Response Time of Reactive Power Based on Different Definitions and Algorithms

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***Abstract*—Dynamic reactive power compensation equipment typically requires a fast response to output the necessary reactive power. The term "dynamic response time of reactive power" is often used but has never been clearly defined. This paper summarizes the reactive power calculations under different definitions and algorithms and considers these calculations in terms of signal processing to simulate and analyze the step response. This paper subsequently focuses on the widely used instantaneous reactive power algorithm and finally concludes that the dynamic reactive power response time closely depends on the reactive power calculation method itself. The single-phase instantaneous reactive power algorithm has the fastest response time. The reactive power response time of dynamic reactive devices in power systems is a minimum of a quarter of one cycle time for the well-known and widely used single-phase reactive power algorithms.**

***Index Terms*—Response time, reactive power, instantaneous reactive power, Hilbert transform, phase shift, synchronous reference frame.**

I. Introduction

THE recent increase in the proportion of renewable energy resources within power systems has increased the significance of the role of power electronic devices. The reactive power characteristics of power electronic equipment are essential in power quality and stability issues and most devices considered as reactive power sources require a fast response. The "dynamic response time of reactive power" is often used to assess the ability of fast reactive power regulation [1]-[3]. However, a clear definition of the dynamic response time of reactive power has not yet been established, despite the discussion regarding the concepts and definitions of reactive power, which have developed into instantaneous reactive power (IRP) theory since the 1980s [4].

Fast reactive power response is critical in high-voltage direct current (HVDC) transmission systems, which are rapidly developing in concurrence with renewable energy utilization. For conventional line-commutated current-sourced converter (LCC) based HVDC transmission systems, the fast reactive power compensation provided by a static synchronous compensator (STATCOM) can reduce the commutation failure probability of LCC thyristor valves [5]. The poor dynamics of active and reactive power control in voltage source converter (VSC) based HVDC transmission systems can lead to instability when the converter is connected to a weak AC system [6], [7]. Furthermore, fast reactive power control on the millisecond scale can enhance the low-voltage ride through (LVRT) ability of a wind turbine generator during critical low-voltage scenarios [8]. For industrial applications such as arc furnace power quality improvement, the response time of reactive power in flicker compensation is critical to meet power quality requirements. Thus, several milliseconds delay in control may be crucial for flicker attenuation [9].

The conventional reactive power concept has been defined to represent of the quantity of electric power resulting from the phase angle difference between the current and voltage. The power calculation is based on a steady sinusoidal AC system, i.e., the root mean square (RMS) concept. If the reactive power calculation method is considered in terms of signal processing, it can be concluded that the calculated response time for reactive power based on traditional reactive power theory would be one cycle time (T), thus $T=20$ ms for 50 Hz or $T=16.7$ ms for 60 Hz. Hence, the same results would be obtained as the Budeanu's reactive power definition, which describes the reactive power under non-sinusoidal conditions.

The concept of IRP was firstly proposed in 1983 to deal with arbitrary voltage and current waveforms without any restriction on the three-phase three- or four-wire systems [4], [10]. Some experts propose that q-power is not associated with any physical phenomenon in the system and the consequences of q-power are constrained by various factors such as the asymmetry or harmonics of supply voltage [11]. The concept of IRP has been established particularly in the power electronics field for its instantaneous compensation of distorted current waveforms [12], [13]. A modified instantaneous power theory was introduced in 1999, which calculates the instantaneous active and reactive power in polar coordinates without complex calculation of p and q from p-q theory [14]. A generalized IRP theory was proposed in 2009 for poly-phase power systems [15]. Three-phase instantaneous power theory and its modified forms have been wide-

Manuscript received: February 12, 2019; accepted: April 16, 2020. Date of CrossCheck: April 16, 2020. Date of online publication: September 3, 2020.

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DOI: 10.35833/MPCE.2019.000085

ly used in active filters for fast current compensation since their emergence. However, instantaneous power theory does not provide credible interpretations of the power properties in power systems, and is often considered to be a quantity defined for convenience, rather than a coherent part of power theory [16].

Even when considered as only a control algorithm, the three-phase instantaneous theory is not applicable to single- or individual-phase cases as the calculations require Clarke's transform among the three phases and algebraic operations for diverse phase quantities. Thus, this theory has been confined to the field of three-phase low-voltage power electronics and has little advantage in the field of power transmission and distribution. Fast single-phase reactive power control is a typical and important requirement of the electromagnetic transient processes in power systems. In general, the fast single-phase reactive power algorithm is more applicable to power systems.

The response time according to three-phase IRP theory is zero, but this is insignificant as the calculated IRP q does not have the common meaning of traditional reactive power [16]. Comparative results and an explanation are illustrated in Section III of this paper. Thus, it is necessary to develop fast single-phase reactive power algorithms, which can be further used to study the three-phase reactive power responses [17], [18].

The reactive power calculation time of the conventional numerical integration method is equal to that of one cycle for single-phase power. It is noted that a long delay in response time is a major drawback of this method. Reference [19] demonstrates that a longer calculation time delay would cause excessive power to flow into or out of the active filter, and proposes a method with half cycle integration to reduce the calculation time to half a cycle, resulting in a relatively quick DC voltage control response in active filtering experiments [19].

A definition for single-phase instantaneous active and reactive power based on the Hilbert transform for operation in the transient state is proposed in [20]. Also, based on the Hilbert transform, [21] provides a digital measuring method for reactive power with low order infinite impulse response (IIR) Hilbert filters, instead of finite impulse response (FIR) filters, to obtain the accurate phase shift for digital measuring [21]. However, Hilbert filtering is used to calculate the reactive power in instrumentation and measurement fields as it can deal with electric quantities under non-sinusoidal conditions. If it is applied to fast control filed, the time delay caused by data processing of filters would make the performance of dynamic response unsatisfactory [22].

Some improved reactive power definitions and algorithms for single phase, also referred to as IRP, have been presented and applied in engineering since the 1990s [18], [23], [24]. They are inspired by Hirofumi Akagi's IRP theory [4], but the definitions differ predominantly in their calculation of the instantaneous value of β, the component in Clarke's transform, using the electrical quantities of the same phase. In these approaches, however, the β component is not directly available and needs to be synthesized via a 90° phase shift operation or cross correlation at the fundamental frequency. In order to reduce the response time, 60°, 45°, and even zero phase shift operations, derived from the differentiator of two adjacent sets of sampling data, have also been proposed [18], [25]. Although fast dynamic response could be obtained with a smaller phase shift of the β component, this would be more likely to cause overshoot or instability of the control variables. In engineering applications, the 90° phase shift algorithm is widely used for reactive power calculations, and will be analyzed with other algorithms in this paper.

Another quick reactive power calculation algorithm addressed in this paper is the synchronous reference frame *dq* (SRF-*dq*) algorithm [17], [26]. The current *dq* components of the SRF algorithm are generated using an $\alpha\beta$-dq transformation, where the three-phase current control strategy can be applied and requires a synchronous angle normally related to voltage phase, provided by phase lock loop (PLL). There would be an additional time delay in the response time when computing PLL, which is neglected in the following sections of this paper as it is not the focus of this study.

The transient response time of single-phase reactive power has not been fully investigated and clarified under different reactive theories or definitions. The problem becomes more complicated when a required response time is given by grid regulations. The required response time of reactive power was roughly defined by the State Grid Corporation of China in 2013, when an increasing number of wind farms were integrated into the grid [27], [28]. Similar requirements have not been reported by European or North American utilities till recently [29], [30]. According to clause 4.10 of the specification document "Q/GDW 11064-2013: specification for technical performance and test of reactive power compensation equipment in wind farm", the system response time of reactive power compensation equipment in wind farms should be less than 30 ms, as shown in Appendix A. However, the response time calculation method is not mentioned in the document. This paper proves that the response time deduced from various methods may differ by dozens of milliseconds. Thus, different response time may lead to technical confusion and even economic disputes between wind farms and grids.

In this paper, the calculation of reactive power response time will be analyzed for single-phase power. Section II will summarize the calculation of reactive power, including the Hilbert transform algorithm, digital phase shift method, IRP algorithm, and SRF-*dq* algorithm. Section III will compare the differences of reactive power calculated using four different reactive power methods with different current changes, and provide further analysis. Section IV will study the influence of the reactive power algorithms on the response time in the control equipment. Finally, conclusions are given in Section V.

II. Calculation of Reactive Power

A. Definitions of Reactive Power in IEEE and IEC Standards for Measurement

The electric power quantities, including reactive power under sinusoidal, non-sinusoidal, balanced, and unbalanced conditions, are defined in IEEE 1459 [31] along with the relevant mathematical expressions. For sinusoidal conditions, the reactive power is expressed via an integral quantity [31].

$$Q=UI\sin\theta=\frac{\omega}{kT}\int_{\tau}^{\tau+kT} i\left(\int u\mathrm{d}t\right)\mathrm{d}t \tag{1}$$

where Q is the reactive power; U is the root mean square (RMS) value of voltage; I is the RMS value of current; $T=1/f$ is the cycle time; k is the positive integral number; τ is the moment when the measurement starts; u is the voltage; and θ is the phase angle between the voltage and the current.

The concept of reactive power for metering is defined in IEC 62053-24 [32].

$$Q=U_1 I_1 \sin\varphi_1 \tag{2}$$

where U_1 is the RMS value of the fundamental frequency component of the voltage; I_1 is RMS value of the fundamental frequency component of the current; and φ_1 is the phase angle between the fundamental frequency voltage and current.

In engineering applications, reactive power measurement instruments use the following phase shift equation to calculate reactive power [32].

$$Q=\frac{1}{T}\int_0^T u(t)i\left(t+\frac{T}{4}\right)\mathrm{d}t \tag{3}$$

Both the IEEE and IEC reactive power calculations are integral algorithms over one or multiple cycles, derived from the traditional definition of reactive power under sinusoidal conditions. It is clear that the response time of reactive power using these algorithms will be one cycle or more due to the integration cycle when the voltage or current variables change. However, in electric energy metering applications, the response time of reactive power is insignificant.

B. Common Reactive Power Algorithm in Reactive Power Control

In reactive power control apparatus such as STATCOM and SVC, the ability to output fast reactive power is critical. Thus, the reactive power itself must be measured quickly in order to maintain the fast control of the system. With the development of digital processing technologies, calculations of the reactive power devices often use the following specific algorithms to achieve fast reactive power calculations:

1) Hilbert Transform Algorithm

The Hilbert transform algorithm mainly solves reactive power calculations when the voltage or current has harmonic components. The theoretical basis is Budeanu's definition of reactive power.

$$Q=\sum_{n=1}^{\infty} U_n I_n \sin\varphi_n \tag{4}$$

where U_n is the RMS value of the n^{th} harmonic voltage; I_n is the RMS value of the n^{th} harmonic current; and φ_n is the phase difference of the n^{th} harmonic voltage and current.

For all harmonics, the Hilbert transform shifts the harmonic components by 90° and calculates the reactive power according to Budeanu's expression.

In engineering applications, the Hilbert transform is widely used for reactive power calculations, the schematic diagram is shown in Fig. 1, despite the lack of harmonic content in voltage and current. A/D is the analog to digital conversion. The Hilbert algorithm is studied as a reactive power calculation method, since its effect on reactive power response time and its advantages in dealing with harmonics are ignored.

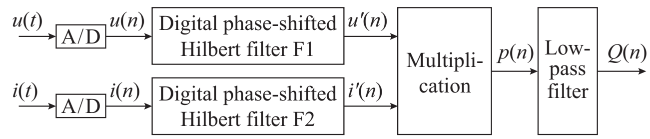

Fig. 1. Schematic diagram of reactive power calculation based on Hilbert transform algorithm.

A digital phase shift Hilbert filter consists of two sets of digital filters, F1 and F2, which satisfy:

$$\frac{H_{F1}(\mathrm{e}^{\mathrm{j}\omega})}{H_{F2}(\mathrm{e}^{\mathrm{j}\omega})}=-\mathrm{j} \quad \omega_0<\omega<\pi-\omega_0 \tag{5}$$

where H_{F1} is the digital phase-shifted Hilbert filter F1; H_{F2} is the digital phase-shifted Hilbert filter F2; ω is the angular frequency; and $(\omega_0, \pi-\omega_0)$ is the target angular frequency range.

2) Digital Phase Shift Method

The digital phase shift method delays the sampling value of voltage or current by a 1/4 cycle and multiplies the sampling value of current or voltage to obtain reactive power.

$$Q=\frac{1}{N}\sum_{k=0}^{N-1} u\left(k+\frac{N}{4}\right)i(k) \tag{6}$$

where N is the total number of sampling points per cycle; k is the number of sampling points; and $u(k)$ and $i(k)$ are the voltage and the current sampling points, respectively.

3) IRP Algorithm

In three-phase three- or four-wire systems, IRP results from the following equations [12], [13]:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix}=\sqrt{2/3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}=\sqrt{2/3}\begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{8}$$

where U_a, U_b, U_c and i_a, i_b, i_c are the voltages and the currents of phases a, b, and c, respectively; U_α, U_β and i_α, i_β are the α and β values of three-phase voltage and current after Clark's transformation, respectively. The three-phase IRP can be obtained via the following expression:

$$q=u_\beta^* i_\alpha - u_\alpha^* i_\beta \tag{9}$$

where q is the three-phase IRP.

For single-phase IRP calculations, a similar expression is proposed in [22], [33]. Shifting the voltage and current angles by 90°, a pair of orthogonal components, α and β, are obtained. Thus, the single-phase IRP would be calculated using (9).

$$\begin{cases} u=\sqrt{2}\,U\sin\omega t \\ i=\sqrt{2}\,I\sin(\omega t-\varphi) \end{cases} \tag{10}$$

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix}=\frac{\sqrt{2}}{2}\begin{bmatrix} \sqrt{2}\,U\sin\omega t \\ \sqrt{2}\,U\sin\left(\omega t-\frac{\pi}{2}\right) \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}=\frac{\sqrt{2}}{2}\begin{bmatrix} \sqrt{2}\,I\sin(\omega t-\varphi) \\ \sqrt{2}\,I\sin\left(\omega t-\varphi-\frac{\pi}{2}\right) \end{bmatrix} \tag{12}$$

In discrete sampling and digital data processing, the above expressions can be written as:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix}=\frac{\sqrt{2}}{2}\begin{bmatrix} u(k) \\ u\left(k+\frac{N}{4}\right) \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}=\frac{\sqrt{2}}{2}\begin{bmatrix} i(k) \\ i\left(k+\frac{N}{4}\right) \end{bmatrix} \tag{14}$$

Equation (9) can be rewritten as (15) with discrete sampling data.

$$q=\frac{1}{2}\left(u\left(k+\frac{N}{4}\right)i(k)-i\left(k+\frac{N}{4}\right)u(k)\right) \tag{15}$$

It is noted that this expression for IRP does not contain integral or sum operations over a single cycle, thus the response time would be less than one cycle.

4) SRF-dq Algorithm

In order to simplify the calculation and analysis of a synchronous machine or three-phase inverter, the Park transform is widely applied as it can rotate the reference frames of AC quantities and transform them into DC quantities [22], [33].

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}\xrightarrow{\text{Clark}}\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix}\xrightarrow{\text{Park}}\begin{bmatrix} u_d \\ u_q \\ u_0 \end{bmatrix} \tag{16}$$

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}\xrightarrow{\text{Clark}}\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix}\xrightarrow{\text{Park}}\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \tag{17}$$

The IRP is calculated as:

$$q=u_q^* i_d-u_d^* i_q \tag{18}$$

Similarly, for single-phase, $u_\alpha u_\beta$, $i_\alpha i_\beta$ obtained via phase shift are the same as from (11) and (12).

$$[\,u\,]\xrightarrow{\text{Phase-shift}}\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix}\xrightarrow{\text{Park}}\begin{bmatrix} u_d \\ u_q \end{bmatrix} \tag{19}$$

$$[\,i\,]\xrightarrow{\text{Phase-shift}}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}\xrightarrow{\text{Park}}\begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{20}$$

The key process of the calculation described above is to use the Park transformation to obtain the *dq* component of voltage and current in the *dq* coordinate system. The IRP can be subsequently obtained from (18).

III. Simulation and Analysis

When the current undergoes a step change, the reactive power calculated via different algorithms has various transitional processes. These reactive power algorithms can be regarded as filters whose time delay is expressed by the time response of the filters. In the simulation, the current abruptly increases from 0.9 p.u. to 1.0 p.u. at $t=0$, while the voltage is sinusoidal, which is an ideal step input to the current.

The idealized mathematical expression for current step impact can be regarded as the characteristic of an ideal reactive power control device, which can instantaneously change the output current. Thus, the simulation is a mathematic model for the ideal fastest reactive power device, whose current output has zero time delay when a control order is received according to Fig. A1 in Appendix A.

However, the reactive power calculations can be understood in terms of signal processing of current and voltage data. Considering different algorithms as filters, the single-phase reactive power algorithms should have a step response to the step current input. The response time of reactive power calculation algorithms should be identified and included.

$$\begin{cases} u(t)=\sin(\omega t+\varphi) \\ i(t)=\begin{cases} 0.9\sin(\omega t+\varphi+\theta) & t\le 0 \\ \sin(\omega t+\varphi+\theta) & t>0 \end{cases} \end{cases} \tag{21}$$

where φ is the phase angle of voltage at the instant $t=0$. The cases studied are listed in Table I.

TABLE I
SIMULATION ARRANGEMENT

θ	Simulation with $\varphi=0$	Simulation with $\varphi=\pi/4$	Simulation with $\varphi=\pi/2$
$-\pi/2$	Fig. 2	Fig. 3	Fig. 4
$-\pi/3$	Fig. 5	Fig. 6	Fig. 7

In the studied cases, the voltage is unchanging sinusoidal, and the current undergoes a step change. Although the cases are simplified and idealized, it can be considered approximate to the typical function of the reactive power control apparatus in power systems. The reactive power control apparatus such as SVC or STATCOM equipment, may have high current output variation but still has little influence on the voltage owing to the significant short circuit capacity of power systems. The alternative case, where the voltage and current vary simultaneously, is also studied in this section.

A. Step Response of Reactive Power

The single-phase voltage and current and the reactive power calculation that result from various algorithms are shown in Figs. 2-4, where the current lags the voltage by π/2.

$$\begin{cases} u(t)=\sin(\omega t+\varphi) \\ i(t)=\begin{cases} 0.9\sin\left(\omega t+\varphi-\frac{\pi}{2}\right) & t\le 0 \\ \sin\left(\omega t+\varphi-\frac{\pi}{2}\right) & t>0 \end{cases} \end{cases} \tag{22}$$

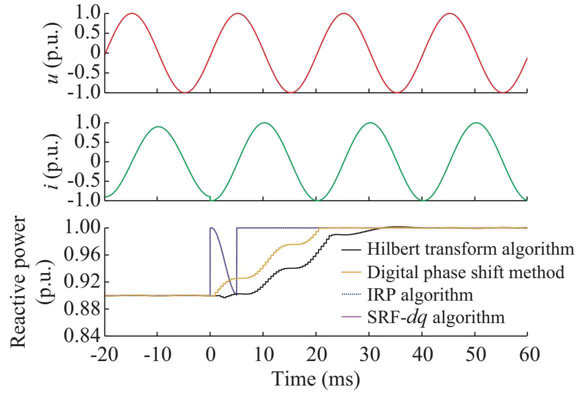

Fig. 2. Step response of different reactive power calculations as current undergoes a step change at $\varphi = 0$.

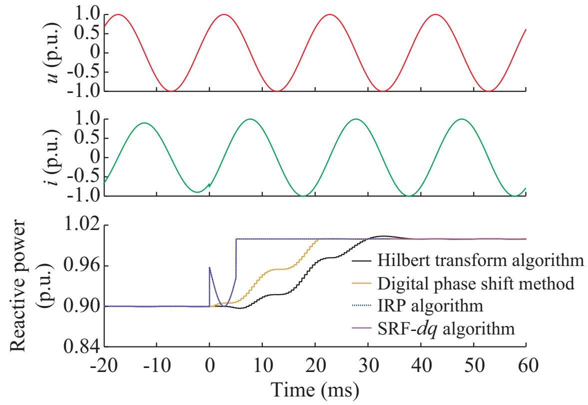

Fig. 3. Step response of different reactive power calculations as current undergoes a step change at $\varphi = \pi/4$.

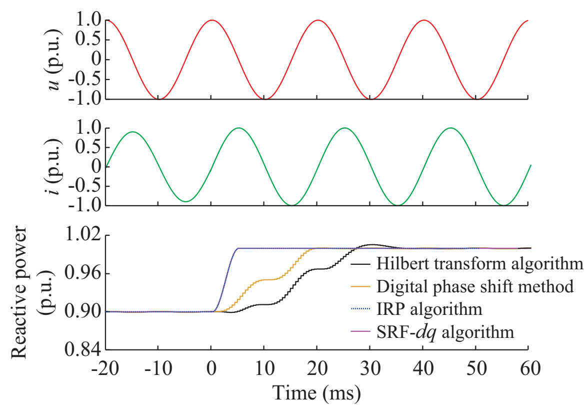

Fig. 4. Step response of different reactive power calculations as current undergoes a step change at $\varphi = \pi/2$.

The calculation results of single-phase voltage, current and the reactive power from the various methods are shown in Figs. 5-7, where the voltage and the current have a phase lag of $\pi/3$.

$$\begin{cases} u(t) = \sin(\omega t + \varphi) \\ i(t) = \begin{cases} 0.9\sin\left(\omega t + \varphi - \dfrac{\pi}{3}\right) & t \le 0 \\ \sin\left(\omega t + \varphi - \dfrac{\pi}{3}\right) & t > 0 \end{cases} \end{cases} \tag{23}$$

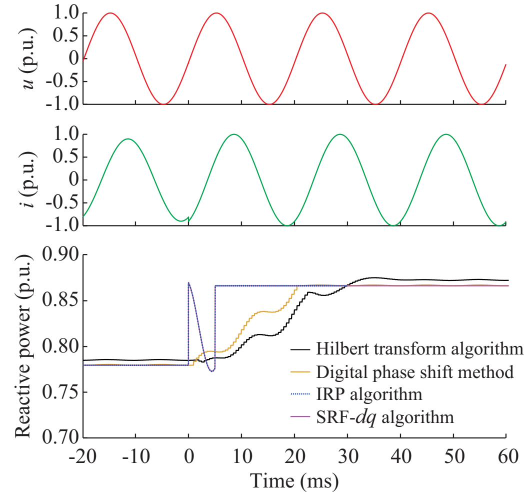

Fig. 5. Step response of different reactive power calculations with $\varphi = 0$.

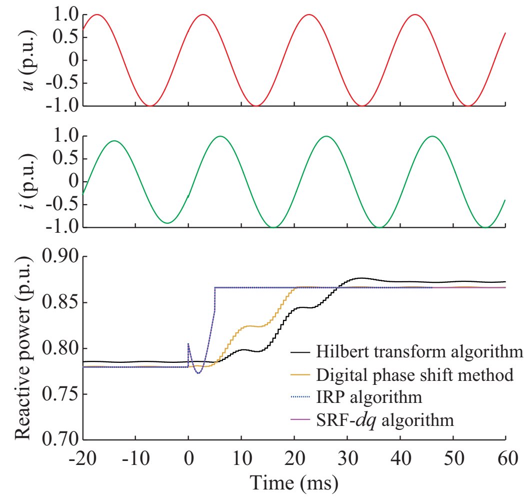

Fig. 6. Step response of different reactive power calculations with $\varphi = \pi/4$.

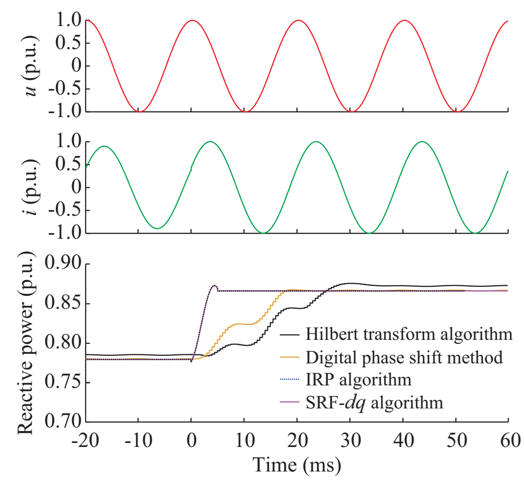

Fig. 7. Step response of different reactive power calculations with $\varphi = \pi/2$.

In general, it can be stated that IRP and SRF-*dq* algorithms have shorter transition processes and a faster step response time of 5 ms, whereas the phase shift method has a response time of 20 ms and the Hilbert transform algorithm has a response time of nearly 30 ms. The response time for diverse reactive power calculation methods can be explained via the filter time delay. For example, using data sampled a quarter of cycle ($N/4$) before, the IRP and SRF-*dq* algorithms have a time delay of nearly 5 ms ($T/4$).

As mentioned above, (21) is mathematical expression for the current step output of an idealized reactive power device. While the current abruptly changes from 0.9 p.u. to 1.0 p.u. at $t=0$, the idealized reactive power device can be regarded as the fastest; i. e., no other reactive power device can respond faster than the idealized one. Alternatively, among these normally used reactive power algorithms, IRP and SRF-*dq* algorithms are the fastest methods for single-phase reactive power calculations, with a response time of $T/4$. Therefore, the conclusion can be reached that the response time of reactive power in a power system is a minimum of $T/4$, using single-phase reactive power algorithms.

If the voltage and the current change simultaneously, the conclusion remains valid, as expressed by (24) and shown in Fig. 8.

$$\begin{cases} u(t)=\begin{cases}\sin(\omega t+\varphi) & t\le 0\\ 0.9\sin(\omega t+\varphi) & t>0\end{cases} \\ i(t)=\begin{cases}0.5\sin\left(\omega t+\varphi-\dfrac{\pi}{2}\right) & t\le 0\\ \sin\left(\omega t+\varphi-\dfrac{\pi}{2}\right) & t>0\end{cases}\end{cases} \tag{24}$$

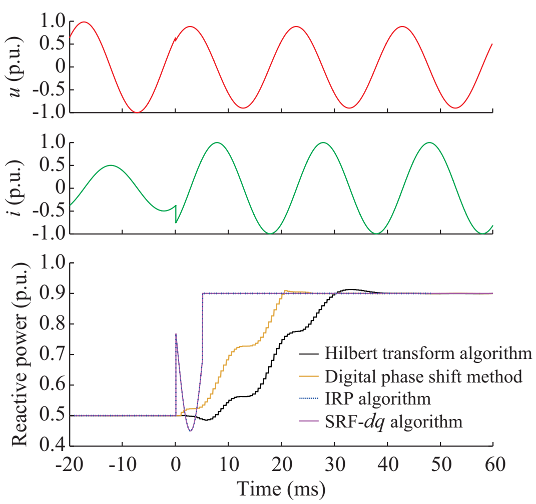

Fig. 8. Step response of different reactive power calculations if voltage and current change simultaneously while $\varphi=\pi/4$.

B. Difference Between Single- and Three-phase IRP Algorithms

According to the above analysis, the step response time of reactive power of the single phase ranges from 5 to 30 ms, with different methods applied to a 50 Hz system. In particular, the response time of the IRP and SRF-*dq* algorithms is 5 ms. Thus, the IRP and SRF-*dq* algorithms are applied in most reactive control devices for fast performance.

However, in three-phase power systems, the response time of the three-phase IRP algorithm using (7)-(9) would be different. An example is shown in (25) and (26).

$$\begin{cases} u_a(t)=\sin(\omega t+\varphi)\\ u_b(t)=\sin\left(\omega t+\dfrac{2\pi}{3}+\varphi\right)\\ u_c(t)=\sin\left(\omega t-\dfrac{2\pi}{3}+\varphi\right)\end{cases} \tag{25}$$

$$\begin{cases} i_a(t)=\begin{cases}0.9\sin\left(\omega t+\varphi-\dfrac{\pi}{2}\right) & t\le 0\\ \sin\left(\omega t+\varphi-\dfrac{\pi}{2}\right) & t>0\end{cases}\\ i_b(t)=\begin{cases}0.9\sin\left(\omega t+\dfrac{2\pi}{3}+\varphi-\dfrac{\pi}{2}\right) & t\le 0\\ \sin\left(\omega t+\dfrac{2\pi}{3}+\varphi-\dfrac{\pi}{2}\right) & t>0\end{cases}\\ i_c(t)=\begin{cases}0.9\sin\left(\omega t-\dfrac{2\pi}{3}+\varphi-\dfrac{\pi}{2}\right) & t\le 0\\ \sin\left(\omega t-\dfrac{2\pi}{3}+\varphi-\dfrac{\pi}{2}\right) & t>0\end{cases}\end{cases} \tag{26}$$

Note that the current of phase a has the same expression as in (22), the currents of the three phases change simultaneously and symmetrically, and the response time of three-phase IRP is approximately zero.

It is noted that the step response time using single-phase IRP indicated in Fig. 2 is different to that using the three-phase IRP indicated in Fig. 9, despite the identical current and voltage calculations for each phase. This is because the single-phase IRP uses historical data for $T/4$ but three-phase IRP uses newly sampled data.

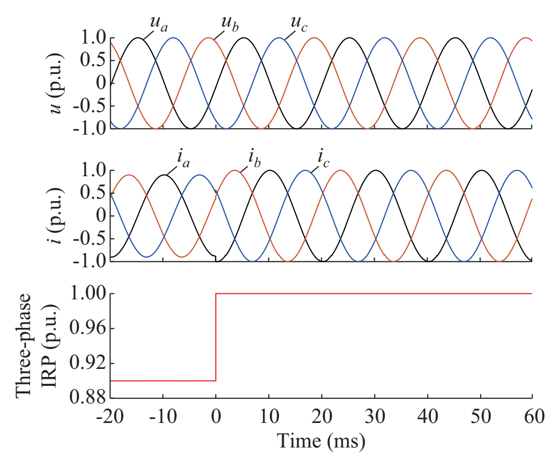

Fig. 9. Step response of three-phase IRP calculation while $\varphi=0$.

A further difference between these two reactive power algorithms occurs under asymmetric current. For example, the current of phase a undergoes a step change, whereas the currents of phases b and c remain constant as follows.

$$\begin{cases} u_a(t)=\sin(\omega t+\varphi)\\ u_b(t)=\sin\left(\omega t+\dfrac{2\pi}{3}+\varphi\right)\\ u_c(t)=\sin\left(\omega t-\dfrac{2\pi}{3}+\varphi\right)\end{cases} \tag{27}$$

$$\begin{cases} i_a(t)=\begin{cases} 0.9\sin\left(\omega t+\varphi-\dfrac{\pi}{2}\right) & t\le 0 \\ \sin\left(\omega t+\varphi-\dfrac{\pi}{2}\right) & t>0 \end{cases} \\ i_b(t)=0.9\sin\left(\omega t+\dfrac{2\pi}{3}+\varphi-\dfrac{\pi}{2}\right) \\ i_c(t)=0.9\sin\left(\omega t-\dfrac{2\pi}{3}+\varphi-\dfrac{\pi}{2}\right) \end{cases} \tag{28}$$

As shown in Fig. 10, the single-phase IRP outputs three unchanging reactive power for each phase, whereas the three-phase IRP outputs an oscillating reactive power for all the three phases. However, the sum of three single-phase reactive power outputs calculated by the single-phase IRP is not equal to that of three-phase reactive power calculated by the three-phase IRP.

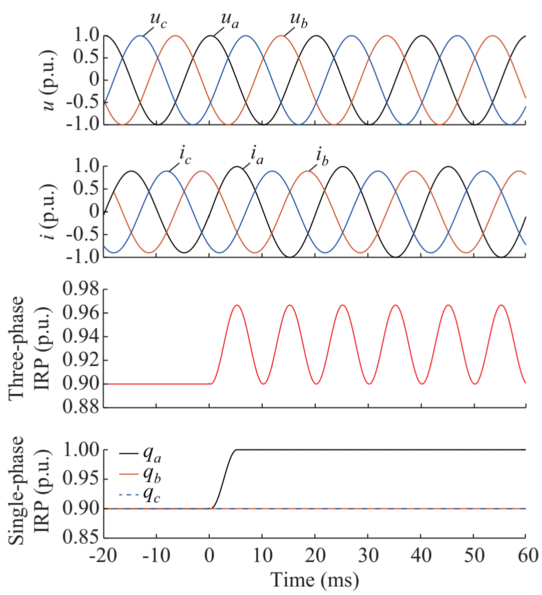

Fig. 10. Step response of IRP of three-phase calculation while $\varphi=\pi/2$.

In fact, the single- and three-phase IRPs are two completely different algorithms [10]. The voltages or currents are often asymmetrical in transient processes in power system. Hence, reactive power devices need to output reactive power according to each phase separately. Thus, single-phase IRP is required. The single-phase reactive power can even be used in symmetrical cases to analyze and solve the problems. Therefore, the single-phase reactive power algorithm is more applicable for reactive power control and the response time of single-phase reactive power algorithms is meaningful and practical under transient conditions.

IV. Response Time for Reactive Power Control

The main function of reactive power compensation systems is to provide fast reactive power compensation, stablize system voltages, and damp power oscillation. Different systems have different response time requirements regarding reactive power and voltage control for reactive power compensation equipment. In a strong system, the response time requirement is relatively slow, but in a weak system, the response time of the reactive power compensation device is required to be fast enough to support the system voltage.

Taking the 500 kV 3000 MW Yongan-Funing LCC-HVDC transmission project of China as an example, the short-circuit ratio of the receiving Funing terminal system is approximately 3. The terminal system has the problem of a shortage of dynamic reactive power, and it is possible that commutation failure may occur after the recovery of an AC system fault, which affects the safe and stable operation of the HVDC transmission system. Therefore, three sets of 100 Mvar STATCOMs have been installed at Funing convert station to provide dynamic reactive power support. Figure 11 shows the single-line diagram of the system. When the fault of the 500 kV AC system is cleared, the STATCOM outputs reactive power to control the AC bus voltage, reducing the probability of commutation failure during the recovery of DC power. The requirement for the reactive power response time of the project is no more than 20 ms.

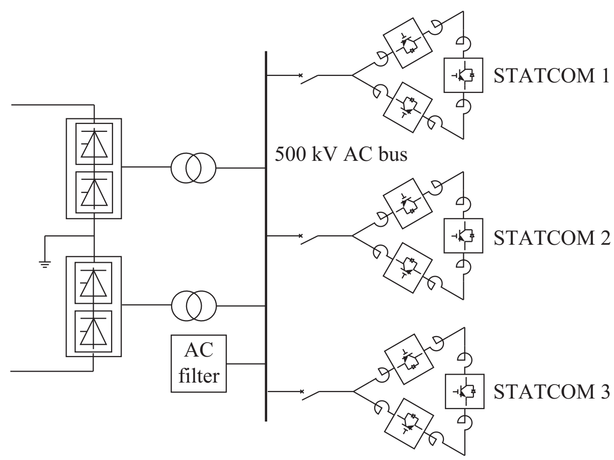

Fig. 11. Single-line diagram.

A significant number of tests on the performance of STATCOM devices have been conducted in the Funing project. The step test for reactive response time is typically used to determine the reactive power requirement from 0 to 100 Mvar. The result is shown in Fig. 12.

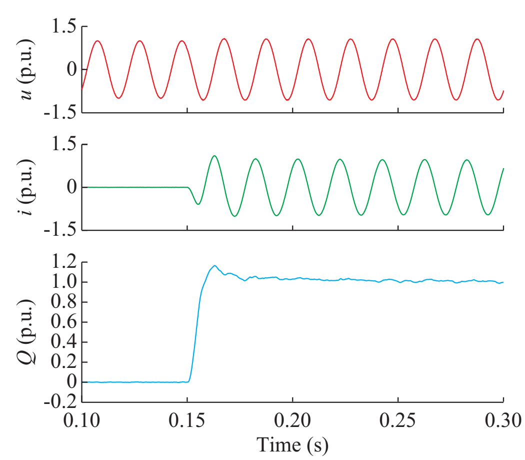

Fig. 12. Test result of reactive power step from 0 to 100 Mvar.

The response time of reactive power is about 6 ms, satisfying the specification requirements, and the reactive power calculation algorithm used for STATCOMs in this project is the single-phase IRP.

Furthermore, this section will demonstrate the influence of the reactive power algorithm itself on the response time of a dynamic reactive power control equipment.

The typical control diagram of STATCOM reactive power control equipment can be described with equivalent transfer functions, while the reactive power algorithm is removed for research.

In Fig. 13, $K_p + K_I/s$ is the expression of proportional-integral (PI) regulation characteristics of reactive power devices such as STATCOM; K is the general gain for the control loop 1; $1/(R + sL_f)$ is the transfer function of the controlled object, with an interface inductance L_f and series resistance R. In the studied cases of this paper, $K = 1$; $R = 0$ p.u.; $L_f =$ 0.12 p.u.; θ_{ref} is the referee variable of reactive power; and q_{cal} is the calculated reactive power. PWM is the pulse width modulation.

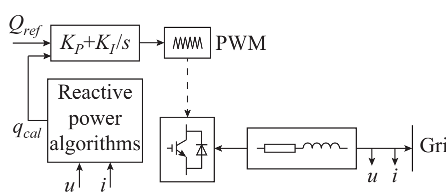

Fig. 13. Control block of reactive power control equipment.

In this section, two cases with relatively fast and slow response characteristics of an equivalent transfer function are studied. The instantaneous current i and calculated reactive power q_{cal} are recorded for the transient processes.

As shown in Fig. 14, for case 1, the step response time of q_{cal} is about 7.3 ms using the IRP and SRF-*dq* algorithms, 20 ms using the phase shift method, and 28 ms using the Hilbert transformation algorithm, respectively. The overshoots of q_{cal} are acceptable for all the algorithms.

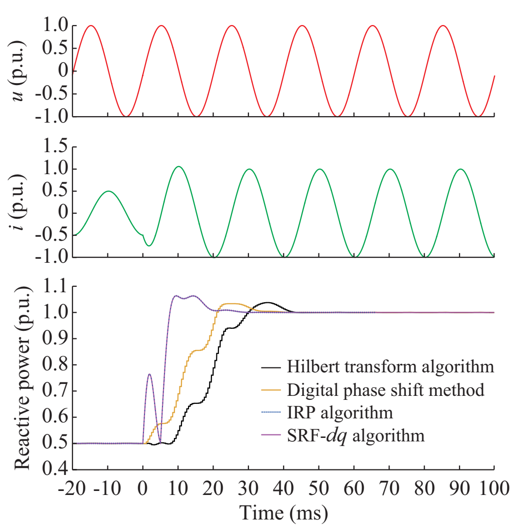

Fig. 14. Step response of reactive power control equipment for case 1 ($K_p = 2.243$, $K_I = 336$).

As shown in Fig. 15, for case 2, the step response time of q_{cal} is about 9.2 ms using the IRP and SRF-*dq* algorithms, 20 ms using the phase shift methods, and 29 ms using the Hilbert transformation algorithm, respectively. In this case, the overshoots of q_{cal} are also acceptable.

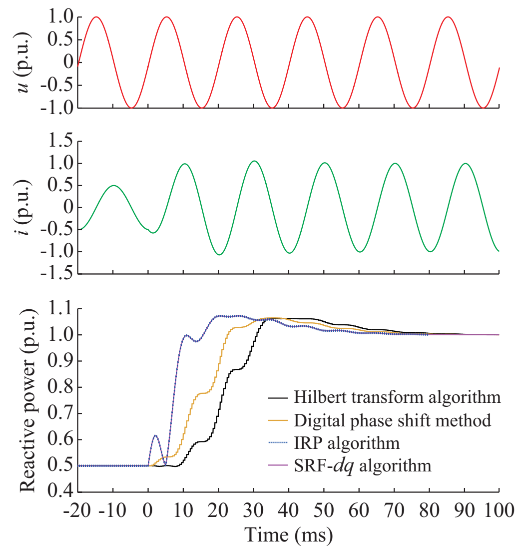

Fig. 15. Step response of reactive power control equipment for case 2 ($K_p = 0.776$, $K_I = 38.84$).

From the above cases, it is verified that IRP and SRF-*dq* algorithms are the fastest of the four algorithms, and that the step response of the reactive power calculation using IRP and SRF-*dq* algorithms is more than 5 ms due to their inner time delay, which is discussed in Section III. It can be further seen that if the control characteristic is very fast, as in case 1, the response time of reactive power control equipment is mainly decided by the reactive power algorithm itself. If the control characteristic is relatively slow, as in case 2, the difference in the response time from various reactive power algorithms is slightly shortened.

In general, for fast dynamic reactive power compensation applications, the reactive power algorithm itself has a significant influence on reactive power regulation. This implies that when evaluating reactive power response characteristics, for example, in Fig. A1 in Appendix A, the reactive power calculation method should be stated. Considering both the response speed and overshoot control, the IRP and SRF-*dq* methods are suggested for dynamic reactive power compensation applications.

V. Conclusion

This paper summarized the definitions of reactive power and the corresponding calculation methods. The reactive power response time, which is very important for dynamic reactive power compensation, is analyzed. The following conclusions are drawn.

1) Reactive power response time is normally used without an accurate definition, and its value is highly dependent on the reactive power calculation method. Among the frequently used single-phase reactive power calculation methods, the IRP and SRF-*dq* algorithms have the fastest response time.

The phase shift method and Hilbert transform algorithm are relatively slower. The response time of the IRP and SRF-*dq* algorithms is *T*/4.

2) The single-phase IRP method is different to the three-phase IRP algorithm in reactive power calculation. The response time of single-phase IRP is meaningful and practical under transient conditions.

3) Among the most well-known and widely used methods for calculating single-phase reactive power, the IRP and SRF-*dq* algorithms are currently the fastest. If a step change to the current is regarded as the fastest ideal control output, the response time of an ideal control device using the IRP and SRF-*dq* algorithms is a minimum of *T*/4.

In conclusion, when determining the reactive power response time of dynamic reactive power compensation equipment, it is necessary to identify the reactive power calculation method. It is meaningless to specify the reactive power response time without the specific calculation method.

APPENDIX A

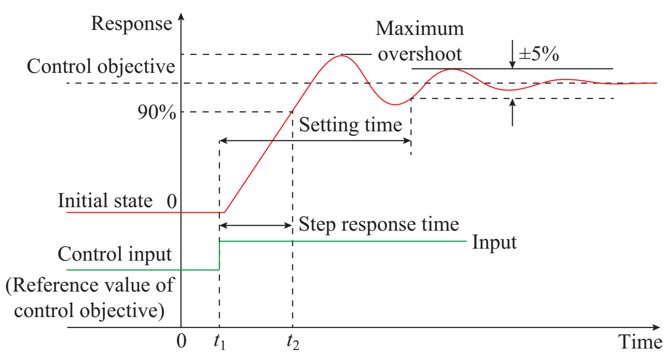

Fig. A1. Regulation characteristics of reactive power compensation equipment.

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