test this hypothesis. As shown in Figs. 11 and 12, a higher conversion factor causes the consumption of natural gas by the electric power firms to decrease albeit the increase in total generation levels of electric power. Therefore, regulators and policymakers may suggest investing in developing more efficient converters to lower the natural-gas consumption as well as the dependency of the electric power markets to natural gas.

TABLE VII
ANALYTICAL EXPRESSION OF MARKET CLEARING PRICE IN ELECTRIC POWER MARKET

Equilibrium	P^P	Equilibrium	P^P
1	$\frac{1}{3}\left(\gamma^P + \frac{2P^{G*}}{\alpha^{PG}}\right)$	9	$\frac{1}{3}\left(\gamma^P + \frac{2P^{G*}}{\alpha^{PG}}\right)$
2	$\dfrac{\gamma^P + \frac{\beta^P}{\beta^P + c_2^P}(a_2^P + \alpha^{PG}\beta^P q_2^{PG,\max}) + \frac{P^{G*}}{\alpha^{PG}} - \alpha^{PG}\beta^P q_2^{PG,\max}}{2 + \frac{\beta^P}{\beta^P + c_2^P}}$	10	$\dfrac{\gamma^P + \frac{a_2^P \beta^P}{\beta^P + c_2^P} + \frac{\alpha^{PG}(\beta^P)^2 q_2^{PG,\max}}{\beta^P + c_2^P} - \alpha^{PG}\beta^P q_2^{PG,\max} + \frac{P^{G*}}{\alpha^{PG}}}{2 + \frac{\beta^P}{\beta^P + c_2^P}}$
3	$\frac{1}{3}\left(\gamma^P + \frac{2P^{G*}}{\alpha^{PG}}\right)$	11	$\frac{1}{3}\left(\gamma^P + \frac{2P^{G*}}{\alpha^{PG}}\right)$
4	$\frac{1}{2}\left(\gamma^P - \beta^P q_2^{P,\max} - \alpha^{PG}\beta^P q_2^{PG,\max} + \frac{P^{G*}}{\alpha^{PG}}\right)$	12	$\frac{1}{2}\left(\gamma^P - \beta^P q_2^{P,\max} - \alpha^{PG}\beta^P q_2^{PG,\max} + \frac{P^{G*}}{\alpha^{PG}}\right)$
5	$\dfrac{\gamma^P + \frac{a_1^P \beta^P}{\beta^P + c_1^P} + \frac{\alpha^{PG}(\beta^P)^2 q_1^{PG,\max}}{\beta^P + c_1^P} - \alpha^{PG}\beta^P q_1^{PG,\max} + \frac{P^{G*}}{\alpha^{PG}}}{2 + \frac{\beta^P}{\beta^P + c_1^P}}$	13	$\frac{1}{2}\left(\gamma^P - \beta^P q_1^{P,\max} - \alpha^{PG}\beta^P q_1^{PG,\max} + \frac{P^{G*}}{\alpha^{PG}}\right)$
6	$\frac{1}{1 + \frac{\beta^P}{\beta^P + c_1^P} + \frac{\beta^P}{\beta^P + c_2^P}}\left[\gamma^P + \beta^P\left(\frac{a_1^P}{\beta^P + c_1^P} + \frac{a_2^P}{\beta^P + c_2^P}\right) + \alpha^{PG}(\beta^P)^2\left(\frac{q_1^{PG,\max}}{\beta^P + c_1^P} + \frac{q_2^{PG,\max}}{\beta^P + c_2^P}\right) - \alpha^{PG}\beta^P(q_1^{PG,\max} + q_2^{PG,\max})\right]$	14	$\frac{1}{1 + \frac{\beta^P}{\beta^P + c_2^P}}\left[\gamma^P + \frac{a_2^P \beta^P}{\beta^P + c_2^P} + \frac{\alpha^{PG}(\beta^P)^2 q_2^{PG,\max}}{\beta^P + c_2^P} - \beta^P q_1^{P,\max} - \alpha^{PG}\beta^P(q_1^{PG,\max} + q_2^{PG,\max})\right]$
7	$\dfrac{\gamma^P + \frac{\beta^P}{\beta^P + c_1^P}(a_1^P + \alpha^{PG}\beta^P q_1^{PG,\max}) + \frac{P^{G*}}{\alpha^{PG}} - \alpha^{PG}\beta^P q_1^{PG,\max}}{2 + \frac{\beta^P}{\beta^P + c_1^P}}$	15	$\frac{1}{2}\left(\gamma^P - \beta^P q_1^{P,\max} - \alpha^{PG}\beta^P q_1^{PG,\max} + \frac{P^{G*}}{\alpha^{PG}}\right)$
8	$\dfrac{\gamma^P + \frac{a_1^P \beta^P}{\beta^P + c_1^P} + \frac{\alpha^{PG}(\beta^P)^2 q_1^{PG,\max}}{\beta^P + c_1^P} - \beta^P q_2^{P,\max} - \alpha^{PG}\beta^P(q_1^{PG,\max} + q_2^{PG,\max})}{1 + \frac{\beta^P}{\beta^P + c_1^P}}$	16	$\gamma^P - \beta^P(q_1^{P,\max} + q_2^{P,\max}) - \alpha^{PG}\beta^P(q_1^{PG,\max} + q_2^{PG,\max})$

D. Future Research

In future, we would like to expand on the findings of this research, by carrying out detailed numerical analysis using the actual data of real-world natural-gas and electric power markets, considering capacity additions in the proposed model, and using a supply-function equilibrium model instead of the Cournot model.

IV. CONCLUSION

We propose a Nash-Cournot equilibrium model pertaining for interdependent natural-gas and electric power markets. This model allows us to comprehend the impact on market outcomes with an increasing integration of the natural gas and electric power markets. The general analytical results derived based on this model are useful to inform the decision-making processes of both regulators and operators. In a simple double-duopoly case, we identify a large number of equilibria, which reveals the inherent complexity of the considered problem. We illustrate the evolution of such equilibria via sensitivity analysis, which renders some counter-intuitive market behavior outcomes.

REFERENCES

[1] J. Gil, A. Caballero, and A. J. Conejo, "Power cycling: CCGTs: the critical link between the electricity and natural gas markets," *IEEE Power and Energy Magazine*, vol. 12, no. 6, pp. 40-48, Oct. 2014.

[2] S. Chen and A. J. Conejo, "Strategic-agent equilibria in the operation of natural gas and power markets," *Energies*, vol. 13, no. 4, p. 868, Feb. 2020.

[3] M. Gil, P. Dueñas, and J. Reneses, "The interdependency of electricity and natural gaas markets: coupling of models," in *Proceedings of 2013 10th International Conference on the European Energy Market (EEM)*, Stockholm, Sweden, May 2013, pp. 1-6.

[4] J. A. de Gouw, D. D. Parrish, G. J. Frost *et al*., "Reduced emissions of CO_2, NO_X, and SO_2 from U.S. power plants owing to switch from coal to natural gas with combined cycle technology," *Earth's Future*, vol. 2, no. 2, pp. 75-82, Feb. 2014.

[5] P. Jaramillo, W. M. Griffin, and H. S. Matthews, "Comparative life-cycle air emissions of coal, domestic natural gas, LNG, and SNG for electricity generation," *Environmental Science & Technology*, vol. 41, no. 17, pp. 6290-6296, Sept. 2007.

[6] Q. Zeng, J. Fang, J. Li *et al*., "Steady-state analysis of the integrated natural gas and electric power system with bi-directional energy conversion," *Applied Energy*, vol. 184, pp. 1483-1492, Dec. 2016.

[7] B. C. Erdener, K. A. Pambour, R. B. Lavin *et al*., "An integrated simulation model for analysing electricity and gas systems," *International Journal of Electrical Power & Energy Systems*, vol. 61, pp. 410-420, Oct. 2014.

TABLE VIII
ANALYTICAL EXPRESSION OF GENERATION LEVELS IN ELECTRIC POWER MARKET

Equilibrium	$q_1^{P*}+\alpha^{PG}q_1^{PG*}$	$q_2^{P*}+\alpha^{PG}q_2^{PG*}$
1	$q_1^{P,\max}+\dfrac{\alpha^{PG}P^P-P^G-\alpha^{PG}\beta^P q_1^{P,\max}}{\alpha^{PG}\beta^P}$	$q_2^{P,\max}+\dfrac{\alpha^{PG}P^P-P^G-\alpha^{PG}\beta^P q_2^{P,\max}}{\alpha^{PG}\beta^P}$
2	$q_1^{P,\max}+\dfrac{\alpha^{PG}P^P-P^G-\alpha^{PG}\beta^P q_1^{P,\max}}{\alpha^{PG}\beta^P}$	$\alpha^{PG}q_2^{PG,\max}+\dfrac{P^P-a_2^P-\alpha^{PG}\beta^P q_2^{PG,\max}}{\beta^P+c_2^P}$
3	$q_1^{P,\max}+\dfrac{\alpha^{PG}P^P-P^G-\alpha^{PG}\beta^P q_1^{P,\max}}{\alpha^{PG}\beta^P}$	$\dfrac{P^G-\alpha^{PG}a_2^P}{\alpha^{PG}c_2^P}+\alpha^{PG}\dfrac{P^P-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_2^P}\right)P^G+\dfrac{\beta^P a_2^P}{c_2^P}}{\beta^P\alpha^{PG}}$
4	$q_1^{P,\max}+\dfrac{\alpha^{PG}P^P-P^G-\alpha^{PG}\beta^P q_1^{P,\max}}{\alpha^{PG}\beta^P}$	$q_2^{P,\max}+\alpha^{PG}q_2^{PG,\max}$
5	$\alpha^{PG}q_1^{PG,\max}+\dfrac{P^P-a_1^P-\alpha^{PG}\beta^P q_1^{PG,\max}}{\beta^P+c_1^P}$	$q_2^{P,\max}+\dfrac{\alpha^{PG}P^P-P^G-\alpha^{PG}\beta^P q_2^{P,\max}}{\alpha^{PG}\beta^P}$
6	$\alpha^{PG}q_1^{PG,\max}+\dfrac{P^P-a_1^P-\alpha^{PG}\beta^P q_1^{PG,\max}}{\beta^P+c_1^P}$	$\alpha^{PG}q_2^{PG,\max}+\dfrac{P^P-a_2^P-\alpha^{PG}\beta^P q_2^{PG,\max}}{\beta^P+c_2^P}$
7	$\alpha^{PG}q_1^{PG,\max}+\dfrac{P^P-a_1^P-\alpha^{PG}\beta^P q_1^{PG,\max}}{\beta^P+c_1^P}$	$\dfrac{P^G-\alpha^{PG}a_2^P}{\alpha^{PG}c_2^P}+\alpha^{PG}\dfrac{P^P-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_2^P}\right)P^G+\dfrac{\beta^P a_2^P}{c_2^P}}{\beta^P\alpha^{PG}}$
8	$\alpha^{PG}q_1^{PG,\max}+\dfrac{P^P-a_1^P-\alpha^{PG}\beta^P q_1^{PG,\max}}{\beta^P+c_1^P}$	$q_2^{P,\max}+\alpha^{PG}q_2^{PG,\max}$
9	$\dfrac{P^G-\alpha^{PG}a_1^P}{\alpha^{PG}c_1^P}+\alpha^{PG}\dfrac{P^P-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_1^P}\right)P^G+\dfrac{\beta^P a_1^P}{c_1^P}}{\beta^P\alpha^{PG}}$	$q_2^{P,\max}+\dfrac{\alpha^{PG}P^P-P^G-\alpha^{PG}\beta^P q_2^{P,\max}}{\alpha^{PG}\beta^P}$
10	$\dfrac{P^G-\alpha^{PG}a_1^P}{\alpha^{PG}c_1^P}+\alpha^{PG}\dfrac{P^P-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_1^P}\right)P^G+\dfrac{\beta^P a_1^P}{c_1^P}}{\beta^P\alpha^{PG}}$	$\alpha^{PG}q_2^{PG,\max}+\dfrac{P^P-a_2^P-\alpha^{PG}\beta^P q_2^{PG,\max}}{\beta^P+c_2^P}$
11	$\dfrac{P^G-\alpha^{PG}a_1^P}{\alpha^{PG}c_1^P}+\alpha^{PG}\dfrac{P^P-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_1^P}\right)P^G+\dfrac{\beta^P a_1^P}{c_1^P}}{\beta^P\alpha^{PG}}$	$\dfrac{P^G-\alpha^{PG}a_2^P}{\alpha^{PG}c_2^P}+\alpha^{PG}\dfrac{P^P-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_2^P}\right)P^G+\dfrac{\beta^P a_2^P}{c_2^P}}{\beta^P\alpha^{PG}}$
12	$\dfrac{P^G-\alpha^{PG}a_1^P}{\alpha^{PG}c_1^P}+\alpha^{PG}\dfrac{P^P-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_1^P}\right)P^G+\dfrac{\beta^P a_1^P}{c_1^P}}{\beta^P\alpha^{PG}}$	$q_2^{P,\max}+\alpha^{PG}q_2^{PG,\max}$
13	$q_1^{P,\max}+\alpha^{PG}q_1^{PG,\max}$	$q_2^{P,\max}+\dfrac{\alpha^{PG}P^P-P^G-\alpha^{PG}\beta^P q_2^{P,\max}}{\alpha^{PG}\beta^P}$
14	$q_1^{P,\max}+\alpha^{PG}q_1^{PG,\max}$	$\alpha^{PG}q_2^{PG,\max}+\dfrac{P^P-a_2^P-\alpha^{PG}\beta^P q_2^{PG,\max}}{\beta^P+c_2^P}$
15	$q_1^{P,\max}+\alpha^{PG}q_1^{PG,\max}$	$\dfrac{P^G-\alpha^{PG}a_2^P}{\alpha^{PG}c_2^P}+\alpha^{PG}\dfrac{P^P-\left(\dfrac{1}{\alpha^{PG}}+\dfrac{\beta^P}{\alpha^{PG}c_2^P}\right)P^G+\dfrac{\beta^P a_2^P}{c_2^P}}{\beta^P\alpha^{PG}}$
16	$q_1^{P,\max}+\alpha^{PG}q_1^{PG,\max}$	$q_2^{P,\max}+\alpha^{PG}q_2^{PG,\max}$

[8] G. Byeon and P. van Hentenryck, "Unit commitment with gas network awareness," *IEEE Transactions on Power Systems*, vol. 35, no. 2, pp. 1327-1339, Sept. 2020.
[9] S. Chen, A. J. Conejo, R. Sioshansi *et al*., "Unit commitment with an enhanced natural gas-flow model," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 3729-3738, Apr. 2019.
[10] R. Chen, J. Wang, and H. Sun, "Clearing and pricing for coordinated gas and electricity day-ahead markets considering wind power uncertainty," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 2496-2508, Sept. 2018.
[11] C. He, X. Zhang, T. Liu *et al*., "Distributionally robust scheduling of integrated gas-electricity systems with demand response," *IEEE Transactions on Power Systems*, vol. 34, no. 5, pp. 3791-3803, Mar. 2019.
[12] Y. He, M. Yan, M. Shahidehpour *et al*., "Decentralized optimization of multi-area electricity-natural gas flows based on cone reformulation," *IEEE Transactions on Power Systems*, vol. 33, no. 4, pp. 4531-4542, Dec. 2018.
[13] J. Yang, N. Zhang, C. Kang *et al*., "Effect of natural gas flow dynamics in robust generation scheduling under wind uncertainty," *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 2087-2097, Jul. 2018.
[14] L. Bai, F. Li, T. Jiang *et al*., "Robust scheduling for wind integrated energy systems considering gas pipeline and power transmission n−1 contingencies," *IEEE Transactions on Power Systems*, vol. 32, no. 2, pp. 1582-1584, Jun. 2017.
[15] A. Zlotnik, L. Roald, S. Backhaus *et al*., "Coordinated scheduling for

interdependent electric power and natural gas infrastructures," *IEEE Transactions on Power Systems*, vol. 32, no. 1, pp. 600-610, Mar. 2017.

[16] S. Chen, Z. Wei, G. Sun *et al*., "Adaptive robust day-ahead dispatch for urban energy systems," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 2, pp. 1379-1390, Dec. 2019.

[17] H. Ameli, M. Qadrdan, and G. Strbac, "Value of gas network infrastructure flexibility in supporting cost effective operation of power systems," *Applied Energy*, vol. 202, pp. 571-580, Sept. 2017.

[18] A. Antenucci and G. Sansavini, "Gas-constrained secure reserve allocation with large renewable penetration," *IEEE Transactions on Sustainable Energy*, vol. 9, no. 2, pp. 685-694, Sept. 2018.

[19] C. Ordoudis, P. Pinson, and J. M. Morales, "An integrated market for electricity and natural gas systems with stochastic power producers," *European Journal of Operational Research*, vol. 272, no. 2, pp. 642-654, Jan. 2019.

[20] X. Zheng, Y. Xu, Z. Li *et al*. (2020, Nov.). Co-optimisation and settlement of power-gas coupled system in day-ahead market under multiple uncertainties. [Online]. Available: https://arxiv.org/abs/2011.07320v1

[21] Y. Wen, X. Qu, W. Li *et al*., "Synergistic operation of electricity and natural gas networks via ADMM," *IEEE Transactions on Smart Grid*, vol. 9, no. 5, pp. 4555-4565, Feb. 2017.

[22] M. Gil, P. Dueñas, and J. Reneses, "Electricity and natural gas interdependency: comparison of two methodologies for coupling large market models within the European regulatory framework," *IEEE Transactions on Power Systems*, vol. 31, no. 1, pp. 361-369, Feb. 2015.

[23] S. Chen, A. J. Conejo, R. Sioshansi *et al*., "Operational equilibria of electric and natural gas systems with limited information interchange," *IEEE Transactions on Power Systems*, vol. 35, no. 1, pp. 662-671, Jul. 2019.

[24] C. Wang, W. Wei, J. Wang *et al*., "Equilibrium of interdependent gas and electricity markets with marginal price based bilateral energy trading," *IEEE Transactions on Power Systems*, vol. 33, no. 5, pp. 4854-4867, Jan. 2018.

[25] C. Wang, W. Wei, J. Wang *et al*., "Strategic offering and equilibrium in coupled gas and electricity markets," *IEEE Transactions on Power Systems*, vol. 33, no. 1, pp. 290-306, Apr. 2017.

[26] C. Ordoudis, S. Delikaraoglou, J. Kazempour *et al*., "Market-based coordination of integrated electricity and natural gas systems under uncertain supply," *European Journal of Operational Research*, vol. 287, no. 3, pp. 1105-1119, Dec. 2020.

[27] B. Zhang, W. Hu, J. Li *et al*., "Dynamic energy conversion and management strategy for an integrated electricity and natural gas system with renewable energy: deep reinforcement learning approach," *Energy Conversion and Management*, vol. 220, p. 113063, Sept. 2020.

[28] V. Grimm, J. Grübel, L. Schewe *et al*., "Nonconvex equilibrium models for gas market analysis: failure of standard techniques and alternative modeling approaches," *European Journal of Operational Research*, vol. 273, no. 3, pp. 1097-1108, Mar. 2019.

[29] C. Gong, D. Wu, N. Gong *et al*., "Multi-agent mixed complementary simulation of natural gas upstream market liberalization in China," *Energy*, vol. 200, p. 117535, Jun. 2020.

[30] J.-Z. Xiao, W.-C. Kong, X.-L. Wang *et al*., "Numerical modeling and assessment of natural gas pipeline separation in China: the data from Henan province," *Petroleum Science*, vol. 17, no. 1, pp. 268-278, Feb. 2020.

[31] S. Mousavian, A. J. Conejo, and R. Sioshansi, "Equilibria in investment and spot electricity markets: a conjectural-variations approach," *European Journal of Operational Research*, vol. 281, no. 1, pp. 129-140, Feb. 2020.

[32] V. Dvorkin, J. Kazempour, and P. Pinson, "Electricity market equilibrium under information asymmetry," *Operations Research Letters*, vol. 47, no. 6, pp. 521-526, Nov. 2019.

[33] R. Li, W. Wei, S. Mei *et al*., "Participation of an energy hub in electricity and heat distribution markets: an MPEC approach," *IEEE Transactions on Smart Grid*, vol. 10, no. 4, pp. 3641-3653, May 2018.

[34] S. Chen, G. Sun, Z. Wei *et al*., "Dynamic pricing in electricity and natural gas distribution networks: an EPEC model," *Energy*, vol. 207, p. 118138, Sept. 2020.

[35] C. Wang, W. Wei, J. Wang *et al*., "Equilibrium of interdependent gas and electricity markets with marginal price based bilateral energy trading," *IEEE Transactions on Power Systems*, vol. 33, no. 5, pp. 4854-4867, Jan. 2018.

[36] A. Ladjici and M. Boudour, "Nash-Cournot equilibrium of a deregulated electricity market using competitive coevolutionary algorithms," *Electric Power Systems Research*, vol. 81, no. 4, pp. 958-966, Apr. 2011.

[37] B. E. Hobbs, "Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets," *IEEE Transactions on Power Systems*, vol. 16, no. 2, pp. 194-202, May 2001.

[38] P. I. Helgesen and A. Tomasgard, "An equilibrium market power model for power markets and tradable green certificates, including Kirchhoff's laws and Nash-Cournot competition," *Energy Economics*, vol. 70, pp. 270-288, Feb. 2018.

[39] J. P. Molina, J. M. Zolezzi, J. Contreras *et al*., "Nash-Cournot equilibria in hydrothermal electricity markets," *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1089-1101, Oct. 2011.

[40] S. A. Gabriel, S. A. Siddiqui, A. J. Conejo *et al*., "Solving discretely-constrained Nash-Cournot games with an application to power markets," *Networks and Spatial Economics*, vol. 13, no. 3, pp. 307-3266, Sept. 2013.

[41] J. Yao, S. S. Oren, and I. Adler, "Two-settlement electricity markets with price caps and Cournot generation firms," *European Journal of Operational Research*, vol. 181, no. 3, pp. 1279-1296, Sept. 2007.

[42] I. Abada, S. Gabriel, V. Briat *et al*., "A generalized Nash-Cournot model for the northwestern European natural gas markets with a fuel substitution demand function: the GaMMES model," *Networks and Spatial Economics*, vol. 13, pp. 1-42, Mar. 2013.

[43] S. Spiecker, "Modeling market power by natural gas producers and its impact on the power system," *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 3737-3746, May 2013.

[44] D. de Wolf and Y. Smeers, "A stochastic version of a Stackelberg-Nash-Cournot equilibrium model," *Management Science*, vol. 43, no. 2, pp. 190-197, Feb. 1997.

[45] S. A. Gabriel, J. Zhuang, and S. Kiet, "A large-scale linear complementarity model of the north American natural gas market," *Energy Economics*, vol. 27, no. 4, pp. 639-665, Jul. 2005.

[46] M. T. Devine, S. A. Gabriel, and S. Moryadee, "A rolling horizon approach for stochastic mixed complementarity problems with endogenous learning: application to natural gas markets," *Computers & Operations Research*, vol. 68, pp. 1-15, Apr. 2016.

[47] D. G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, vol. 2, Berlin: Springer, 1984.

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