

# Optimal Combined Heat and Power Economic Dispatch Using Stochastic Fractal Search Algorithm

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**Abstract**—Combined heat and power (CHP) generation is a valuable scheme for concurrent generation of electrical and thermal energies. The interdependency of power and heat productions in CHP units introduces complications and non-convexities in their modeling and optimization. This paper uses the stochastic fractal search (SFS) optimization technique to treat the highly non-linear CHP economic dispatch (CHPED) problem, where the objective is to minimize the total operation cost of both power and heat from generation units while fulfilling several operation interdependent limits and constraints. The CHPED problem has bounded feasible operation regions and many local minima. The SFS, which is a recent metaheuristic global optimization solver, outranks many current reputable solvers. Handling constraints of the CHPED is achieved by employing external penalty parameters, which penalize infeasible solution during the iterative process. To confirm the strength of this algorithm, it has been tested on two different test systems that are regularly used. The obtained outcomes are compared with former outcomes achieved by many different methods reported in literature of CHPED. The results of this work affirm that the SFS algorithm can achieve improved near-global solution and compare favorably with other commonly used global optimization techniques in terms of the quality of solution, handling of constraints and computation time.

**Index Terms**—Combined heat and power (CHP), economic dispatch, global optimization, metaheuristic algorithms, non-convex optimization problem, power systems, stochastic fractal search.

## I. INTRODUCTION

**E**CONOMIC dispatch (ED) of electric power generation units is among the most imperative operational optimization issues in power systems. This ED-constrained non-linear problem must be solved to minimize the total operation cost of committed units while satisfying the load demand and obeying physical limits of units. It can also be augmented to consider other concerns such as losses in transmission network and pollution quantity made by generation units.

The combined heat and power (CHP) generation (cogene-

ration) refers to the joint generation of electrical/mechanical power and advantageous thermal energy from the same source of energy for heating and cooling intentions. The CHP generation is the most effective model for concurrent production of both electrical energy and thermal energy [1]. As an environmentally friendly system, cogeneration offers considerable savings of the generation cost compared to the heat-only boilers and traditional thermal units. The production of CHP is restricted by the feasible operation region (FOR), as productions of heat and power in these units are jointly connected.

Nowadays, optimal dispatch of generation mix involving CHP units is an attractive and crucial optimization issue in power system operation. The major worldwide shift within the energy sector caused by launching microgrid initiatives and technologies has increased the interest in CHP units. Generation units in CHP ED (CHPED) comprise classical thermal power-only supplying units, CHP generation units, and heat-only generation units (boilers). The complexity of this dispatch originates from the FORs of CHP units, which indicates dual non-linear dependency between heat and power productions. CHPED aims to find an optimal schedule of heat and power generation while respecting various electrical and operational constraints and limits of the generation units. The optimization problem is non-linear and non-convex which requires global solvers.

Solution techniques of the CHPED are broadly classified as conventional mathematical methods and recent methods. Conventional methods include Lagrangian relaxation (LR) [2], Benders decomposition [3], [4], mixed-integer non-linear programming [5], and branch and bound (B&B) algorithm [6]. The classical techniques cannot perform effectively for solving CHPED problems as they have shortcomings such as sensitivity to initial estimates, convergence into local optimal solution, computational complexity, and difficulties in handling discontinuities and non-smooth functions, especially when the dispatch problem is highly non-linear.

In the past decade, various recent optimization solvers have been disclosed in the literature to reach global or near-global solutions of non-linear optimization problems. Recent methods include nature-inspired metaheuristic optimization algorithms. Application of these algorithms for CHPED problems has received research attention in the last few years, as they can deal with discontinuities, high non-linearities, and

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non-convexities in objective functions and constraints. Many metaheuristic techniques have been positively utilized to treat CHPED problems such as genetic algorithm (GA), harmony search (HS), particle swarm optimization (PSO), firefly algorithm (FA), cuckoo search algorithm (CSA), artificial bee colony (ABC), gravitational search algorithm (GSA), group search optimization (GSO), grey wolf optimization (GWO), bat algorithm (BA), differential evolution (DE), ant swarm optimization (ASO), invasive weed optimization (IWO), teaching learning based optimization (TLBO), artificial immune system (AIS), krill herd (KH), and evolutionary programming (EP). The most recent applied heuristic optimization methods to attain optimum results of CHPED problem were reported in [1].

Reference [7] presented a general review of modeling, planning and energy management in a microgrid that used combined cooling, heating and power (CCHP). Reference [8] provided an ample review of recent trend in CCHP schemes and optimization techniques used to improve their performance. Reference [9] presented a review on definition, benefits, characteristics, and various configurations of CCHP systems. Reference [10] presented the development, benefits and analysis of CCHP schemes, and reviewed sizing, control, optimization procedure, and management of these systems. Reference [11] reviewed the current and future trends in micro CHP systems to improve system efficiencies and reduce gas emissions, and investigated such systems for residential applications including modeling and simulation.

In [12], the self-adaptive real-coded genetic algorithm (SARGA) was utilized to solve the non-convex CHPED optimization problem considering the inequality and equality constraints, and penalty technique was suggested to handle the constraints. In [13], an improved GA technique utilizing a multiplier updating was presented to solve the CHPED with a small population size. In [14], CHPED with losses in transmission network and valve-point effects was considered, where a mutation operator of real coded GA was used to improve the convergence time and optimal operation cost. In [15], the researcher presented a solution of a multi-objective economic-environmental CHPED problem using non-dominated sorting real coded GA.

In [16], PSO was employed to deal with a multi-objective CHPED problem considering operation cost, gas emissions, and wind power resources. A PSO using time-varying acceleration factors was used in [17] to obtain the optimal productions of power and heat units in CHPED considering valve-point effects and network losses. An improved PSO (IPSO) technique was employed to solve a stochastic model of CHPED problem in [18], where both demands of heat and power in the system were treated as random variables.

In [19], DE was used to find the optimal schedule of CHP units considering valve-point effects and losses in transmission network. In [20], a DE utilizing a Gaussian mutation operator was employed to solve the CHPED considering valve-point effects and network losses. In [21], the DE was integrated with the sequential quadratic programming to solve a short-term scheduling of CHP generating units. A combination of continuous grasp algorithm and DE for solving non-

smooth non-convex CHPED problem was introduced in [22] to enhance the global search ability and avoid convergence to local minima. Reference [23] introduced a hybrid DE with multiplier updating to obtain the optimal solution of a CHPED within the FOR of CHP units.

The GSO was presented in [24] to deal with the non-convex non-smooth CHPED considering transmission losses, valve-point effects and prohibited operating zones of classical thermal generation units. A modified GSO utilizing the B-Spline wavelet theory was introduced in [25] to solve the CHPED problem to avoid premature convergence of the solution. The GSA was used in [26] to solve the CHPED including transmission network losses and valve-point effects.

The CSA was used in [27], [28] to solicit the optimal schedule of generation in CHP units considering transmission losses and valve-point effects. A CSA technique utilizing an external penalty function was used in [29] for the CHPED. Reference [30] used the GWO for different formulations of CHPED optimizations considering ramp-rate limits, transmission losses, valve-point effects, and spinning reserve.

In [33], a hybrid HS-GA was used for the CHPED problem, where GA features were used to handle the difficulties of non-linearity and non-convergence, and HS features were used to increase the probability of global optimal solution.

The IWO algorithm was used in [34] for solving CHPED problem. The bee colony optimization (BCO) was used in [35] for solving non-convex CHPED problem considering power transmission losses. The TLBO integrated with opposition-based learning (OBL) for improved convergence characteristics was used in [36] to solve the non-linear CHPED optimization problem, and the AIS algorithm was suggested in [37]. The FA was proposed in [38] to treat the CHPED problem with the two objective functions of the total fuel cost and gas emission, considering the spinning reserves, where the FA was used to attain a series of non-dominated solutions utilizing chaotic mechanism and new mutation procedures. In [39], the FA was used for the CHPED problem, with an improved random search process. The KH algorithm was used in [40] to obtain the power and heat generation scheduling in the CHPED, considering transmission network losses and valve-point effects. In [41], the crisscross optimization (CSO) algorithm was used for solving CHPED problem. The algorithm was very competent in both accuracy and convergence rate compared to other algorithms.

The metaheuristic global optimizer, stochastic fractal search (SFS), was a nature-inspired algorithm proposed in [42]. Based on diffusion and fractal properties, this algorithm overcame the shortcomings of the other commonly used metaheuristic solvers. By employing uncomplicated operations, it can realize a global or a better near-global solution with fewer iterations, larger accuracy, and less convergence time [42]. The SFS replicated the natural growth phenomenon by using the fractal mathematic concept [42].

The work in this paper utilizes the SFS technique to deal with the CHPED problem. Handling equality and inequality constraints is achieved in this paper using penalty parameters that penalize infeasible solution during the iterative pro-

cess. Through these parameters, the constrained CHPED problem is transformed into an unconstrained optimization. To confirm its efficient performance, the algorithm has been used for two distinct systems, which are commonly used as test cases in CHPED literature. Comparisons of results presented in this paper disclose that the optimal SFS-based solution can decrease the production costs with a short computation time. It also reveals that the optimal SFS-based solutions are superior to some of the frequently employed global optimization solutions

The remainder of this paper is organized as follows. Section II presents the mathematical formulation of CHPED. Section III presents a detailed description and the mathematical modeling of the SFS. Section IV presents the test systems, optimization results, and discussion of results. Section V presents summary of the main findings, concluding statements, and recommendations for future effort.

## II. MATHEMATICAL FORMULATION OF CHPED OPTIMIZATION PROBLEM

The optimization problem of CHPED is non-linear and non-convex, whose intent is to achieve the optimal mix of heat and power generations from three different types of resources: conventional power-only units, co-generation (CHP) units, and heat-only units. Accordingly, the objective function, which is the total production cost, is a combination of three different cost functions with equality and inequality constraints. This optimization problem has two equality constraints. The first is that the total electrical power produced from all power generation units meets the total power demand  $p_d$ , and the second is that the total heat produced from all heat generation units meets the total heat demand  $h_d$ . The inequality constraints relate to the CHP units, which require that optimal productions of CHP units should lie within the FORs. The limits indicate the upper and lower limits of all units participating in the CHPED. Figure 1 shows three possible types of heat-power plane of a CHP unit, which represent possible FOR of any CHP unit [3], [12], [17], [23], [29], [32]-[34], [39], where  $P$  and  $H$  indicate real power production and heat production of a CHP unit, respectively.

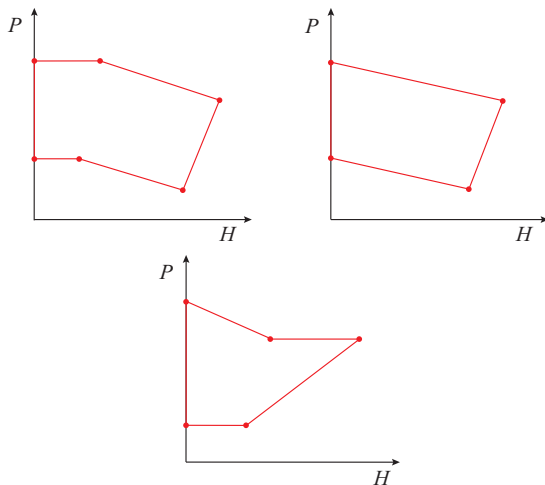


Fig.1. Possible shapes of FOR of a CHP.

The CHPED optimization problem can be mathematically formulated as follows [3], [12]-[14], [23], [27], [29], [32]-[34], [39]:

$$\min C = \sum_{i=1}^{N_p} C_i(p_i) + \sum_{j=1}^{N_c} C_j(p_j, h_j) + \sum_{k=1}^{N_h} C_k(h_k) \quad (1)$$

s.t.

$$\sum_{i=1}^{N_p} p_i + \sum_{j=1}^{N_c} p_j = p_d \quad (2)$$

$$\sum_{j=1}^{N_c} h_j + \sum_{k=1}^{N_h} h_k = h_d \quad (3)$$

$$p_i^{\min} \leq p_i \leq p_i^{\max} \quad i = 1, 2, \dots, N_p \quad (4)$$

$$p_j^{\min(h_j)} \leq p_j \leq p_j^{\max(h_j)} \quad j = 1, 2, \dots, N_c \quad (5)$$

$$h_j^{\min(p_j)} \leq h_j \leq h_j^{\max(p_j)} \quad j = 1, 2, \dots, N_c \quad (6)$$

$$h_k^{\min} \leq h_k \leq h_k^{\max} \quad k = 1, 2, \dots, N_h \quad (7)$$

where the cost functions are given by:

$$C_i(p_i) = a_i + b_i p_i + c_i p_i^2 \quad (8)$$

$$C_j(p_j, h_j) = a_j + b_j p_j + c_j p_j^2 + d_j h_j + e_j h_j^2 + f_j p_j h_j \quad (9)$$

$$C_k(h_k) = a_k + b_k h_k + c_k h_k^2 \quad (10)$$

where  $C$  is the total fuel (production) cost;  $C_i$ ,  $C_j$ ,  $C_k$  are the fuel costs of the conventional power-only unit, co-generation unit, and heat-only unit, respectively;  $a_i$ ,  $b_i$ ,  $c_i$  are the fuel cost coefficients of the  $i^{\text{th}}$  conventional power-only unit;  $a_j$ ,  $b_j$ ,  $c_j$ ,  $d_j$ ,  $e_j$ ,  $f_j$  are the fuel cost coefficients of the  $j^{\text{th}}$  co-generation unit;  $a_k$ ,  $b_k$ ,  $c_k$  are the fuel cost coefficients of the  $k^{\text{th}}$  heat-only unit;  $p_i$  and  $p_j$  are the power productions of conventional power and co-generation units, respectively;  $h_j$  and  $h_k$  are the heat productions of co-generation and heat-alone units, respectively;  $h_d$  and  $p_d$  are the heat and power demands, respectively;  $N_p$ ,  $N_c$ ,  $N_h$  are the numbers of conventional power units, co-generation units and heat-alone units, respectively;  $p_i^{\min}$  and  $p_i^{\max}$  are the minimum and maximum power generation limits of the  $i^{\text{th}}$  conventional unit, respectively;  $p_j^{\min}$  and  $p_j^{\max}$  are the minimum and maximum power generation limits of the  $j^{\text{th}}$  conventional unit, respectively;  $h_j^{\min}$  and  $h_j^{\max}$  are the minimum and maximum heat generation limits of the  $j^{\text{th}}$  co-generation unit, respectively; and  $h_k^{\min}$  and  $h_k^{\max}$  are the minimum and maximum heat generation limits of the  $k^{\text{th}}$  heat-only unit, respectively.

The constrained CHPED problem is transformed into an unconstrained one by handling equality and inequality constraints utilizing the approach of penalty parameters [12], [29], which penalizes the infeasible solution during the iterative process. The proper settings of the penalty factors must be cautiously selected after some trials towards improved optimal solutions while respecting the constraints [29]. Because various constraints have various orders of magnitude, it is more appropriate to initially normalize each equality and inequality constraint. The  $i^{\text{th}}$  equality constraint,  $g'_i(\mathbf{x}) - a_i = 0$ , and the  $j^{\text{th}}$  inequality constraint,  $h'_j(\mathbf{x}) - b_j \leq 0$ ,

can be normalized to be  $g_i(\mathbf{x}) \equiv g'_i(\mathbf{x})/a_i - 1 = 0$  and  $h_j(\mathbf{x}) \equiv h'_j(\mathbf{x})/b_j - 1 \leq 0$ , respectively. After normalizing the constraints, the optimization problem can be expressed in compact form as follows:

$$\min C(x_1, x_2, \dots, x_n) \quad (11)$$

s.t.

$$g_i(x_1, x_2, \dots, x_n) = 0 \quad i = 1, 2, \dots, N_{ec} \quad (12)$$

$$h_j(x_1, x_2, \dots, x_n) \leq 0 \quad j = 1, 2, \dots, N_{ic} \quad (13)$$

$$x_k^{\min} \leq x_k \leq x_k^{\max} \quad k = 1, 2, \dots, n \quad (14)$$

where  $n$ ,  $N_{ec}$  and  $N_{ic}$  are the number of state variables (unknown), number of equality constraints and number of inequality constraints, respectively. The penalized objective function becomes:

$$f = C(x_1, x_2, \dots, x_n) + r \left[ \sum_{i=1}^{N_{ec}} |g_i(x_1, x_2, \dots, x_n)|^2 + \sum_{j=1}^{N_{ic}} \max(0, h_j(x_1, x_2, \dots, x_n)) \right] \quad (15)$$

where  $r$  is the penalty factor.

### III. SFS ALGORITHM

The nature-inspired SFS technique is a new metaheuristic global optimization algorithm, which uses the concept of fractals to imitate the natural growth. The diffusion property frequently used in random fractals is utilized by the particles in this technique to efficiently explore the search space [42].

Random fractals can be produced by adjusting the iteration process using stochastic rules. The SFS algorithm uses a random walk to model the diffusion process, where the diffusing particle stays connected with the seed particle which produces it. This process is repeated until a cluster is established [42].

The diffusion and the updating are the two major processes that occur in the SFS technique. Figure 2 presents the flowchart of the SFS and summarizes the diffusion phase and the updating process which involve two updating processes. The two processes will be detailed next. In this figure,  $GW_1$  and  $GW_2$  represent the Gaussian walks participating in the diffusion process,  $BP$  is the best point location in the group, and  $P_i$  represents the  $i^{\text{th}}$  point in a group, which will be discussed later in the paper. In the diffusion process, every particle diffuses around its current position to guarantee exploitation characteristic. The diffusion process prevents entrapment in local minima and improves the opportunity of achieving the global solution. In the updating process, the SFS shows how a point among a group revises its position based on the positions of other points in the same group. The SFS adopts a static diffusion process, where the best produced particle achieved from the diffusing process is the only particle taken into consideration, while the other particles are neglected. The SFS utilizes random methods as the

updating processes.

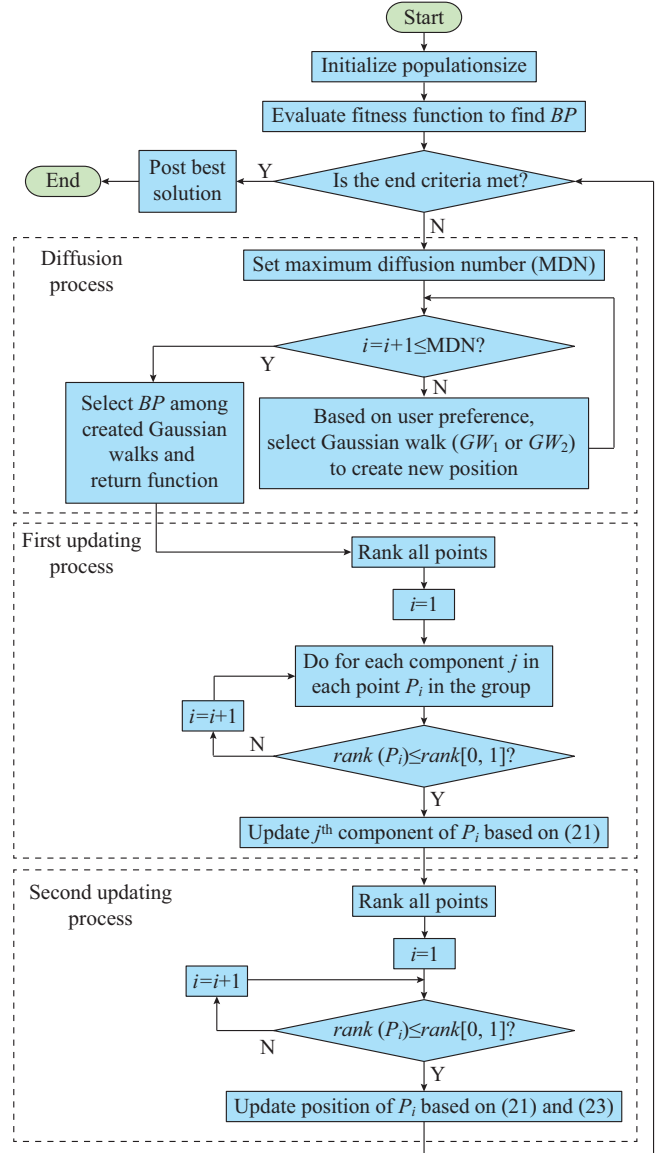


Fig. 2. Flowchart of SFS algorithm.

Suppose  $\varepsilon$  and  $\varepsilon'$  are two random numbers which are distributed uniformly in the range  $[0, 1]$ . A series of Gaussian walks ( $GW_1$  and  $GW_2$ ) engaged in the diffusion course are defined by [42]:

$$GW_1 = \text{Gaussian}(\mu_{BP}, \sigma) + \varepsilon \cdot BP - \varepsilon' P_i \quad (16)$$

$$GW_2 = \text{Gaussian}(\mu_p, \sigma) \quad (17)$$

where  $\mu_{BP}$ ,  $\mu_p$  and  $\sigma$  are the Gaussian means and standard deviation, respectively;  $\mu_{BP} = |BP|$  in (16), and  $\mu_p = |P_i|$  in (17). If  $g$  refers to the generation (iteration) number,  $\sigma$  in (16) and (17) is determined as follows [42]:

$$\sigma = \left| \frac{P_i - BP}{g} \log g \right| \quad (18)$$

For an optimization problem with dimension  $D$ , every spe-



cific particle assumed to solve the problem has been made based on a  $D$ -dimensional vector. In the initialization phase as shown in Fig. 2, each point is randomly initialized based on its maximum and minimum bounds. The initial value of the  $j^{\text{th}}$  point ( $P_j$ ) is determined as follows [42]:

$$P_j = LB + \varepsilon(UB - LB) \quad (19)$$

where  $LB$  and  $UB$  refer to the upper and the lower bounds of problem variables, respectively.

After the initialization of all particles, the fitness value of each particle is evaluated to reach the best point  $BP$  among all particles. For consistency with the exploitation ability in the diffusion process, all points roam around their current location to exploit the problem search space [42].

Due to the exploration property, SFS uses two statistical actions to improve the better space exploration. The first one is applied for each individual vector index, while the other one is then performed for all points. Initially, the first action ranks all points based on fitness values. Each point  $i$  in the group is then designated a probability value ( $P_{ai}$ ) which obeys a uniform distribution determined by [42]:

$$P_{ai} = \text{rank}(P_i) / N \quad (20)$$

where  $\text{rank}(P_i)$  is the rank of point  $P_i$  among other points in the group; and  $N$  is the number of all points in the group. Equation (20) indicates that the better the rank of point, the higher the probability to be selected. As illustrated in Fig. 2, for each point  $P_i$  in a group, if  $P_{ai} < \varepsilon$  is met, the  $j^{\text{th}}$  component of  $P_i$  is revised based on the following relation; and if it is not met, it stays unaltered [42].

$$P'_i(j) = P_r(j) - \varepsilon(P_i(j) - P_r(j)) \quad (21)$$

where  $P'_i(j)$  is the new updated location of point  $P_i$ ; and  $P_r$  and  $P_i$  are the points selected randomly in the group.

The preceding discussion indicates that the first statistical process is performed for the components of the points. As shown in Fig. 2, the other statistical process adjusts the location of a point considering the position of other points in the group to enhance the quality of exploration and to fulfill the diversification property. Ahead of starting the second statistical process, all points achieved from the first statistical process are ranked based on (20). As in the first process, if  $P_{ai} < \varepsilon$  is satisfied for a new point  $P'_i$ , the existing position of  $P'_i$  is revised conforming to (22) and (23); and if  $P_{ai} < \varepsilon$  is not satisfied, no update takes place [42].

$$P''_i = P'_i - \beta(P'_i - BP) \quad \beta \leq 0.5 \quad (22)$$

$$P''_i = P'_i + \beta(P'_i - P_r) \quad \beta > 0.5 \quad (23)$$

where  $P_r$  and  $P'_i$  are the randomly chosen points achieved from the first statistical process; and  $\beta$  is the number produced randomly from the Gaussian normal distribution.  $P'_i$  replaces the new point  $P''_i$  if the fitness value of  $P''_i$  is superior to that of  $P'_i$  [42].

## IV. RESULTS AND DISCUSSION

### A. Test System 1

This system is the well-known four-unit test system in the literature of CHPED, which is presented in [1] - [3], [12] - [14], [17], [23], [27], [29], [31] - [34], [39] and many other related references. It comprises one conventional power-only unit (unit 1), two CHP units (units 2 and 3), and one heat-only unit (unit 4). The minimum and maximum limits of the conventional power unit are 0 and 150 MW, respectively. The minimum and maximum limits of the heat-only unit are 0 and 2690 MWth, respectively. The FORs of the two CHP units are illustrated in Fig. 3. The system power demand  $p_d$  and the heat demand  $h_d$  are 200 MW and 115 MWth, respectively.

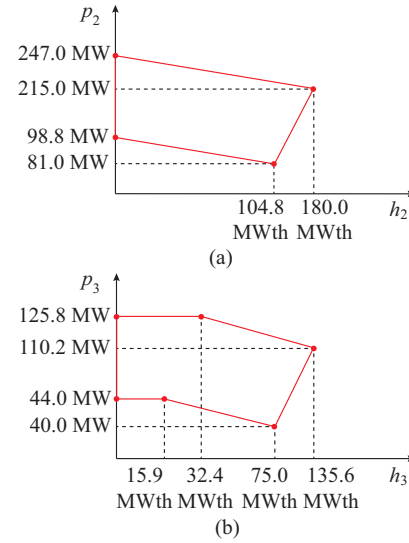


Fig. 3. FORs of CHP units of test system 1. (a) First CHP unit (unit 2). (b) Second CHP unit (unit 3).

The four units has the following cost functions [1] - [3], [12] - [14], [17], [23], [27], [29], [31] - [34], [39]:

$$\begin{cases} C_1(p_1) = 50p_1 \\ C_2(p_2, h_2) = 2650 + 14.5p_2 + 0.0345p_2^2 + 4.2h_2 + 0.030h_2^2 + 0.031p_2h_2 \\ C_3(p_3, h_3) = 1250 + 36.0p_3 + 0.0435p_3^2 + 0.6h_3 + 0.027h_3^2 + 0.011p_3h_3 \\ C_4(h_4) = 23.4h_4 \end{cases} \quad (24)$$

The optimization problem of CHPED of this system is formulated as:

$$\min C = C_1(p_1) + C_2(p_2, h_2) + C_3(p_3, h_3) + C_4(h_4)$$

The constraints are as follows:

1) Equality constraints:

$$\begin{cases} p_1 + p_2 + p_3 = 200 \\ h_2 + h_3 + h_4 = 115 \end{cases} \quad (25)$$

2) Inequality constraints which represent the FOR of the two CHP units:

$$\begin{cases} 1.781914894h_2 - p_2 - 105.7446809 \leq 0 \\ 0.177777778h_2 + p_2 - 247.0000000 \leq 0 \\ -0.169847328h_2 - p_2 + 98.8000000 \leq 0 \\ 1.158415842h_3 - p_3 - 46.88118818 \leq 0 \\ 0.151162791h_3 + p_3 - 130.6976744 \leq 0 \\ -0.067681895h_3 - p_3 + 45.07614213 \leq 0 & h_3 \geq 15.9 \\ 44 - p_3 \leq 0 & h_3 < 15.9 \end{cases} \quad (26)$$

3) The limits:

$$\begin{cases} 0 \leq p_1 \leq 150 \\ 81 \leq p_2 \leq 247 \\ 0 \leq h_2 \leq 180 \\ 40 \leq p_3 \leq 125.8 \\ 0 \leq h_3 \leq 135.6 \\ 0 \leq h_4 \leq 2695.2 \end{cases} \quad (27)$$

For this system, the results achieved by the proposed SFS method for the above CHPED equations will be put in comparison with the previously obtained results reported in [1], [2], [12]-[14], [17], [27], [29], [31]-[34], [39], [41] using LR, B&B, improved ant colony search (IACS) algorithm, GA-based penalty function (GAPF) method, PSO, EP, FA, improved genetic algorithm with multiplier updating (IGAMU), HS, self-adaptive real-coded genetic algorithm (SARGA), artificial bee colony (ABC), DE, mesh adaptive direct search and particle swarm optimization (MADS-PSO), mesh adaptive direct search and Latin hypercube sampling (MADS-LHS), IWO, GSA, CSA, and CSO.

The SFS algorithm is coded in MATLAB and executed using a 1.8 GHz, 8 GB RAM Pentium Core i5 PC. The results of the other methods are taken from literature. The user-sup-

plied parameter setting of the SFS are population size, maximum generation, and MDN, which are selected to be 120, 1000, and 4, respectively.

The characteristics curve of the SFS convergence for test system 1 is shown in Fig. 4, which illustrates a fast convergence.

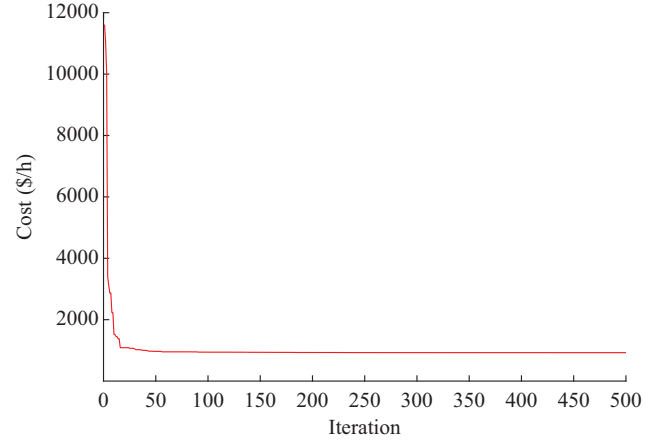


Fig. 4. Convergence characteristics of SFS algorithm for test system 1.

The state variables, fuel costs, and average CPU times are selected to examine the performance of the SFS with the other techniques. Table I summarizes the results obtained for this system using SFS method and the other methods. As can be seen from Table I, the SFS algorithm obtains the lowest minimum cost with less computation time. The computation time is among the shortest compared to that of other algorithms.

TABLE I  
COMPARISON OF RESULTS FOR TEST SYSTEM 1

Algorithm	$p_1$ (MW)	$p_2$ (MW)	$p_3$ (MW)	$h_2$ (MWth)	$h_3$ (MWth)	$h_4$ (MWth)	Total cost (\$/h)	Computation time (s)
IACS [14]	0.0800	150.9300	49.0000	48.8400	65.7900	0.3700	9452.20	5.26
MADS-PSO [27]	0.0092	157.9392	42.0516	42.4459	72.5522	0.0019	9301.38	7.56
MADS-LHS [27]	0.0017	159.8000	40.2014	42.4042	72.3904	0.2054	9277.13	7.04
ABC [29]	0.2400	158.7800	40.9600	39.5800	75.2300	0.1800	9276.70	NA
GAPF [12]	0	159.2300	40.7700	39.9400	75.0600	0	9267.28	4.32
PSO [12]	0.0500	159.4300	40.5700	39.9700	75.0300	0	9265.10	3.09
DE [29]	0.0200	159.9400	39.9300	40.0200	74.9900	0.0600	9258.90	NA
LR [12]	0	160.0000	40.0000	40.0000	75.0000	0	9257.10	3.98
B&B [12]	0	160.0000	40.0000	40.0000	75.0000	0	9257.10	4.27
EP [12]	0	160.0000	40.0000	40.0000	75.0000	0	9257.10	7.96
FA [39]	0.0014	159.9986	40.0000	40.0000	75.0000	0	9257.10	NA
IGAMU [12]	0	160.0000	40.0000	39.9900	75.0000	0	9257.09	5.53
HS [12]	0	160.0000	40.0000	40.0000	75.0000	0	9257.07	4.21
SARGA [12]	0	159.9900	40.0100	39.9900	75.0000	0	9257.07	3.76
CSO [41]	0	160.0000	40.0000	40.0000	75.0000	0	9257.07	1.18
IWO	0	160.0000	40.0000	40.0000	75.0000	0	9257.07	11.21
GSA	0.0003	159.4494	40.5494	38.8850	75.4736	0.6414	9269.14	7.26
CSA	0	160.0000	40.0000	40.0000	75.0000	0	9257.07	5.62
SFS	0	160.0000	40.0000	40.0000	75.0000	0	9257.07	3.78

Figure 5 illustrates comparisons of the convergence characteristics of SFS and other nine algorithms used in this test system and the next test system. These comparisons demonstrate the convergence speed and efficiency of the SFS to reach the optimal solution.

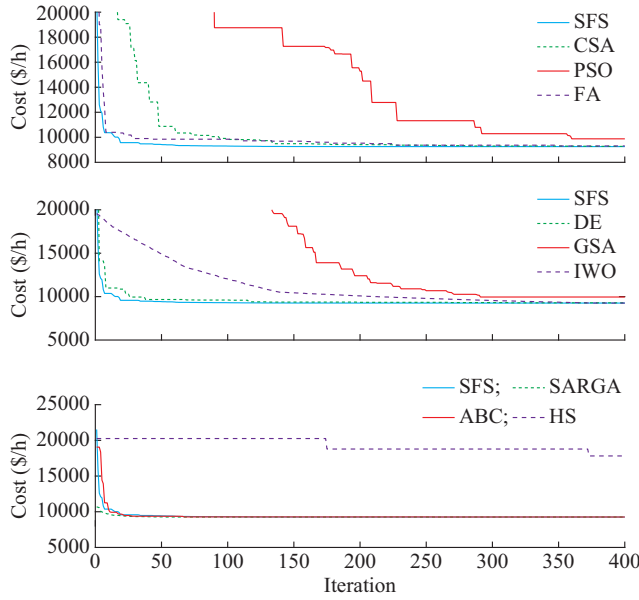


Fig. 5. Comparison of convergence characteristics of SFS and other nine algorithms for test system 1.

The computation time of the SFS algorithm can be reduced by selecting proper values of the SFS parameters including population size, maximum generation, and MND. Table II shows improvements in calculation time of test system 1 for selected sets of SFS parameters. Note that most of the computation time is much better than those of all algorithms compared in Table I.

TABLE II  
REDUCTIONS IN COMPUTATION TIME OF TEST SYSTEM 1 FOR SELECTED VALUES OF SFS PARAMETERS

Population size	Maximum generation	MDN	Total cost (\$/h)	Computation time (s)
16	500	1	9257.07	0.897198
16	600	1	9257.07	0.968269
16	700	1	9257.07	1.112184
16	800	1	9257.07	1.272459
18	500	1	9257.07	0.894139
18	600	1	9257.07	1.067141
18	700	1	9257.07	1.241997
18	800	1	9257.07	1.417746
20	500	1	9257.07	0.987465
20	600	1	9257.07	1.181672
20	700	1	9257.07	1.383194
20	800	1	9257.07	1.570533
22	500	1	9257.07	1.079808
22	600	1	9257.07	1.290776
22	700	1	9257.07	1.512136
22	800	1	9257.07	1.731690

## B. Test System 2

This system is a five-unit system, which is also a well-known system available in many references such as [1], [3], [23], [25], [29], [31]–[34], [39]. It consists of one conventional power-only unit (unit 1), three CHP units (units 2, 3 and 4), and one heat-only unit (unit 5). The maximum and minimum limits of the conventional power unit are 135 MW and 35 MW, respectively. The minimum and maximum limits of the heat-only unit are 0 and 60 MWth, respectively. The FORs of the three CHP units are illustrated in Fig. 6.

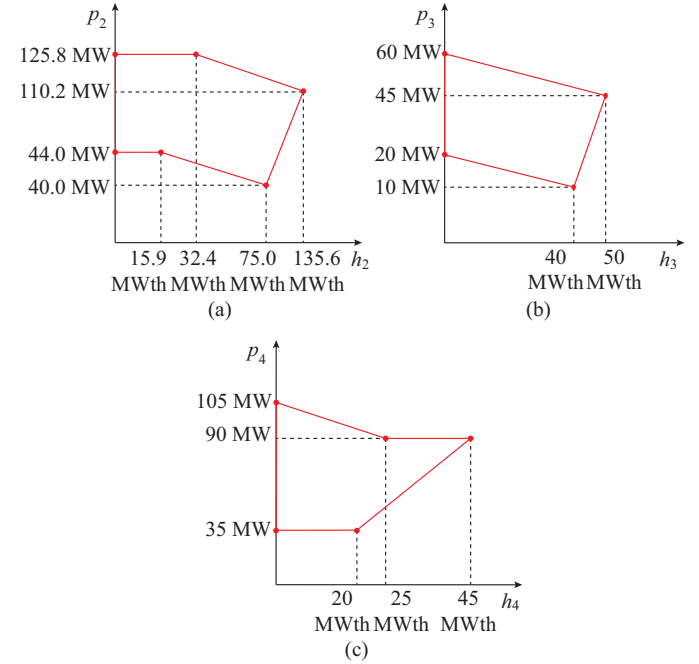


Fig. 6. FORs of CHP units of test system 2. (a) First CHP unit (unit 2). (b) Second CHP unit (unit 3). (c) Third CHP unit (unit 4).

This test system will be examined for three different cases of power and heat demands:

- 1) Case 1:  $p_d = 300$  MW,  $h_d = 150$  MWth.
- 2) Case 2:  $p_d = 250$  MW,  $h_d = 175$  MWth.
- 3) Case 3:  $p_d = 150$  MW,  $h_d = 220$  MWth.

The cost functions of the five units are given by [1], [3], [23], [25], [29], [31]–[34], [39]:

$$\begin{cases}
 C_1(p_1) = 254.8863 + 7.6997p_1 + 0.00172p_1^2 + 0.00115p_1^3 \\
 C_2(p_2, h_2) = 1250 + 36.0p_2 + 0.0435p_2^2 + 0.6h_2 + 0.027h_2^2 + 0.011p_2h_2 \\
 C_3(p_3, h_3) = 2650 + 34.5p_3 + 0.1035p_3^2 + 2.203h_3 + 0.025h_3^2 + 0.051p_3h_3 \\
 C_4(p_4, h_4) = 1565 + 20p_4 + 0.072p_4^2 + 2.3h_4 + 0.02h_4^2 + 0.04p_4h_4 \\
 C_5(h_5) = 950 + 2.0109h_5 + 0.038h_5^2
 \end{cases} \quad (28)$$

The optimization problem of the CHPED of this system is formulated as:

$$\min C = C_1(p_1) + C_2(p_2, h_2) + C_3(p_3, h_3) + C_4(p_4, h_4) + C_5(h_5) \quad (29)$$

The constraints are as follows:

1) Equality constraints:

$$\begin{cases} p_1 + p_2 + p_3 + p_4 = p_d \\ h_2 + h_3 + h_4 + h_5 = h_d \end{cases} \quad (30)$$

2) Inequality constraints which represent the FOR of the CHP units:

$$\begin{cases} p_2 + 0.151162791h_2 - 130.6976744 \leq 0 & h_2 \geq 32.4 \\ p_2 - 125.8 \leq 0 & h_2 \leq 32.4 \\ -p_2 + 1.158415842h_2 - 46.88118818 \leq 0 \\ -p_2 - 0.067681895h_2 + 45.07614213 \leq 0 & h_2 \geq 15.9 \\ 44 - p_2 \leq 0 & h_2 \leq 15.9 \\ p_3 + 0.272727272h_3 - 60.00000000 \leq 0 \\ -p_3 - 0.250000000h_3 + 20.00000000 \leq 0 \\ p_3 + 2.333333333h_3 - 83.33333333 \leq 0 \\ p_4 + 0.60h_4 - 105.0 \leq 0 & h_4 \leq 25 \\ -p_4 + 2.20h_4 - 9.0 \leq 0 \end{cases} \quad (31)$$

3) The limits:

$$\begin{cases} 35 \leq p_1 \leq 135 \\ 40 \leq p_2 \leq 125.8 \\ 10 \leq p_3 \leq 60 \\ 35 \leq p_4 \leq 105 \\ 0 \leq h_2 \leq 135.6 \\ 0 \leq h_3 \leq 55 \\ 0 \leq h_4 \leq 45 \\ 0 \leq h_5 \leq 60 \end{cases} \quad (32)$$

The computation time to solve the above optimization problem using SFS is 10.76 s, 10.74 s, and 10.83 s for cases 1, 2, and 3, respectively, which is considered reasonable. The characteristic curve of the SFS convergence for case 1 of test system 2 is shown in Fig. 7. For the second test system, the results achieved by the proposed SFS technique will be compared with those results obtained using GA [32], RCGA [15], HS [32], classic PSO (CPSO [17], TVAC-PSO [17], COA [28], GSA [26], RCGA-IMM [14], CSA [27], IWO [34], FA [39], DE, ABC, and SARGA. The results obtained for this system using SFS and the other techniques (as reported in literature), are summarized in Tables III - V for the three cases, respectively. Again, as can be observed from these tables, the SFS technique is able to obtain the lowest possible minimum cost with small computation time.

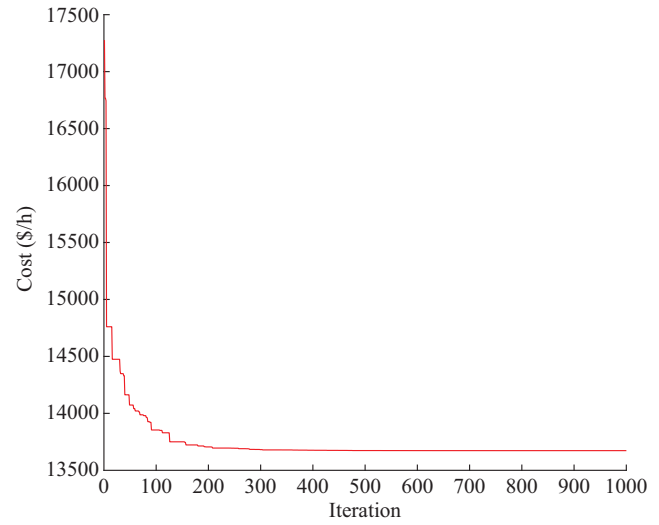


Fig. 7. Convergence characteristics of SFS algorithm for case 1 of test system 2.

TABLE III  
COMPARISON OF CHPED RESULTS OF TEST SYSTEM 2 FOR CASE 1

Algorithm	$p_1$ (MW)	$p_2$ (MW)	$p_3$ (MW)	$p_4$ (MW)	$h_2$ (MWth)	$h_3$ (MWth)	$h_4$ (MWth)	$h_5$ (MWth)	Total cost (\$/h)
GA [32]	135.0000	70.8100	10.8400	83.2800	80.5400	39.8100	0.0000	29.6400	13779.50
RCGA [15]	134.9904	49.9525	25.0827	89.9744	73.5089	35.8519	1.2916	39.3476	13776.14
HS [32]	134.7400	48.2000	16.2300	100.8500	81.0900	23.9200	6.2900	38.7000	13723.20
CPSO [17]	135.0000	40.7309	19.2728	105.0000	64.4003	26.4119	0.0000	59.1955	13692.52
IWO [34]	134.7300	40.0000	20.8600	104.4100	75.0000	37.6000	0.0000	37.4000	13683.65
FA [39]	134.7400	40.0000	20.2500	105.0000	75.0000	27.8700	0.0000	47.1200	13683.22
TVAC-PSO [17]	135.0000	41.4019	18.5981	105.0000	73.3562	37.4295	0.0000	39.2143	13672.89
COA [28]	135.0000	40.7687	19.2313	105.0000	73.5956	36.7760	0.0000	39.6284	13672.83
SFS	135.0000	40.7689	19.2311	105.0000	73.5955	36.7766	0.0000	39.6279	13672.83

The results summarized in Tables III-V show that the SFS solver outperforms many other methods in terms of the total cost  $C_T$ , but presents the same best values using the rest of the methods presented in the tables.

Figure 8 illustrates the comparisons of convergence characteristics of SFS and other nine algorithms for case 2 of test system 2. It shows that the SFS has fast convergence, which is one of the best algorithms used in the comparison.



TABLE IV  
COMPARISON OF CHPED RESULTS OF TEST SYSTEM 2 FOR CASE 2

Algorithm	$p_1$ (MW)	$p_2$ (MW)	$p_3$ (MW)	$p_4$ (MW)	$h_2$ (MWth)	$h_3$ (MWth)	$h_4$ (MWth)	$h_5$ (MWth)	Total cost (\$/h)
GA [32]	119.2200	45.1200	15.8200	69.8900	78.9400	22.6300	18.4000	54.9900	12327.37
HS [32]	134.6700	52.9900	10.1100	52.2300	85.6900	39.7300	4.1800	45.4000	12284.45
IWO [34]	134.5900	40.0000	10.9400	64.4700	75.0000	38.9800	8.8100	52.2100	12134.33
CPSO [17]	135.0000	40.3446	10.0506	64.6060	70.9318	39.9918	4.0773	60.0000	12132.86
FA [39]	134.8100	40.0000	10.0000	65.1800	75.0000	40.0000	16.9700	43.0200	12119.86
TVAC-PSO [17]	135.0000	40.0118	10.0391	64.9491	74.8263	39.8443	16.1867	44.1428	12117.39
GSA [26]	135.0000	39.9998	10.0000	64.9807	74.9844	40.0000	17.8939	42.1095	12117.37
COA [28]	135.0000	40.0000	10.0000	64.9910	75.0000	40.0000	14.4001	45.6000	12116.60
CSA [27]	135.0000	40.0000	10.0000	65.0000	75.0000	40.0000	14.4046	45.5954	12116.60
DE	135.0000	40.0000	10.0000	64.9998	75.0000	39.9998	14.3984	45.6018	12116.61
ABC	135.0000	40.0488	16.2528	58.7008	74.3491	42.6795	18.7334	39.2381	12178.49
SARGA	135.0000	40.2096	10.2792	64.5112	71.9024	38.8834	16.0870	48.1273	12123.81
SFS	135.0000	40.0000	10.0000	65.0000	75.0000	40.0000	14.4043	45.5957	12116.60

TABLE V  
COMPARISON OF CHPED RESULTS OF TEST SYSTEM 2 FOR CASE 3

Algorithm	$p_1$ (MW)	$p_2$ (MW)	$p_3$ (MW)	$p_4$ (MW)	$h_2$ (MWth)	$h_3$ (MWth)	$h_4$ (MWth)	$h_5$ (MWth)	Total cost (\$/h)
GA [32]	37.9800	76.3900	10.4100	35.0300	106.0000	38.3700	15.8400	59.9700	11837.40
HS [32]	41.4100	66.6100	10.5900	41.3900	97.7300	40.2300	22.8300	59.2100	11810.88
CPSO [35]	35.5972	57.3554	10.0070	57.0587	89.9767	40.0025	30.0232	60.0000	11781.37
RCGA-IMM [14]	42.1660	64.6523	10.0000	43.1817	96.2810	40.0000	23.7190	60.0000	11758.64
TVAC-PSO [17]	42.1433	64.6271	10.0001	43.2295	96.2593	40.0001	23.7404	60.0000	11758.06
COA [28]	42.1497	64.6342	10.0000	43.2161	96.2654	40.0000	23.7346	60.0000	11758.06
CSA [27]	42.2652	64.7630	10.0112	42.9706	96.3766	40.0005	23.6230	59.9990	11758.09
SFS	42.1454	64.6294	10.0000	43.2252	96.2613	40.0000	23.7387	60.0000	11758.06

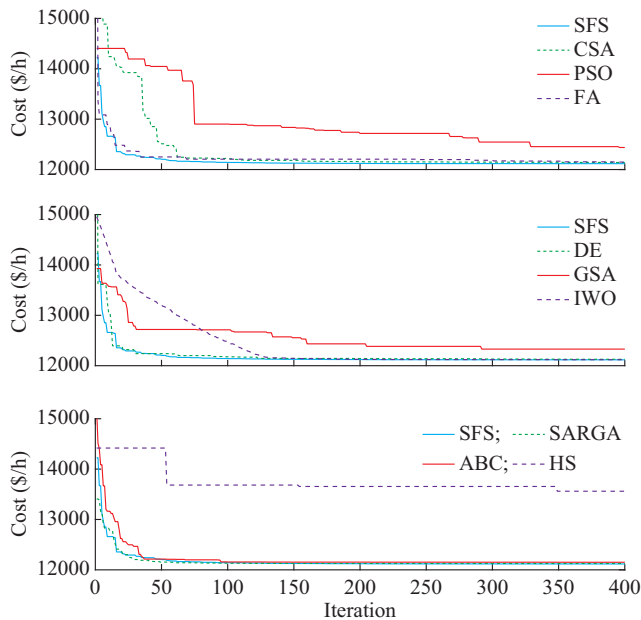


Fig. 8. Comparison of convergence characteristics of SFS and other algorithms of test system 2 for case 2.

The advantages of SFS can be summarized as follows:

- 1) From comparisons with the reported best cost results

obtained by many other solvers, SFS can reach all best solutions reported in literature for all test cases investigated in the paper. This indicates that the algorithm is capable of overcoming the local minima of the problem found by some other algorithms.

- 2) The algorithm demonstrates robust behavior when its user-supplied parameters are changed or when the number of decision variables changes.

- 3) Shorter computation time for the same best solution can be achieved using the SFS by the proper selection of its parameters.

- 4) The algorithm guarantees fast convergence and accuracy in smaller numbers of iterations compared to many other solvers.

- 5) The SFS has uncomplicated mathematical operations.

## V. CONCLUSION AND RECOMMENDATION

This paper presents the application of the SFS algorithm to solve the CHPED optimization problem which is currently a crucial issue in power system operations. The SFS technique is among the promising and powerful global optimization solvers and can outperform some present well-known metaheuristic optimization techniques. The CHPED formulation is a non-convex non-linear optimization problem that models concurrent production of both electrical power and

thermal energy, whose objective is minimizing the production cost of heat and power generation units while fulfilling various inequality and equality constraints and interdependent limits. The equality and inequality constraints are handled in this paper by employing penalty parameters, which are able to penalize infeasible solution during the iterative process, where the constrained CHPED problem is transformed into an unconstrained one. The algorithm has been tested on two different well-known test systems used in the literature of CHPED. The SFS-based results are compared with those obtained by many other commonly used and efficient global optimization techniques. The results have verified that the optimal solutions obtained using the SFS algorithm perform better than many of these frequently used methods. It is also revealed that the optimal SFS-based solution has lowered the system operation costs and achieves the best feasible solution obtained by the rest of other optimization techniques reported in literature. The proposed algorithm is robust in obtaining the best reported feasible solutions for different systems and case studies, and has accomplished improved near-global optimal solutions with very reasonable computation time.

For further work, a variety of ideas can be recommended such as study cases with higher dimension, using SFS to solve optimal power flow (OPF) incorporating CHP units, renewable energy resources in the CHPED problem, application of SFS to the CHPED problem with valve-point effects of thermal power units, gas emission levels in the objective function for environmental concerns, and multi-objective CHPED problems, etc.

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