# Fully Distributed State Estimation for Power System with Information Propagation Algorithm

Qiao Li, Lin Cheng, Wei Gao, and David Wenzhong Gao

*Abstract*—**In this paper, a new fully distributed state estimation (DSE) based on weighted least square (WLS) method and graph theory is proposed for power system. The proposed method is fully distributed so that the centralized facilities, e.g., supervisory control and data acquisition (SCADA) and centralized estimators, are not required. Also, different from the existing DSE methods, the proposed method is a bus-level DSE method, in which the power system is not required to be partitioned into several areas. In order to realize the proposed fully distributed DSE method, a novel information propagation algorithm is developed in this paper. This algorithm has great potential in future applications since it is useful to broadcast the local information of the nodes to the entire system in a fully distributed network. The proposed DSE method is compared with the conventional centralized state estimation method and existing multi-area DSE method in different models in this paper. The results show that the proposed method has better performance than the traditional methods.**

*Index Terms*—**Consensus protocol, state estimation, smart grid communication, graph theory, distributed network.**

## I. INTRODUCTION

STATE estimation is an important technique for power system operation. It is usually performed by the energy management system (EMS) [1]-[3] in control centers to obtain the states of the power system. The major objectives of state estimation are monitoring the states of power systems, improving measurement accuracy, and detecting bad measurements. Currently, some state estimation methods have been applied in power systems such as weighted least square (WLS) method [4]-[7] and Kalman filter [8]-[10]. These methods have good performance in conventional centralized power systems. However, due to the development of renewable energy, microgrid, and smart grid, the number of distributed generators (DGs) and sensors in power systems grows dramatically, thus the future power systems tend to be more and more distributed. As a result, in these distributed power systems, the conventional centralized methods may not be efficient enough to work with the distributed devices. Also, distributed methods are usually more robust than centralized ones. Therefore, distributed state estimation (DSE) method is needed for future power systems.

Some efforts have been made in recent years to develop the DSE method. The most commonly used method is the distributed multi-area state estimation as in [11]-[16]. In this type of methods, the entire power system is partitioned into several small areas. Each area has a local state estimator to estimate the states of its own buses, and communicates with the nearby estimators to ensure the observability of the estimation. The state estimation in this method is realized by decomposing the centralized state estimation problem into several smaller problems. However, since there are some requirements for the area partitioning such as the size of the areas, the way to divide the system and the overlapping between the areas, this method may not be compatible with many power systems. Also, this method is not fully distributed since it has centralized structure (all sensors in an area are connected to the local estimator) inside each area and the state estimation algorithm is partially centralized (all data in an area are assembled at the estimator for calculation). In addition, other methods are also proposed in [17], [18] to address the DSE problem. However, the centralized facilities such as global positioning system (GPS) and supervisory control and data acquisition (SCADA) are required in these methods. Therefore, these methods are not totally distributed as well.

The latest works on the DSE are [19]-[21]. Reference [19] proposes a fully distributed robust bilinear state-estimation (D-RBSE) method for multi-area power systems. The method is an extension of bi-linear state estimation [22] into multi-area power systems. In this method, the state estimation problem in each area is solved locally and a small amount of data is transmitted between different areas to achieve the globally accurate estimation. In [20], a three-stage DSE method for AC-DC hybrid distribution network is proposed. In the first stage, the alternating direction multiplier method (ADMM) is used to perform the preliminary state estimation over the distributed multi-area system. The second stage includes a centralized nonlinear conversion system which calculates the intermediate variables from the results of the first stage considering the nonlinearity of the system. The third stage includes a centralized state estimation system

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which computes the final estimated states using the intermediate variables from the second stage. Finally, a DSE method based on parallelized stream computing on the real-time cloud platform is proposed in [21]. In the computation, the interconnected power grids with tie lines are decoupled into sub-regions to realize the state estimation. In sum, the latest works on DSE are based on the multi-area method, in which they are still not totally decentralized.

According to the above discussion, the existing DSE methods are not totally distributed. This drawback makes the distributed power system less robust since the centralized structure still exists in the system. Also, due to the same reason, these methods are not compatible with some other distributed techniques in power systems, e. g. distributed economic dispatch methods [23], so that the development of more advanced distributed techniques in power systems is limited.

To overcome this drawback of the current DSE methods, a true DSE method without any centralized structure or facility, e.g. SCADA, local control center or GPS, is proposed in this paper so that the state estimation can be realized with only the distributed smart meters. Also, a novel information propagation algorithm is proposed as the basis of the proposed DSE method. The information propagation algorithm can help the smart meters broadcast their local data to the entire system with a distributed communication network. In addition, the distributed WLS method for normal sensor network [24], DC power flow model and AC power flow model [25] are adopted to develop the DSE method.

The major contributions of this paper are summarized as follows.

1) The fully distributed, meter-level DC and AC state estimation methods for power systems are developed without the need for centralized structures or facilities. Unlike the existing DSE method, the system is not required to be partitioned into small areas.

2) A novel information propagation algorithm is proposed to broadcast the local data of each node to others in a distributed system.

3) The proposed method has been tested in different power system simulation models. The results show that the estimation of the proposed distributed method is more accurate than the conventional centralized method.

This paper is organized as follows. In Section II, the obstacle to apply the WLS-based DSE for sensor network [24] in power systems is explained at first. The information propagation algorithm is introduced as a useful tool to make the system observable. Based on the proposed information propagation method, the proposed DSE is developed. In Section III, the proposed algorithms are tested in several case studies in MATLAB/Simulink software. Finally, Section IV concludes this paper.

## II. Proposed Approach

### A. Problem Statement

This paper will focus on the DC state estimation first. Based on the DC power flow model, the measurement process in the power system can be modeled as:

$$z = \boldsymbol{H}\boldsymbol{\theta} + \boldsymbol{\eta} \tag{1}$$

where $z$ is the measurement vector of readings on all meters; $\boldsymbol{H}$ is the observation matrix that represents the characteristic of the power system; $\boldsymbol{\theta}$ is the state of the power system such as the phase angle and voltage; and $\boldsymbol{\eta}$ is the noise or error in the measurement.

For example, the power system shown in Fig. 1 has four buses, in which bus 4 is set as the reference. There are four meters, i.e., $M_{12}$, $M_{24}$, $M_{31}$ and $M_{43}$, measuring the real power on the transmission lines. The meters are connected by the communication lines as shown by the dashed lines in Fig. 1.

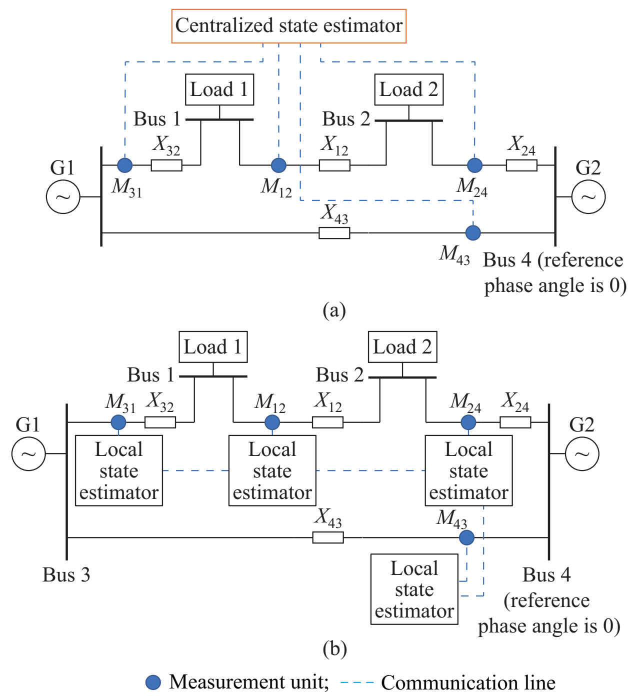


Fig. 1. An example of state estimation for power system with four buses. (a) Centralized state estimation. (b) Proposed DSE.

According to the DC power flow model and measurement model (1), the system can be modeled as:

$$\begin{bmatrix} M_{12} \\ M_{31} \\ M_{24} \\ M_{43} \end{bmatrix} = \begin{bmatrix} \frac{1}{X_{12}} & -\frac{1}{X_{12}} & 0 \\ -\frac{1}{X_{31}} & 0 & \frac{1}{X_{31}} \\ 0 & \frac{1}{X_{24}} & 0 \\ 0 & 0 & -\frac{1}{X_{43}} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + \boldsymbol{\eta} \tag{2}$$

where $X_{ij}$ is the reactance of the line between the bus $i$ and bus $j$; and $\theta_i$ is the phase angle of the bus $i$.

In this system, the measurement is the measured real power on the transmission lines. The states to be estimated are the phase angles on the buses except for the slack bus. The observation matrix $\boldsymbol{H}$ is created according to the transmission line parameters.

Typically, a centralized state estimation method as shown in Fig. 1(a) using WLS method [25] is adopted to estimate the states in power system. In this method, the readings from all meters are collected by the centralized state estimator. Then, the centralized state estimator performs the WLS to estimate the states with the noisy measurement $\boldsymbol{z}$ by:

$$\hat{\boldsymbol{\theta}}=\left(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{z} \tag{3}$$

where $\hat{\boldsymbol{\theta}}$ is the estimation of the state $\boldsymbol{\theta}$; and $\boldsymbol{R}$ is the covariance matrix of measurement errors, which are dependent on the accuracy of meters.

In a power system using DSE as shown in Fig. 1(b), the DSE is performed in the nodes of the system. A node is the combination of a meter and a local state estimator as shown in Fig. 1(b). The local state estimator performs the proposed information propagation and the proposed state estimation from this paper. Also, each local state estimator connects to its neighbors by the communication lines (the dash lines in the figure). In this system, since there is no centralized state estimator, the WLS method and other centralized state estimation methods cannot be directly applied. For example, the node $M_{31}$ is only connected to the node $M_{12}$, so the measured values $M_{24}$ and $M_{43}$ are unknown for $M_{31}$. Then, in order to perform centralized state estimation, the measurement model for the local state estimator in node $M_{31}$ is built as:

$$\begin{bmatrix}M_{12}\\M_{31}\end{bmatrix}=\begin{bmatrix}\dfrac{1}{X_{12}} & -\dfrac{1}{X_{12}} & 0\\ -\dfrac{1}{X_{31}} & 0 & \dfrac{1}{X_{31}}\end{bmatrix}\begin{bmatrix}\theta_1\\\theta_2\\\theta_3\end{bmatrix}+\boldsymbol{\eta} \tag{4}$$

It is easy to verify that the term $\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}$ from (3) is singular with the $\boldsymbol{H}$ matrix from (4), thus (3) cannot be solved accurately. This is to say that the system is unobservable for the node $M_{31}$ [25] and the state estimation cannot simply proceed in a distributed fashion. To solve this problem, an information propagation algorithm is proposed to broadcast the local measurement data (local information) of each node to the entire system, so that the system will become observable for all nodes.

### *B. Information Propagation Algorithm*

The function of the proposed information propagation algorithm is to broadcast the local information, i.e., the measurements from the meter in power system, of each node to the entire system without a centralized communication coordinator.

The information propagation algorithm is based on the consensus protocol technique [24]. In this method, the communication network of a power system can be modeled as a graph, in which the meters with local state estimators, e.g., $M_{32}$, $M_{12}$, $M_{24}$, and $M_{43}$ in Fig. 1(b), are treated as nodes, while the communication lines between the nodes are the edges of the graph. Suppose that $\boldsymbol{x}_i$ is the information state of the $i^{\text{th}}$ node in the graph. The vector $\boldsymbol{X}=[\boldsymbol{x}_1,\boldsymbol{x}_2,...,\boldsymbol{x}_n]$ includes the information states of all $n$ nodes. Then, the consensus protocol for the $i^{\text{th}}$ node is:

$$\boldsymbol{x}_i(k+1)=\boldsymbol{x}_i(k)+\tau\sum_{j\in\mathcal{N}_i}w_{ij}\left(\boldsymbol{x}_j(k)-\boldsymbol{x}_i(k)\right) \tag{5}$$

where $\boldsymbol{x}_i(k)$ is the information state $\boldsymbol{x}_i$ at the time step $k$; $\tau$ is the time interval between two consecutive update time steps; $w_{ij}$ is a weight coefficient on the edge between node $i$ and node $j$; and $\mathcal{N}_i$ is the set of neighbor nodes of the node $j$. For the entire graph, the information states of all nodes converge to their average value $\sum\boldsymbol{x}_i/n$ if the weight coefficients $w_{ij}$ are chosen based on certain rules in [24].

Now, suppose that each node in the graph has its local information, e.g., $z_i$ is the local information of the node $i$. The goal of the information propagation algorithm is to allow each node in the graph accurately estimate the local information of all nodes, i.e., the vector of all local information $[z_1,z_2,...,z_n]$ can be estimated by each node with the information propagation algorithm.

In order to achieve this goal, the information state $\boldsymbol{x}_i$ in (5) is considered as the estimated local information vector $\boldsymbol{x}_i=[\hat{z}_{i1},\hat{z}_{i2},...,\hat{z}_{in}]$, where $\hat{z}_{ij}$ is the estimation of the $j^{\text{th}}$ node's local information by node $i$. Then, if the consensus protocol (5) can be modified so that the information state $\boldsymbol{x}_i$ does not converge to the average value but the local information vector, i.e., $\lim\limits_{k\to\infty}\boldsymbol{x}_i(k)=[z_1,z_2,...,z_n]$, the information propagation is achieved. Based on this idea, the following modified consensus protocol serves as the proposed information propagation algorithm:

$$\boldsymbol{x}_i(k+1)=\boldsymbol{x}_i(k)+\tau\boldsymbol{I}_i^0\sum_{j\in\mathcal{N}_i}w_{ij}\left(\boldsymbol{x}_j(k)-\boldsymbol{x}_i(k)\right) \tag{6}$$

The above equation can be re-written in vector form as:

$$\begin{bmatrix}\hat{z}_{i1}(k+1)\\\hat{z}_{i2}(k+1)\\\vdots\\\hat{z}_{in}(k+1)\end{bmatrix}=\begin{bmatrix}\hat{z}_{i1}(k)\\\hat{z}_{i2}(k)\\\vdots\\\hat{z}_{in}(k)\end{bmatrix}+\tau\boldsymbol{I}_i^0\sum_{j\in\mathcal{N}_i}w_{ij}\left(\begin{bmatrix}\hat{z}_{j1}(k)\\\hat{z}_{j2}(k)\\\vdots\\\hat{z}_{jn}(k)\end{bmatrix}-\begin{bmatrix}\hat{z}_{i1}(k)\\\hat{z}_{i2}(k)\\\vdots\\\hat{z}_{in}(k)\end{bmatrix}\right) \tag{7}$$

where the initial values $\hat{z}_{ij}(0)$ can be arbitrary if $i\neq j$, but $\hat{z}_{ii}(0)=z_i$ since the local information $z_i$ is available at the $i^{\text{th}}$ node itself; and $\boldsymbol{I}_i^0$ is an $n\times n$ diagonal matrix which is the same as the identity matrix but has a zero at the $i^{\text{th}}$ diagonal entry:

$$\boldsymbol{I}_i^0\triangleq\mathrm{diag}(1,1,...,1,0,1,...,1) \tag{8}$$

For the node $i$, the $i^{\text{th}}$ row of (7) is:

$$\begin{aligned}\hat{z}_{ii}(k+1)=\hat{z}_{ii}(k)+\tau\times0\times\sum_{j\in\mathcal{N}_i}w_{ij}\left(\hat{z}_j(k)-\hat{z}_i(k)\right)=\\ \hat{z}_{ii}(k)+0=\hat{z}_{ii}(k)=\hat{z}_{ii}(0)=z_i\end{aligned} \tag{9}$$

Since the $i^{\text{th}}$ row of the matrix $\boldsymbol{I}_i^0$ is 0, the second term in (9) is always 0. Thus, the $i^{\text{th}}$ row of the estimated local information vector remains unchanged for all the times, i.e., $\hat{z}_{ii}(k)=\hat{z}_{ii}(0)=z_i$ for $k=0,1,...,\infty$.

For the $j^{\text{th}}$ row of (9) in which $j\neq i$, since $\hat{z}_{jj}(k)$ on the $j^{\text{th}}$ node is fixed at $z_j$ as discussed above, the consensus protocol works in the leader-follower mode [26]. In this mode, the node $j$ is the leader and other nodes including node $i$ are the followers. Thus, the $j^{\text{th}}$ row $\hat{z}_{ij}(k)$ of the information state $\boldsymbol{x}_i$ on the node $i$ will converge to $z_j$, i.e., $\lim\limits_{k\to\infty}\hat{z}_{ij}(k)=z_j$.

According to the discussion above, the $i^{\text{th}}$ row of the information state $\boldsymbol{x}_i$ is fixed at the local information $z_i$ of the node $i$, and the other rows $\hat{z}_j(k)$ converge to the local information $z_j$ of other nodes. Therefore, the goal $\lim\limits_{k\to\infty}\boldsymbol{x}_i(k)=[z_1,z_2,...,z_n]$ is achieved.

### C. Proposed DSE

With the information propagation algorithm, the DSE for power systems can be established. By implementing the information propagation algorithm in power systems, the local information $z_i$ is the measurement from the meter of the $i^{\text{th}}$ node. The notation $\boldsymbol{Z}_i$ is used to represent the estimated local information vector $\boldsymbol{Z}_i = \boldsymbol{x}_i = [\hat{z}_{i1}, \hat{z}_{i2}, ..., \hat{z}_{in}]$ of node $i$. Equation (6) becomes:

$$\boldsymbol{Z}_i(k+1) = \boldsymbol{Z}_i(k) + \tau \boldsymbol{I}_i^0 \sum_{j \in \mathcal{N}_i} w_{ij} \left( \boldsymbol{Z}_j(k) - \boldsymbol{Z}_i(k) \right) \tag{10}$$

By integrating the WLS method with the information propagation algorithm, the new DSE algorithm at node $i$ is described as:

$$\begin{cases} Z'_{ii}(k) = z_i(k) \\ Z'_{ij}(k) = Z_{ij}(k) \quad j \neq i \\ \boldsymbol{Z}_i(k+1) = \boldsymbol{Z}'_i(k) + \tau \boldsymbol{I}_i^0 \sum\limits_{j \in \mathcal{N}_i} w_{ij}^Z \left( \boldsymbol{Z}'_j(k) - \boldsymbol{Z}'_i(k) \right) \\ \hat{\boldsymbol{\theta}}_i(k+1) = \left( \boldsymbol{H}^{\mathrm{T}} \boldsymbol{R}^{-1} \boldsymbol{H} \right)^{-1} \boldsymbol{H}^{\mathrm{T}} \boldsymbol{R}^{-1} \boldsymbol{Z}_i(k+1) \end{cases} \tag{11}$$

where $\boldsymbol{Z}'_i(k)$ is a modification of $\boldsymbol{Z}_i(k)$ that the $i^{\text{th}}$ entry is replaced by the actual measurement $z_i(k)$; $w_{ij}^Z$ is the a weight coefficient for the communication of information vector on the edge between node $i$ and node $j$; and $\hat{\boldsymbol{\theta}}_i$ is the estimated states, i.e., phase angles, of all buses in the power system calculated by node $i$.

According to the DSE equation (11), the algorithm can work as a low-pass filter which will reduce the impact of the measurement errors. The reason is presented as follows.

Firstly, the consensus protocol (5) can be represented into matrix form [22] as:

$$\boldsymbol{X}(k+1) = \boldsymbol{X}(k) - \tau \boldsymbol{L}_w \boldsymbol{X}(k) \tag{12}$$

where $\boldsymbol{X}(k) = [\boldsymbol{x}_1(k); \boldsymbol{x}_2(k); \boldsymbol{x}_3(k); ...; \boldsymbol{x}_n(k)]$ contains all information states in the system; and $\boldsymbol{L}_w$ is the weighted Laplacian matrix [24]. Similarly, the matrix form of the $i^{\text{th}}$ row of the DSE algorithm (11) can be written as:

$$\boldsymbol{Z}_i(k+1) = \boldsymbol{Z}'_i(k) - \tau \boldsymbol{I}_i^0 \boldsymbol{L}_w \boldsymbol{Z}'_i(k) \tag{13}$$

Let $\boldsymbol{Z}'_i(k) = \boldsymbol{Z}_i(k) + \Delta\boldsymbol{\eta}$, where $\Delta\boldsymbol{\eta} = [0, ..., 0, \mathrm{d}\eta_i, 0, ..., 0]$ and $\mathrm{d}\eta_i$ is the difference of the measurement error on the meter of node $i$ between two time steps. The equation becomes:

$$\begin{aligned} &\boldsymbol{Z}_i(k+1) = \boldsymbol{Z}_i(k) + \Delta\boldsymbol{\eta} - \tau \boldsymbol{I}_i^0 \boldsymbol{L}_w (\boldsymbol{Z}_i(k) + \Delta\boldsymbol{\eta}) \Rightarrow \\ &\frac{\boldsymbol{Z}_i(k+1) - \boldsymbol{Z}_i(k)}{\tau} = \frac{\Delta\boldsymbol{\eta}}{\tau} - \boldsymbol{I}_i^0 \boldsymbol{L}_w \boldsymbol{Z}_i(k) \end{aligned} \tag{14}$$

where $\boldsymbol{I}_i^0 \boldsymbol{L}_w \Delta\boldsymbol{\eta} = \boldsymbol{0}$ since $\Delta\boldsymbol{\eta}$ only has non-zero value at its $i^{\text{th}}$ row and the elements at $i^{\text{th}}$ row of $\boldsymbol{I}_i^0$ are all zeros. Then, (14) is rewritten into continuous form by letting $\tau \rightarrow \infty$, and the DSE algorithm can be represented in the state space form by adding the last sub-equation of (11):

$$\begin{cases} \dot{\boldsymbol{Z}}_i(t) = -\boldsymbol{I}_i^0 \boldsymbol{L}_w \boldsymbol{Z}_i(t) + \dot{\boldsymbol{\eta}} \\ \hat{\boldsymbol{\theta}}_i(t) = \left( \boldsymbol{H}^{\mathrm{T}} \boldsymbol{R}^{-1} \boldsymbol{H} \right)^{-1} \boldsymbol{H}^{\mathrm{T}} \boldsymbol{R}^{-1} \boldsymbol{Z}_i(t) \end{cases} \tag{15}$$

This is a state space model where the input $\boldsymbol{u} = \dot{\boldsymbol{\eta}} = [0, ..., 0, \dot{\eta}_i, 0, ..., 0]$ and $\dot{\eta}_i$ is the derivative of the measurement error; the output is $\boldsymbol{y} = \hat{\boldsymbol{\theta}}_i(t)$; $\boldsymbol{A} = -\boldsymbol{I}_i^0 \boldsymbol{L}_w$; $\boldsymbol{B} = \boldsymbol{I}$ where $\boldsymbol{I}$ is an identity matrix; $\boldsymbol{C} = \left( \boldsymbol{H}^{\mathrm{T}} \boldsymbol{R}^{-1} \boldsymbol{H} \right)^{-1} \boldsymbol{H}^{\mathrm{T}} \boldsymbol{R}^{-1}$; and $\boldsymbol{D} = \boldsymbol{0}$. The transfer function of this model is:

$$\frac{\boldsymbol{Y}(s)}{\boldsymbol{U}(s)} = \frac{\hat{\boldsymbol{\theta}}_i(s)}{\dot{\boldsymbol{\eta}}(s)} = \frac{\left( \boldsymbol{H}^{\mathrm{T}} \boldsymbol{R}^{-1} \boldsymbol{H} \right)^{-1} \boldsymbol{H}^{\mathrm{T}} \boldsymbol{R}^{-1}}{s\boldsymbol{I} + \boldsymbol{I}_i^0 \boldsymbol{L}_w} \tag{16}$$

Since the eigenvalues of $\boldsymbol{L}_w$ are non-negative due to the convergence condition of the consensus protocol, the poles of the system (16) are on the left-half plane. Therefore, the model (16) of the DSE algorithm can be considered as a low-pass filter. This is helpful to mitigate the noise in the measurement.

### D. DSE with AC Power Flow

Based on the proposed information propagation algorithm, an AC DSE can be developed as well. The idea of this AC DSE is to use the information propagation algorithm to share the measured data with other nodes in the system, then the AC state estimation algorithm [25] is performed when the information is well shared. To determine if the information is well shared, the information mismatch on the node $i$ can be calculated as:

$$\delta_i^{info} = \left\| \sum_{j \in \mathcal{N}_i} \left( \boldsymbol{Z}'_j(k) - \boldsymbol{Z}'_i(k) \right) \right\|_\infty \tag{17}$$

where $\|\cdot\|_\infty$ is the infinity norm which gives the maximal value in the vector. When the information mismatch $\delta_i^{info}$ is lower than a threshold value, the AC state estimation will be executed.

Different from the DC DSE discussed above, the AC DSE measures not only the real power on the transmission line, but also the reactive power on the transmission line. Thus, the information propagation algorithm will be applied to share both the measured real power and reactive power. Let $z_i^P$ and $z_i^Q$ be the real power and reactive power measured on the meter of node $i$, respectively. The estimated states are the phase angle and voltage magnitude on the buses in the system. The algorithm of the AC DSE is listed in Algorithm I.

**Algorithm I** AC DSE

Initialize all variables
**while** $\delta_i^{info}$ is larger than the threshold **do**
  Update the measurements $z_i^P$ and $z_i^Q$
  Run (11) to share $z_i^P$
  Run (11) to share $z_i^Q$
  Calculate information mismatch $\delta_i^{info}$ by (17)
**end while**
Perform AC state estimation [23]

## III. Simulation Results

### A. Information Propagation Algorithm in Example Graphs

In order to verify that the proposed information propagation algorithm can share the local value of each vertex to the others, the simulations are conducted in the communication networks with two different graphs shown in Fig. 2, which are the cycle graph and the tree graph, respectively [27], where V1 to V6 are the vertices of the graph.

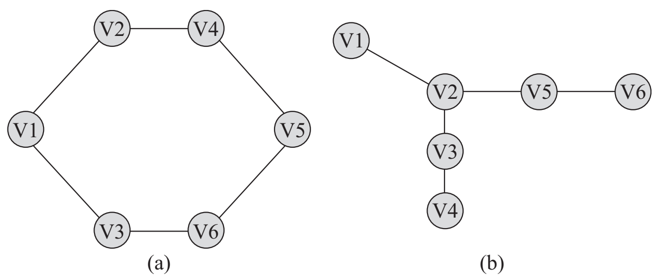


Fig. 2. Two different graphs. (a) Cycle graph. (b) Tree graph.

In the two case studies, the vertices V1, V2, V3, V4, V5 and V6 have initial local values 8, 2, 25, 12, 13 and 3, respectively. Then, at $t=0.5$ s, the local values are turned into 4, 6, 14, 9, 21 and 11. By applying the information propagation algorithm, the results are shown in Figs. 3 and 4.

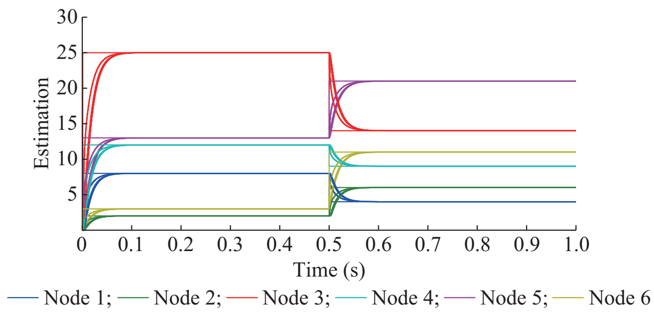


Fig. 3. Information propagation algorithm in cycle graph.

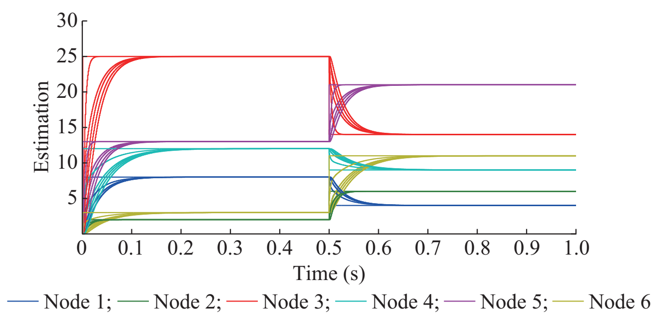


Fig. 4. Information propagation algorithm in tree graph.

The results show that local data can be successfully propagated in the graphs by the information propagation algorithm. According to Figs. 3 and 4, the estimations can converge to the correct values in about 0.15 s. Also, the information propagation algorithm converges faster in the cycle graph than in the tree graph. This is because the Laplacian matrix for a cycle graph usually has larger eigenvalues, which means that the algorithm can converge more quickly. The average value of the eigenvalues of the modified Laplacian matrix of the cycle graph is 1.6667, and it is 1.3889 for the tree graph, where the modified Laplacian matrix is the Laplacian matrix of the information propagation algorithm. Therefore, the larger eigenvalues for cycle graph allow it to converge faster.

*B. State Estimation in Western System Coordinating Council (WSCC) 9-bus Power System*

In this work, the WSCC 3-generator 9-bus system [28] is adopted to test the proposed DSE method. WSCC 9-bus system is an approximated model of the WSCC power system in 60 Hz, as shown in Fig. 5, where the generators G1 and G3 in the power system are hydro plant and thermal plant, respectively.

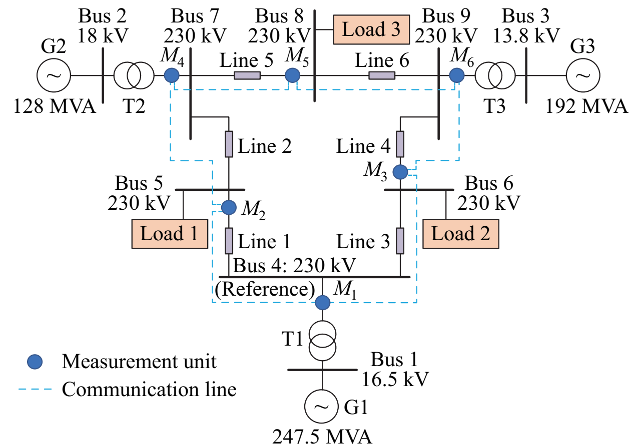


Fig. 5. WSCC 9-bus system.

Considering the rapid development of renewable energy, the generator G2 is modeled as an inverter-based generator. Three transformers T1, T2 and T3 step up the voltages of the generators to 230 kV. The impedances (in p. u.) of the transmission lines are: 0.01+j0.085 for line 1, 0.032+j0.161 for line 2, 0.017+j0.092 for line 3, 0.039+j0.17 for line 4, 0.0085+j0.072 for line 5, and 0.0119+j0.1008 for line 6. There are three loads, load 1 (187.5 MVA), load 2 (153 MVA) and load 3 (170 MVA), on bus 5, bus 6 and bus 8, respectively. In addition, six meters, $M_1$, $M_2$, ..., $M_6$, are installed to measure the real power on the transmission lines in the system. In real systems, the meters can be built by the embedded systems such as advanced reduced instruction set computing (RISC) machine (ARM), field-programmable gate array (FPGA) or digital signal processor (DSP). Each node is connected with its neighbor nodes by the communication lines. The communication lines should be bi-directional with low latency. In practice, TCP/IP protocol based communication methods such as Ethernet network and Wifi can be used to build the communication lines. Also, in the simulation, in order to simulate the delay in communication, a delay component is added on every communication line.

In a 25 s simulation, load 1 is increased by 30 MW at 5 s, while the load 3 is decreased by 20 MW at 15 s. The Gaussian noise with zero mean and standard derivation $\sigma=0.01$ is added on the meters. The proposed DSE algorithm (11) is used to estimate the phase angle on each bus. Figure 6 shows the phase angle estimation results by the distributed algorithm executed at node $M_1$. The result is compared with the centralized state estimation and the actual value. It shows the DSE has less noise than the centralized method. This is because the consensus protocol is a recursive algorithm that has the inherent filtering effect, which reduces the noise.

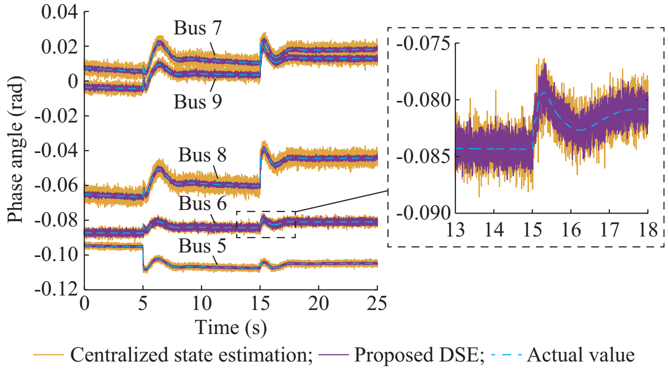


Fig. 6. Comparison among actual value, estimation by centralized method and estimation by proposed DSE method at node $M_1$.

In order to evaluate the performance of the proposed method, the mean squared error (MSE) is adopted to compute the accuracy of the estimation. The MSE is defined by $MSE=\frac{1}{T}\sum_{k=0}^{T}\left(a(k)-\hat{a}(k)\right)^2$, where $a(k)$ is the actual value; $\hat{a}(k)$ is the estimated value; and $T$ is the total time steps. The results are shown in Fig. 7.

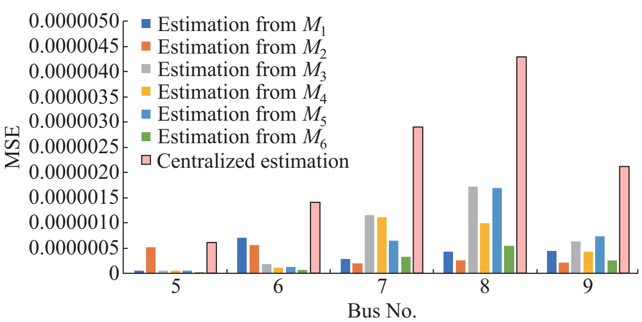


Fig. 7. Comparison of MSE between different methods.

In the bar chart, the bars are categorized into five groups, in which each group represents the results for each bus in the power system. Meanwhile, inside each group, the bars show the estimation errors of different nodes and methods. Since the proposed DSE algorithm can be performed on every node, each node has its own state estimation results. Therefore, the left six bars in each group are the results of the proposed DSE executed on the six different nodes, and the last bar represents the result of the centralized method performed by the control center. According to the results, the error of our proposed method is much smaller than the centralized method. On the other hand, the estimations of some buses have higher errors in some meters, e.g., the estimation of bus 5 by the $M_2$ (the second bar in the first group in Fig. 7) has higher error. This is because the meter $M_2$ can directly measure the data from bus 5 without going through the information propagation algorithm, which means that the measurement noise is directly input into the state estimation algorithm in $M_2$. However, for other meters, the data of bus 5 is transmitted by the information propagation algorithm so that the measurement errors are reduced during the communication due to the low-pass filter characteristic of the information propagation algorithm as discussed above. Therefore, the estimation of the state of bus 5 on meter $M_2$ is less accurate than on the other meters. But this problem can be easily addressed by simply adding a low-pass filter to the meters.

### C. State Estimation in IEEE 39-bus System

The IEEE 39-bus system is a power system model with 10 generators and 39 buses, as shown in Fig. 8. The parameters can be found in [29]. In the system, 37 meters running the proposed DSE are installed on the power lines in Fig. 8, which are connected by the communication network. The communication network in the system has multiple cycles, e.g., the meters $M_1$, $M_2$, $M_{25}$, $M_3$, $M_4$, $M_5$, $M_6$, $M_{32}$, $M_7$, $M_8$, $M_9$, $M_{30}$ form a cycle. The existence of the cycles in communication network makes this DSE system more robust. This is because that there are more redundant paths in the network to link any two meters. For example, if the communication line between the meters $M_1$ and $M_2$ is broken, the information can still be transmitted between $M_1$ and $M_2$ by the path $M_2 \rightarrow M_{25} \rightarrow M_3 \rightarrow \ldots \rightarrow M_{30} \rightarrow M_1$. The results of the simulation are shown in Fig. 9. This figure only shows the MSE of the estimation for the states of all buses in the system by the meter $M_1$ since all other meters have similar results. According to the results, it is clear that the proposed DSE is more accurate than the traditional centralized method. This improvement of the estimation is also caused by the low-pass filter characteristic of the information propagation algorithm.

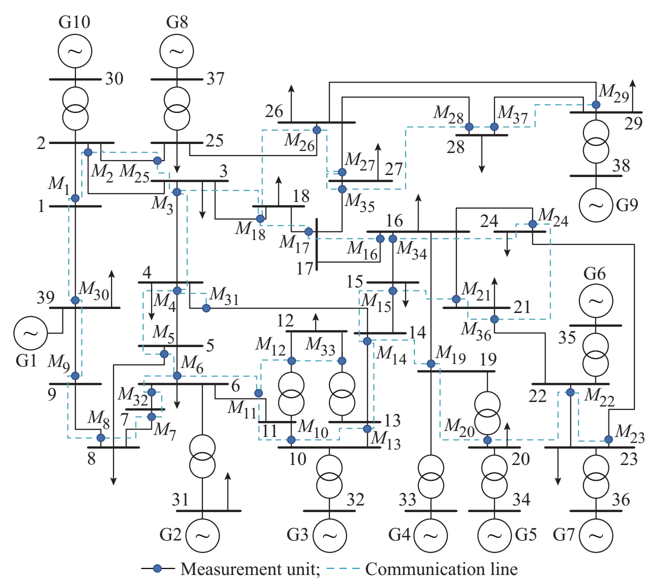


Fig. 8. IEEE 39-bus system.

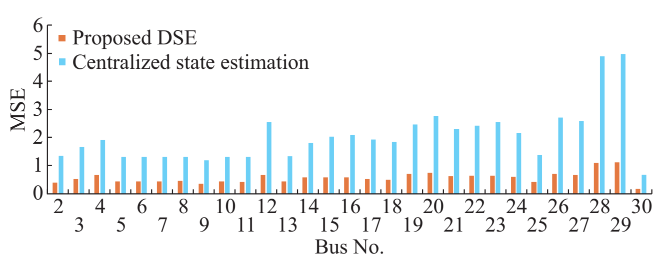


Fig. 9. MSE of different methods for IEEE 39-bus system.

### *D. Comparison of Different Methods in IEEE 14-bus System*

In order to verify the performance of the proposed DSE method, the commonly used multi-area DSE method [11] is adopted for comparison. The comparison is done in the IEEE 14-bus system as shown in Fig. 10. All methods in this comparison use the same meters as shown in the figure. Also, for the multi-area DSE method, the system is partitioned [11] as the figure shows.

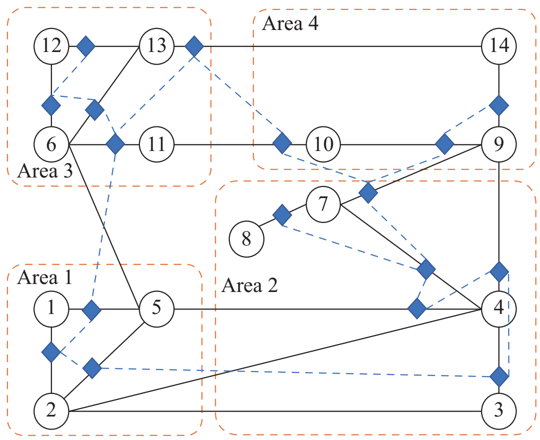


Fig. 10. IEEE 14-bus system.

The results of the simulations are listed in Table I. The results show the errors of the estimated phase angle from each method. The errors are calculated by averaging the errors in 1000 runs, in which the measurement noise follows normal distribution with 0 mean and 0.01 standard deviation. The results show that the proposed DC DSE method has similar accuracy as the multi-area DSE and the centralized state estimation methods. However, like most other DSE methods, the multi-area DSE requires local estimator in each partitioned area and centralized coordinator for the whole system. Therefore, the method is not fully distributed. On the other hand, The AC DSE method in our paper has a much lower error than other methods. This is because the AC power flow model is more accurate than the DC power flow model. In conclusion, the proposed DSE method achieves similar or slightly better performance as the commonly used multi-area DSE, but it is a truly distributed method compared to the existing methods.

TABLE I
ERRORS OF PHASE ANGLE ESTIMATION

| Method | Average error of 1000 tests (p.u.) |
|---|---|
| Centralized state estimation | 0.0163 |
| Proposed DC DSE | 0.0158 |
| Proposed AC DSE | 0.0021 |
| Multi-area DSE | 0.0161 |

### *E. Improving Measured Data*

On the other hand, the estimated states can be used to improve the accuracy of the measurements by the equation $\hat{z}=\boldsymbol{H}\hat{\boldsymbol{\theta}}$, where $\hat{\boldsymbol{\theta}}$ is the estimated phase angle obtained from the proposed DSE method; and $\hat{z}$ is the estimation of the power flow. Using this method, the noise in the raw measurements of the power flow from the nodes can be reduced, thus the measurements are more precise. The case study is done on the IEEE 39-bus system. The estimation results from the meter $M_1$ in IEEE 39-bus system are presented in Fig. 11 as an example.

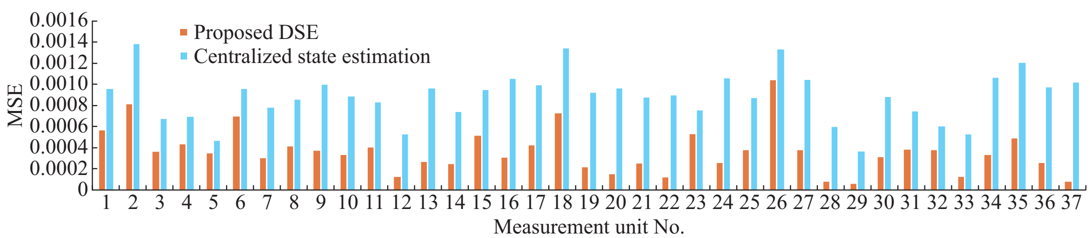


Fig. 11. MSE of power flow estimation for $M_1$ in IEEE 39-bus system.

The results of the IEEE 39-bus system in Fig. 11 compare the MSEs of the power flow estimation between the proposed DSE method and the traditional centralized state estimation method. According to the results, the proposed method has lower MSE for the estimation of each bus, which means that it is much more accurate than the centralized method, which is true for other nodes.

### *F. Bad Measurement Detection*

Bad measurement detection is one of the most important topics in power system estimation [30], [31]. In a large sensor network, the hardware malfunction or cyber-attack may cause the sensors to provide incorrect readings. Therefore, it is very important to identify and eliminate bad measurements. In this case study, the system runs for 8 s and two bad measurements (by increasing the raw measurement reading by 1.0) are injected between 2-3 s and 5-6 s on $M_1$ and $M_{14}$, respectively. Based on the proposed DSE method, the bad measurement can be detected by the $i^{\text{th}}$ meter by calculating the measurement residual $J_i(\hat{\theta}_i)$ [25]:

$$J_i(\hat{\theta}_i)=\sum_{j=1}^{n}\frac{(z_{ij}-\hat{z}_{ij})^2}{\sigma_{ij}^2} \tag{18}$$

where $z_{ij}$ is the $j^{\text{th}}$ raw measurements; $\hat{z}_i=H\hat{\theta}_i$ is the estimation of the measurements; and $\sigma_{ij}$ is the standard deviation of the measurement which is 0.01 in this system. In IEEE 39-bus system, the number of estimated states $\hat{\theta}_i$ is 29 and the number of measurements is 37, hence the degrees of freedom for the state estimation is $37-29=8$. The measurement residual $J_i(\hat{\theta}_i)$ satisfies the $\chi^2$ distribution. Then, according to the probability distribution function (PDF) of $\chi^2$ distribution, by choosing $3\sigma$ confidence, the threshold value of $J_i(\hat{\theta}_i)$ for the bad measurement detection is 23.3.

In the proposed method, bad data detection can be performed by any meter in the system. In Fig. 12, the residuals $J_i(\hat{\theta}_i)$ calculated by all nodes (the lines in different colors) are presented. The results show that the residuals on all nodes exceed the threshold during 2-3 s and 5-6 s. It means that the bad data injection on $M_1$ and $M_{14}$ are detected.

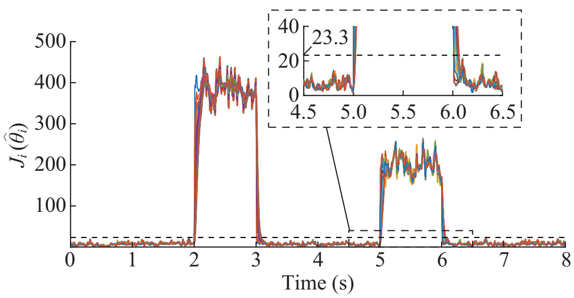


Fig. 12. Values of objective function $J_i(\hat{\theta}_i)$ for bad data detection.

## IV. Conclusion

This paper proposes a new DSE method for power systems. The proposed algorithm can dynamically estimate the states without any centralized facility. An information propagation algorithm is developed to broadcast any information of vertex to the whole system. The information propagation algorithm can be useful in a distributed system since it does not require the centralized unit to gather data. The proposed method is simulated in the WSCC 9-bus power system, IEEE 14-bus system, and IEEE 39-bus system in MATLAB/Simulink. The results show that the proposed DSE algorithm can achieve better performance than the traditional centralized state estimation method. In future work, for a large power system with thousands of meters and buses, a new method to reduce the computation will be explored.

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