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An Extended DC Power Flow Model Considering Voltage Magnitude

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***Abstract*—A quasi-linear relationship between voltage angles and voltage magnitudes in power flow calculation is presented. An accurate estimation of voltage magnitudes can be provided by the quasi-linear relationship when voltage angles are derived by classical DC power flow. Based on the quasi-linear relationship, a novel extended DC power flow (EDCPF) model is proposed considering voltage magnitudes. It is simple, reliable and accurate for both distribution network and transmission network in normal system operation states. The accuracy of EDCPF model is verified through a series of standard test systems.**

***Index Terms*—DC power flow, distribution network, transmission network, voltage magnitude, linear power flow model.**

I. Introduction

RECENTLY, there has been an upsurge of interest in linear power flow models. The non-iterative model has considerable analytical and computation performances compared with the AC power flow (ACPF) model [1]. Therefore, the well-known classical DC power flow (DCPF) model is widely used in transmission expansion planning, economic dispatch and contingency analyses. However, the application of DCPF model is limited owing to its drawbacks such as low accuracy and inability of calculating voltage magnitudes and reactive power. Many versions of improved models are proposed to guarantee the accuracy under tough conditions. There are increasing studies that investigate the power flow model with voltage magnitudes and reactive power. In [2], the voltage magnitudes and voltage angles were not completely decoupled. The linear power flow model proposed in [3] was suitable for radial distribution networks. Reference [4] made a further step toward a V-θ completely-decoupled linear power flow model. To derive a linearized power flow model with voltage magnitudes and reactive power, [5] took the logarithmic transform of voltage magnitudes, and [6] utilized a data-driven method. References [7] and [8] performed detailed analysis and investigation of the existing linear approximations of PF and OPF problems. Valuable insights into the linearization of power flow model were also provided [2]-[8].

A quasi-linear relationship between voltage angles and voltage magnitudes is revealed. On this basis, an extended DCPF (EDCPF) model with an accurate estimation of voltage magnitudes is developed.

II. EDCPF Model

A. From Nodal Complex Power to Voltage Magnitudes

For a power system, let N denote the set of bus indices of PQ buses, and M denote the set of bus indices of PV buses and the slack bus. The complex power $\boldsymbol{S}$, admittance matrix $\boldsymbol{Y}$, voltage angles $\boldsymbol{\theta}$, and voltage magnitudes $\boldsymbol{V}$ can be arranged in the sequence as $\boldsymbol{S}=\begin{bmatrix}\boldsymbol{S}_N\\ \boldsymbol{S}_M\end{bmatrix}$, $\boldsymbol{Y}=\begin{bmatrix}\boldsymbol{Y}_{NN} & \boldsymbol{Y}_{NM}\\ \boldsymbol{Y}_{MN} & \boldsymbol{Y}_{MM}\end{bmatrix}$, $\boldsymbol{\theta}=\begin{bmatrix}\boldsymbol{\theta}_N\\ \boldsymbol{\theta}_M\end{bmatrix}$, $\boldsymbol{V}=\begin{bmatrix}\boldsymbol{V}_N\\ \boldsymbol{V}_M\end{bmatrix}$.

The nodal complex power injection of PQ buses is calculated as:

$$S_i^* = \dot{V}_i^* \sum_{\forall k} Y_{ik} \dot{V}_k \quad i \in N \tag{1}$$

where S_i is the nodal complex power at bus i; the superscript $*$ represents the conjugation; $\dot{V}_i = V_i \mathrm{e}^{\mathrm{j}\theta_i}$ is the complex bus voltage at bus i; and Y_{ik} is the admittance of line i-k.

Since bus voltage magnitudes are close to 1 p.u. in most cases, let ΔV_i denote the value of voltage magnitude deviation from 1.0 p.u. and apply the following transformation (2) in nodal complex power equation (1).

$$V_i = 1 + \Delta V_i \approx \frac{1}{1-\Delta V_i} = \frac{1}{1-(V_i-1)} = \frac{1}{2-V_i} \tag{2}$$

Then (1) can be rewritten as:

$$S_i^* = \frac{1}{(2-V_i)\mathrm{e}^{\mathrm{j}\theta_i}} \sum_{\forall k} Y_{ik} \dot{V}_k \quad i \in N \tag{3}$$

By dividing all buses into N and M types, (3) can be expressed as:

$$\sum_{\forall k \in N} Y_{ik} V_k \mathrm{e}^{\mathrm{j}\theta_k} + S_i^* V_i \mathrm{e}^{\mathrm{j}\theta_i} = 2 S_i^* \mathrm{e}^{\mathrm{j}\theta_i} - \sum_{\forall k \in M} Y_{ik} V_k \mathrm{e}^{\mathrm{j}\theta_k} \quad i \in N \tag{4}$$

where θ_i and θ_k are the i^{th} and k^{th} elements of voltage angles $\boldsymbol{\theta}$, respectively.

The matrix form of (4) can be written as:

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$$(\boldsymbol{Y}_{NN}+[\boldsymbol{S}_N^*])[\mathrm{e}^{\mathrm{j}\boldsymbol{\theta}_N}]\boldsymbol{V}_N=2[\boldsymbol{S}_N^*]\mathrm{e}^{\mathrm{j}\boldsymbol{\theta}_N}-\boldsymbol{Y}_{NM}[\boldsymbol{V}_M]\mathrm{e}^{\mathrm{j}\boldsymbol{\theta}_M} \tag{5}$$

where $[\cdot]$ is used to denote an operator that takes an $n\times 1$ vector and creates the corresponding $n\times n$ diagonal matrix with the vector elements on the diagonal. Equation (5) is a complex linear equation, and the voltage magnitudes of PQ buses $\boldsymbol{V}_N$ can be computed by matrix inversion (6).

$$\boldsymbol{V}_N=[\mathrm{e}^{-\mathrm{j}\boldsymbol{\theta}_N}]\frac{2[\boldsymbol{S}_N^*]}{\boldsymbol{Y}_{NN}+[\boldsymbol{S}_N^*]}\mathrm{e}^{\mathrm{j}\boldsymbol{\theta}_N}-[\mathrm{e}^{-\mathrm{j}\boldsymbol{\theta}_N}]\frac{\boldsymbol{Y}_{NM}[\boldsymbol{V}_M]}{\boldsymbol{Y}_{NN}+[\boldsymbol{S}_N^*]}\mathrm{e}^{\mathrm{j}\boldsymbol{\theta}_M} \tag{6}$$

Considering a special case when all buses are (or converted to) PQ bus, we can write the voltage magnitude deviation $\Delta\boldsymbol{V}$ in a specific form as:

$$\Delta\boldsymbol{V}=[\mathrm{e}^{-\mathrm{j}\boldsymbol{\theta}}]\frac{\boldsymbol{Y}-[\boldsymbol{S}^*]}{\boldsymbol{Y}+[\boldsymbol{S}^*]}\mathrm{e}^{\mathrm{j}\boldsymbol{\theta}} \tag{7}$$

So far, only one approximation (2) has been made. The truncation error caused by the approximation is $O(\Delta V^2)$, i.e., if voltage magnitudes range from 0.95 p.u. to 1.05 p.u., the maximum error would be 0.0025 p.u.. Hence, (6) and (7) are highly accurate in normal system operation states.

B. Quasi-linear Relationship Between V and θ

When calculating the power flow, the loads of PQ buses $\boldsymbol{S}_N$ and the voltage magnitudes of PV buses $\boldsymbol{V}_M$ are given. The voltage magnitude of the slack bus is set to be 1 p.u.. Hence, according to (6), the voltage magnitudes of PQ buses $\boldsymbol{V}_N$ can be expressed as the function of voltage angles $\boldsymbol{\theta}$:

$$\boldsymbol{V}_N=f(\boldsymbol{\theta}) \tag{8}$$

The theoretical value of $f(\boldsymbol{\theta})$ should exactly be a real vector since $\boldsymbol{V}_N$ is a real vector. However, the actual value of $f(\boldsymbol{\theta})$ contains a very small imaginary part resulted from the approximation (2). Hence, the linearization of (6) is based on the following two assumptions.

1) Ignore the imaginary part of $f(\boldsymbol{\theta})$ and only take the real part of $f(\boldsymbol{\theta})$.

$$\boldsymbol{V}_N=\Re\{f(\boldsymbol{\theta})\} \tag{9}$$

where $\Re\{\cdot\}$ is the operator that returns the real part.

2) Classical DCPF assumption: the differences of voltage angle across branches are small enough:

$$\begin{cases}\cos(\theta_i-\theta_k)\approx 1\\ \sin(\theta_i-\theta_k)\approx\theta_i-\theta_k\end{cases} \tag{10}$$

For convenience, four constant matrices $\boldsymbol{C}\in\mathbb{R}^{n\times n}$, $\boldsymbol{D}\in\mathbb{R}^{n\times n}$, $\boldsymbol{E}\in\mathbb{R}^{n\times m}$ and $\boldsymbol{F}\in\mathbb{R}^{n\times m}$ are defined in (11).

$$\begin{cases}\boldsymbol{C}=\Re\{2(\boldsymbol{Y}_{NN}+[\boldsymbol{S}_N^*])^{-1}[\boldsymbol{S}_N^*]\}\\ \boldsymbol{D}=\Im\{2(\boldsymbol{Y}_{NN}+[\boldsymbol{S}_N^*])^{-1}[\boldsymbol{S}_N^*]\}\\ \boldsymbol{E}=\Re\{(\boldsymbol{Y}_{NN}+[\boldsymbol{S}_N^*])^{-1}\boldsymbol{Y}_{NM}[\boldsymbol{V}_M]\}\\ \boldsymbol{F}=\Im\{(\boldsymbol{Y}_{NN}+[\boldsymbol{S}_N^*])^{-1}\boldsymbol{Y}_{NM}[\boldsymbol{V}_M]\}\end{cases} \tag{11}$$

where n and m are the sizes of N and M, respectively; and $\Im\{\cdot\}$ is the operators that return the imaginary part.

The voltage magnitude of the i^{th} PQ bus can be linearized as:

$$\begin{aligned}V_{N(i)}&=\mathrm{e}^{-\mathrm{j}\theta_{N(i)}}\sum_{k=1}^{n}(C_{ik}+\mathrm{j}D_{ik})\mathrm{e}^{\mathrm{j}\theta_{N(k)}}-\mathrm{e}^{-\mathrm{j}\theta_{N(i)}}\sum_{k=1}^{n}(E_{ik}+\mathrm{j}F_{ik})\mathrm{e}^{\mathrm{j}\theta_{M(k)}}\approx\\ &\sum_{k=1}^{n}[C_{ik}\cos(\theta_{N(k)}-\theta_{N(i)})-D_{ik}\sin(\theta_{N(k)}-\theta_{N(i)})]-\\ &\sum_{k=1}^{m}[E_{ik}\cos(\theta_{M(k)}-\theta_{N(i)})-F_{ik}\sin(\theta_{M(k)}-\theta_{N(i)})]\approx\\ &\sum_{k=1}^{n}[C_{ik}-D_{ik}(\theta_{N(k)}-\theta_{N(i)})]-\sum_{k=1}^{m}[E_{ik}-F_{ik}(\theta_{M(k)}-\theta_{N(i)})]=\\ &H_{ii}\theta_{N(i)}+\sum_{k=1}^{n}H_{ik}\theta_{N(k)}+\sum_{k=1}^{n}F_{ik}\theta_{M(k)}+b_i\end{aligned} \tag{12}$$

where i and k are the element indices of matrices $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{F}$; and H_{ii}, H_{ik}, b_i are the auxiliary elements defined as:

$$\begin{cases}H_{ii}=-D_{ii}+\sum\limits_{k=1}^{n}D_{ik}-\sum\limits_{k=1}^{m}F_{ik}\\ H_{ik}=-D_{ik}\\ b_i=\sum\limits_{k=1}^{n}C_{ik}-\sum\limits_{k=1}^{m}E_{ik}\end{cases} \tag{13}$$

The matrix form of (10) can be written as:

$$\boldsymbol{V}_N=\boldsymbol{H}\boldsymbol{\theta}_N+\boldsymbol{F}\boldsymbol{\theta}_M+\boldsymbol{b} \tag{14}$$

C. EDCPF Model

With the quasi-linear relationship between voltage angles and voltage magnitudes, the classical DCPF model can be extended by integrating (14). Let $\boldsymbol{A}=[\boldsymbol{H}\ \boldsymbol{F}]$, the EDCPF model for a power system with n_b buses can be written as:

$$\begin{cases}\boldsymbol{P}=\boldsymbol{B}\boldsymbol{\theta}'\\ \boldsymbol{V}_N=\boldsymbol{A}\boldsymbol{\theta}+\boldsymbol{b}\end{cases} \tag{15}$$

where $\boldsymbol{P}$ is the $(n_b-1)\times 1$ vector for the buses injected with active power without the slack bus; $\boldsymbol{B}$ is the $(n_b-1)\times(n_b-1)$ susceptance matrix without the slack bus; $\boldsymbol{\theta}'$ is the $(n_b-1)\times 1$ vector for voltage angles without the slack bus; $\boldsymbol{\theta}$ is the $n_b\times 1$ vector for all voltage angles; $\boldsymbol{V}_N$ is the $n\times 1$ vector for voltage magnitudes; $\boldsymbol{A}$ is the $n\times n_b$ constant matrix; and $\boldsymbol{b}$ is the $n\times 1$ constant vector.

The matrix $\boldsymbol{B}$ in (15) is the same as the susceptance matrix in the classical DCPF model. Therefore, the accuracy of voltage angles of EDCPF is the same as that of DCPF.

III. Numerical Results

Case studies are carried out on a series of test systems from MATPOWER 6.0 [5] including distribution networks (IEEE 33-bus and IEEE 69-bus) and transmission networks (IEEE 14-bus, IEEE 30-bus, IEEE 118-bus and Polish 3012-bus). The mean ratio of r/x of IEEE 33-bus and IEEE 69-bus are 1.44 and 2.05, respectively. The simulation platform is MATLAB R2017a on a PC with Intel CORE i5 and 8 GB RAM. Taking the results of ACPF model as the benchmark, the average errors of the voltage magnitudes of PQ buses are calculated by:

$$\varepsilon_V=\frac{1}{n}\sum_{i\in\mathcal{N}}|V_i-V_i^{ref}| \tag{16}$$

where V_i^{ref} is the voltage magnitude obtained from ACPF.

A. Accuracy Comparison of Different Methods

Table I shows the performances of the classical DCPF model, the models from [2] and [4], and the proposed EDCPF model, whose computation time are represented by t^{DCPF}, $t^{[2]}$, $t^{[4]}$, and t^{EDCPF}, respectively. For two radial distribution networks, the proposed EDCPF model achieves high precision. For four meshed transmission networks, the proposed EDCPF model also has lower error compared with other models. It can be observed from Table I that the EDCPF model has the best performance in almost all cases including radial distribution networks with high r/x ratio and meshed transmission networks. The detailed performance of EDCPF on voltage magnitude estimation is shown in Figs. 1 and 2.

TABLE I
COMPARISON OF AVERAGE ERRORS OF VOLTAGE MAGNITUDES OF DIFFERENT DCPF MODELS

Test case	ε_V^{DCPF} (p.u.)	$\varepsilon_V^{[2]}$ (p.u.)	$\varepsilon_V^{[4]}$ (p.u.)	ε_V^{EDCPF} (p.u.)
IEEE 33-bus	0.0532	0.0507	0.0037	0.0004
IEEE 69-bus	0.0270	0.0258	0.0019	0.0004
IEEE 14-bus	0.0448	0.0114	0.0120	0.0031
IEEE 30-bus	0.0226	0.0119	0.0017	0.0003
IEEE 118-bus	0.0225	0.0065	0.0016	0.0017
Polish 3012-bus	0.0892	0.0055	0.0034	0.0026

Note: The bus voltage profile is assumed to be flat at 1.0 p.u. in the DCPF model.

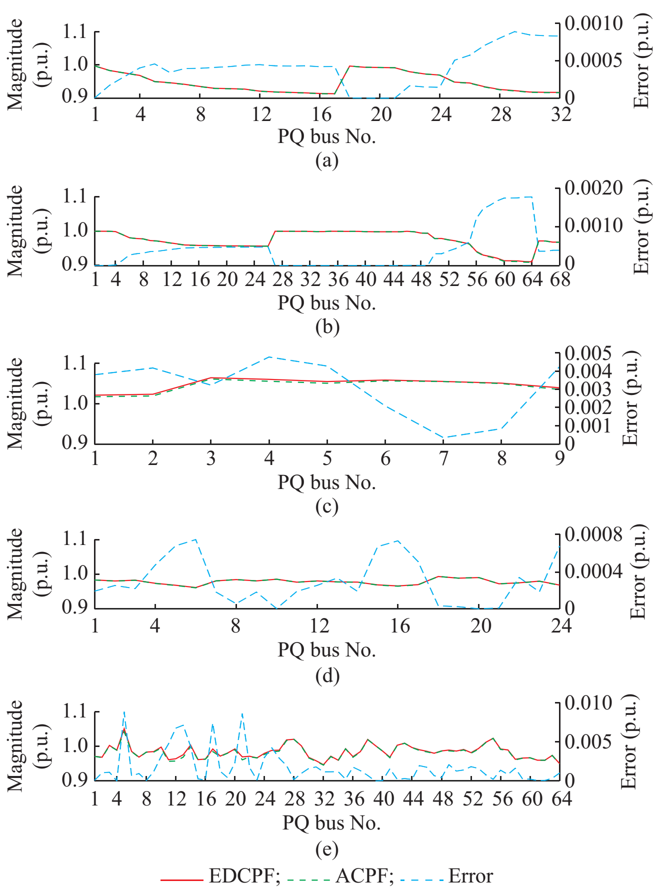

Fig. 1. Performance of EDCPF on voltage magnitudes estimation verified by IEEE test systems. (a) IEEE 33-bus. (b) IEEE 69-bus. (c) IEEE 14-bus. (d) IEEE 30-bus. (e) IEEE 118-bus.

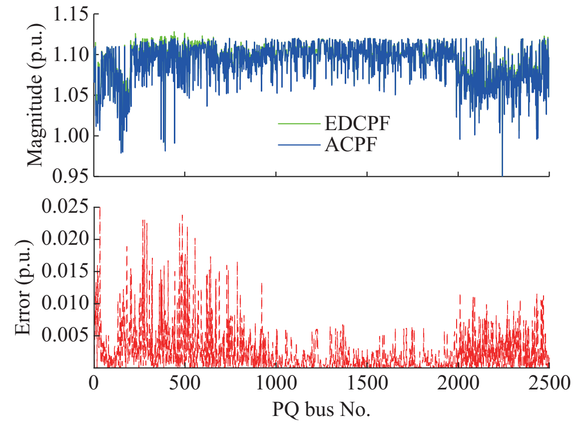

Fig. 2. Performance of EDCPF on voltage magnitudes estimation verified by Polish 3012-bus test system.

For a large test system such as Polish 3012-bus system, the voltage magnitudes of the system range from 0.95 p.u. to 1.1 p.u. or even higher. In EDCPF, although the calculation error of most PQ buses is acceptable (mean error: 0.0026 p. u.), the calculation error of a few PQ buses is more than 0.02 p.u.. Admittedly, there is room for the improvement of the calculation accuracy for large complex systems.

B. Comparison of Computation Time of Different Methods

The computation time of different methods with different test cases is listed in Table II. Overall, the computation time of each method is quite short, and the computation time of single calculation may be inaccurate. Therefore, to avoid random errors, we take the average computation time of 100 calculations for each method.

TABLE II
COMPARISON OF COMPUTATION TIME OF DIFFERENT DCPF MODELS

Test case	t^{DCPF} (ms)	$t^{[2]}$ (ms)	$t^{[4]}$ (ms)	t^{EDCPF} (ms)
IEEE 33-bus	5.28	3.31	4.50	5.32
IEEE 69-bus	5.81	4.77	4.28	6.18
IEEE 14-bus	4.75	2.58	3.86	4.78
IEEE 30-bus	5.17	3.24	3.85	5.35
IEEE 118-bus	7.98	7.31	4.10	8.16
Polish 3012-bus	8.55	1827.35	944.60	987.60

The computation time of the EDCPF model includes that of the classical DCPF model. In general, for the first five test cases, the speeds of all methods are at the same level. In terms of the computation efficiency of the EDCPF model for large systems, the formation of the coefficient matrix needs one time of computation matrix inversion, which dominates the overall computation time. Nevertheless, the EDCPF model can be quickly calculated due to the sparsity of the coefficient matrix.

C. Performance of EDCPF Under Extreme Conditions

When deducing the EDCPF model, we assume that the voltage magnitudes are close to 1.0 p.u.. However, the assumption may not always hold in reality.

To test the EDCPF model when the voltage magnitudes are not close to 1.0 p.u., we change the voltage magnitudes of PV buses and the slack bus V_M with ±0.1 p.u. and ±0.2 p.u., respectively, and then calculate the voltage magnitudes of PQ buses V_N. Table III shows the average errors of V_N of different test systems under extreme conditions. The average errors are calculated according to (16). The base case refers to the case with the original V_M in test systems. The extreme conditions include $V_M-0.2$, $V_M-0.1$, $V_M+0.1$, and $V_M+0.2$. For example, the condition of $V_M-0.2$ means decreasing all voltage magnitudes of PV buses and the slack bus by 0.2 p.u.. Also, Table III lists the original range of V_M in test systems. The power flow of the Polish 3012-bus test system would not converge when the voltage magnitudes of PV buses are changed. Therefore, the Polish 3012-bus test system is not included in Table III. Taking the IEEE 118-bus test system for example, the voltage magnitudes of PQ buses are demonstrated with different voltage magnitudes of PV buses in Fig. 3.

TABLE III
AVERAGE ERRORS OF VOLTAGE MAGNITUDES UNDER EXTREME CONDITIONS (P.U.)

Test case	Original range of V_M	Condition				
		$V_M-0.2$	$V_M-0.1$	Base case	$V_M+0.1$	$V_M+0.2$
IEEE 33-bus	[1.0,1.0]	0.0065	0.0022	0.0004	0.0002	0.0011
IEEE 69-bus	[1.0,1.0]	0.0036	0.0013	0.0004	0.0003	0.0007
IEEE 14-bus	[1.01,1.09]	0.0076	0.0046	0.0031	0.0025	0.0028
IEEE 30-bus	[1.0,1.0]	0.0024	0.0009	0.0003	0.0004	0.0009
IEEE 118-bus	[0.943,1.05]	0.0043	0.0026	0.0017	0.0013	0.0012

Note: The error of IEEE 118-bus test system is 0.0014 p.u. under the condition of $V_M+0.3$.

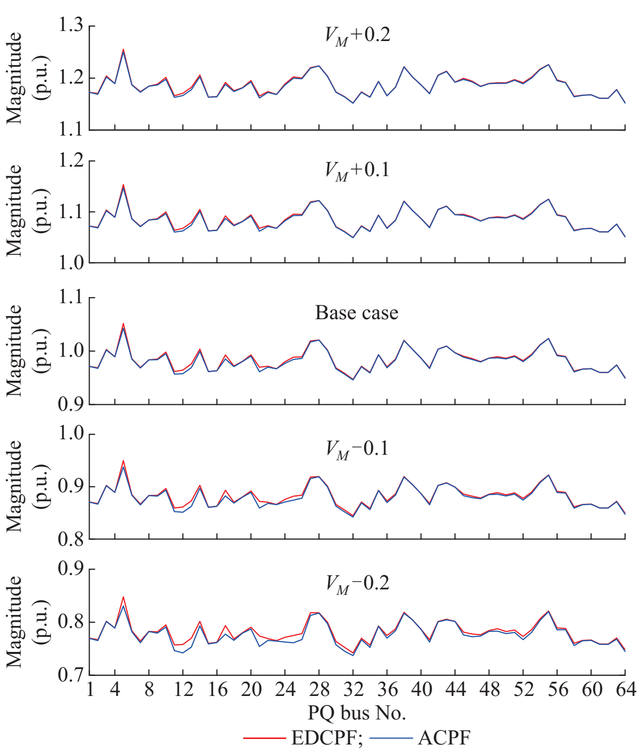

Fig. 3. Performance of EDCPF on voltage magnitudes estimation under extreme conditions verified by IEEE 118-bus test system.

As shown in Table III, in most cases, the error of the EDCPF model increases as the voltage magnitudes deviate from 1.0 p.u.. Compared with the increase of voltage magnitudes, the decrease of voltage magnitudes has greater impacts on calculation accuracy. Although extreme conditions are considered, the overall performance of EDCPF is satisfactory, since the average error does not exceed 0.01 p.u. for all conditions. However, the EDCPF method should be carefully applied under extreme conditions.

IV. CONCLUSION

In this letter, we have introduced a quasi-linear relationship between voltage angles and voltage magnitudes. Besides, the EDCPF model with a relatively precise estimation of voltage magnitudes is proposed. The accuracy of the EDCPF model is verified through a series of standard test systems under normal and extreme conditions. A feature of EDCPF is that the classical DCPF model remains unchanged in the EDCPF model. Therefore, the EDCPF model can be used directly in the problem where the classical DCPF model is used. EDCPF model is inspiring and valuable in many fields. The linear structure of the EDCPF model is suitable for an optimization problem for voltage magnitude consideration when the problem is confined to the usage of DCPF model. EDCPF is also helpful for the initialization of the calculation of ACPF. The convergence of ACPF is sensitive to the initial value of voltage magnitudes. For a large system such as Polish 3012-bus system, ACPF is unable to converge when flat voltage initialization is applied. Thus, the EDCPF model can be used to obtain a good initial value of voltage magnitudes.

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