

A Comprehensive Power Flow Approach for Multi-terminal VSC-HVDC System Considering Cross-regional Primary Frequency Responses

Yida Ye, Ying Qiao, Le Xie, and Zongxiang Lu

Abstract—For the planning, operation and control of multi-terminal voltage source converter (VSC) based high-voltage direct current (HVDC) (VSC-MTDC) systems, an accurate power flow formulation is a key starting point. Conventional power flow formulations assume the constant frequencies for all asynchronous AC systems. Therefore, a new feature about the complex coupling relations between AC frequencies, DC voltages and the exchanged power via VSC stations cannot be characterized if VSC-MTDC systems are required to provide cross-regional frequency responses. To address this issue, this paper proposes a comprehensive frequency-dependent power flow formulation. The proposed approach takes the frequencies of asynchronous AC systems as explicit variables, and investigates the novel bus models of the interlinking buses of VSC stations. The proposed approach accommodates different operation modes and frequency droop strategies of VSC stations, and considers the power losses of VSC stations. The effectiveness and generality of the developed approach are validated by a 6-terminal VSC-HVDC test system. The test system presents the characteristics of the coexistence of numerous VSC operation modes, the absence of slack buses in both AC and DC subsystems, and diversified grid configurations such as point-to-point integration of renewable energy sources and one AC system integrated with multiple VSC stations.

Index Terms—Voltage source converter based high-voltage direct current (VSC-HVDC), multi-terminal VSC based HVDC (VSC-MTDC), cross-regional frequency response, power flow formulation, VSC operation mode.

I. INTRODUCTION

OVER the years, the increasing electricity demand and the renewable energy integration have greatly promoted the rapid development of high-voltage direct current (HVDC) technologies [1], [2]. Compared to the current source converter (CSC) based HVDC or line commutated

converter based HVDC (LCC-HVDC), the voltage source converter (VSC) based technology can flexibly and independently regulate the active power and reactive power at each terminal. Practical experiences demonstrate that the VSC-based technology is appropriate for the integration of renewable energy sources (RESs) and the extension to a complex multi-terminal (MT) configuration connected by several large-scale asynchronous AC grids, e.g., a 7-terminal HVDC grid project in Zhangbei, China [3]. Furthermore, multi-terminal VSC-based HVDC (VSC-MTDC) systems can also actively and adaptively participate in frequency and voltage regulations, which have great potential for future power systems.

As the local synchronous generators (SGs) are replaced by feed-in DC transmissions, a larger rate of change of frequency (RoCoF), a worse frequency nadir and longer frequency recovery time may occur in one single synchronous system with the decreased rotating inertia and frequency regulation capacity [4]–[6]. Thus, a cross-regional frequency response is imperative for VSC-MTDC systems.

To achieve the cross-regional frequency responses and unbalanced power allocation among different asynchronous AC grids, droop-controlled strategies are usually adopted by VSC stations [7]–[10]. In such a system, the frequency of each asynchronous AC system varies under different operation conditions, which results in that the units or VSC stations may change the output power or exchanged power according to their frequency regulation strategies. As a result, a salient feature is introduced to the VSC-MTDC systems that AC frequencies, DC voltages, the power injected into the point of common connection (PCC) of VSC stations and the output power of units are coupled together. Therefore, an investigation into this prominent coupling feature is necessary to accurately analyze the steady state of the entire VSC-MTDC systems.

The conventional power flow approaches for AC grids [11] and LCC-HVDC systems [12]–[14] typically set a slack bus in each AC system/subsystem to hold a constant frequency. But this assumption does not apply to the VSC-MTDC system that provides the cross-regional frequency regulations. Numerous efforts spent on the power flow formulation of VSC-MTDC systems share the same constant-frequency assumption. References [15] and [16] propose an alternating iteration approach and a unified approach to solve the AC

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and DC power flow variables in VSC-MTDC systems, respectively. Because the converters in [15] and [16] adopt constant-power and constant-voltage strategies, neither VSC stations nor units are capable of providing frequency responses. References [17] and [18] propose the power flow algorithms for VSC-MTDC systems with different DC voltage droop relations. These studies conduct the droop-controlled strategies based on only DC variables, i.e., V_{DC} - P_{DC} droop and V_{DC} - I_{DC} droop, aiming at adaptively allocating the power flow in DC networks. However, they fail to explore the coupling relations between the DC variables and the AC frequency or the power injected into the PCC.

To investigate a power flow formulation incorporating the above-mentioned frequency coupling feature, [19]-[22] indirectly couple the AC frequency and DC bus voltages by controlling the exchanged power via converters. However, the control strategy used in these studies applies to a certain VSC operation mode, i.e., active power is controlled in the active current channel of VSC controllers, which cannot be generalized to other possible VSC operation modes in meshed MTDC systems. Also, simple network configurations are designed in these studies, e.g., only one AC subsystem integrates with one DC subsystem [19]-[21], or two AC (or DC) subsystems integrate with one DC (or AC) subsystem via one converter separately [22], all of which fail to consider a real-world configuration of multiple AC/DC subsystems connected by complex MTDC topologies. In addition, [20]-[22] do not consider the converter losses, AC filters and transformer losses. The assumptions made in these studies, e.g., a single VSC operation mode, a simple network topology and a lossless converter, may simplify the computational complexity, but they no longer hold in calculating the power flow of a real large-scale VSC-MTDC system connected by numerous asynchronous AC subgrids. In particular, the bus type classification of the PCC and DC interlinking buses in VSC stations becomes a difficult task due to the facts that the droop coupling variables, converter losses and network losses are unknown and that all the variables of the PCC and DC interlinking buses in VSC stations cannot be pre-specified before the power flow calculation (PFC).

Therefore, a novel comprehensive approach is developed in this paper to formulate the power flow of VSC-MTDC systems considering the cross-regional frequency regulations. The contributions of this paper are as follows: ① different operation modes of VSC stations and versatile frequency droop strategies are accommodated by the proposed approach; ② new bus types are defined to deal with the difficulty that almost all variables related to the PCC and DC interlinking buses of VSC stations are unknown before the PFC; ③ additional power flow variables, i.e., AC system frequencies and the losses of VSC stations are introduced, and the corresponding extra mismatch equations are formulated to guarantee that the number of unknown variables is equal to the number of equations; ④ the challenges are overcome by the proposed power flow approach such as the absence of slack buses in both AC and DC subsystems, and different network configurations, e.g., one AC subsystem integrated with multiple VSC stations.

The remainder of this paper is organized as follows. Section II introduces the model of VSC stations and defines novel bus types considering converter losses and frequency droop strategies. Section III investigates the power flow formulation of VSC-MTDC systems. Section IV presents the case study to demonstrate the effectiveness of the proposed approach. Finally, the conclusions are drawn in Section V.

II. MODEL OF VSC STATIONS CONSIDERING FREQUENCY DROOP STRATEGIES AND CONVERTER LOSSES

A. VSC Station Model

In the steady-state analysis, a VSC station can be represented by a controllable AC voltage source and a controllable DC current source, as shown in Fig. 1. On the AC side, the model consists of a PCC bus, a coupling transformer with a complex impedance $Z_T = R_T + jX_T$, an AC filter with a susceptance B_f , a phase reactor with an impedance $Z_c = R_c + jX_c$ [15] and ω_s is the AC frequency. On the DC side, the model consists of a DC interlinking bus B_2 and DC capacitors with a capacitance C_{DC} .

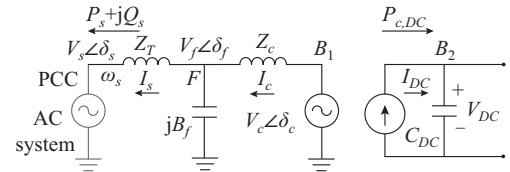


Fig. 1. Steady-state model of a VSC station.

The PCC and DC interlinking buses are directly involved in the PFC, while the internal buses F and B_1 account for the calculation of VSC station losses. The current injected into the PCC is calculated by:

$$I_s = \left(\frac{P_s + jQ_s}{V_s e^{j\delta_s}} \right)^* \quad (1)$$

where P_s and Q_s are the active power and reactive power injected into the PCC bus, respectively; and V_s and δ_s are the voltage magnitude and phase angle of the PCC bus, respectively.

The bus voltage of an AC filter V_f , the passing current of a phase reactor I_c , and the power injected into the DC interlinking bus $P_{c,DC}$ are obtained by (2)-(4), respectively.

$$V_f = \left| V_s \left| e^{j\delta_s} + Z_T I_s \right| \right| \quad (2)$$

$$I_c = I_s + jB_f V_f \quad (3)$$

$$P_{c,DC} = -(P_s + P_{cl}) \quad (4)$$

where P_{cl} is the total loss of the VSC station, which consists of two parts. The first part is the transformer and phase reactor losses P_{cl1} , and the second part is the converter loss P_{cl2} . Equations (6) and (7) indicate that the converter losses are related to the magnitude of the passing currents.

$$P_{cl} = P_{cl1} + P_{cl2} \quad (5)$$

$$P_{cl1} = R_T |I_s|^2 + R_c |I_c|^2 \quad (6)$$

$$P_{cl2} = a_{cl} + b_{cl}|I_c| + c_{cl}|I_c|^2 \quad (7)$$

where a_{cl} , b_{cl} and c_{cl} are the no-load converter losses, linear and quadratic coefficients of converter losses, respectively.

For the modular multilevel converter (MMC), there is no need to use an AC voltage filter, so the corresponding steady-state model can be simplified as (1), (4)-(7) with $I_c = I_s$.

B. Operation Modes and Frequency Droop Strategies

Figure 2 presents a typical VSC double-loop control topology [7], in which either the DC bus voltage V_{DC} or the active power injected into the PCC P_s provides a reference to the active current, and either the PCC bus voltage magnitude V_s or the injected reactive power Q_s provides a reference to the reactive current. The subscripts “mes” and “ref” represent the measured and reference values, respectively.

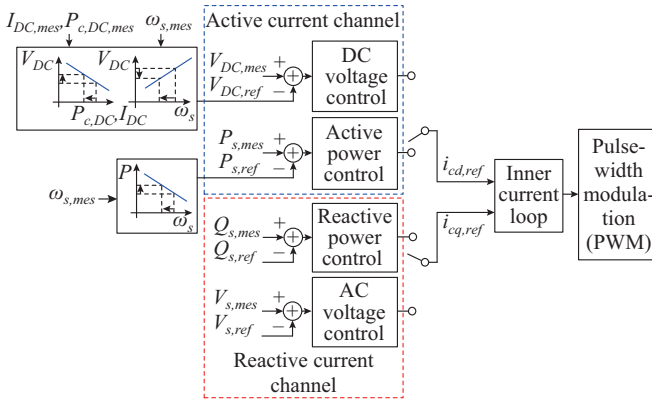


Fig. 2. Operation modes of a VSC station.

Combining different references from active and reactive current channels, different modes of VSC stations are determined such as P - V_{AC} , P - Q , V_{DC} - Q , and V_{DC} - V_{AC} . Specifically, VSC stations can also operate in a V_{AC} - F mode if they are connected to an islanded AC system or an RES power plant, as described in Fig. 3.

Based on the different operation modes of VSC stations, versatile frequency regulation strategies can be designed. In this paper, different frequency-related droop strategies are adopted to enable the units in one subsystem providing cross-

regional responses to the frequency events in other asynchronous AC subsystems. A V_{DC} - ω_s or P_s - ω_s droop strategy allows VSC stations to change the DC bus voltage or the power injected into the PCC by perceiving the frequency deviations of the connected AC subsystem with frequency events, and a ω_s - V_{DC} or ω_s - P_s droop strategy allows VSC stations to change the frequencies of AC subsystems, in which the units can provide frequency responses based on only local measurements, by perceiving the deviations of DC bus voltage or the power injected into the PCC.

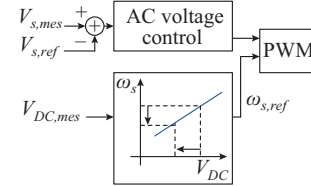


Fig. 3. V_{AC} - F operation mode of a VSC station.

C. New Bus Type Classification Considering Converter Losses and Different VSC Control Strategies

Since the converter losses and droop coupling variables of VSC stations are unknown before PFC, four variables of the PCC bus (active power P_s , reactive power Q_s , voltage magnitude $|V_s|$ and phase angle δ_s), two variables of the DC interlinking bus (active power P_{DC} and bus voltage V_{DC}), and AC frequency ω_s cannot be pre-specified, which contradict the assumption that half of the variables of traditional bus types are known before the PFC such as P - Q , P - V and slack buses in AC systems, or constant V and P buses in DC systems. Thus, new bus types need to be defined.

Taking the converter losses, different VSC operation modes and control strategies into consideration, Table I summarizes the bus type classification of the PCC and DC interlinking buses. In Cases 1-5 and 6-12, VSC stations adopt constant and droop-controlled strategies, respectively. As mentioned before, VSC stations typically operate in a V_{AC} - F mode when they are connected to an islanded AC system or an RES station, so the PCC acts as a V - δ bus in Cases 1 and 6. The bus type classification of PCC and DC interlinking buses is as follows.

TABLE I
BUS TYPE CLASSIFICATION OF PCC AND DC INTERLINKING BUSES

No.	Control strategy in active current channel	Control strategy in reactive current channel	VSC mode	PCC bus type	DC interlinking bus type
1	Constant ω_s	Constant V_s	V_{AC} - F	V - δ bus	DC tie bus
2	Constant P_s	Constant V_s	P - V_{AC}	P - V bus	DC tie bus
3	Constant P_s	Constant Q_s	P - Q	P - Q bus	DC tie bus
4	Constant V_{DC}	Constant V_s	V_{DC} - V_{AC}	P - V tie bus	V_{DC} bus
5	Constant V_{DC}	Constant Q_s	V_{DC} - Q	P - Q tie bus	V_{DC} bus
6	ω_s - V_{DC}/P_s	Constant V_s	V_{AC} - F	V - δ bus	DC tie bus
7	P_s - ω_s/V_{DC}	Constant V_s	P - V_{AC}	P - V tie bus	DC tie bus
8	P_s - ω_s/V_{DC}	Constant Q_s	P - Q	P - Q tie bus	DC tie bus
9	V_{DC} - ω_s/P_s	Constant V_s	V_{DC} - V_{AC}	P - V tie bus	DC tie bus
10	V_{DC} - ω_s/P_s	Constant Q_s	V_{DC} - Q	P - Q tie bus	DC tie bus
11	V_{DC} - P_{DC}/I_{DC}	Constant V_s	V_{DC} - V_{AC}	P - V tie bus	DC tie bus
12	V_{DC} - P_{DC}/I_{DC}	Constant Q_s	V_{DC} - Q	P - Q tie bus	DC tie bus

In Cases 1-3 and 6-12, since the power or/and voltage of the PCC bus is/are unknown, the converter losses cannot be obtained by (1)-(7), and the power injected into the DC interlinking bus is unknown before the PFC. Therefore, if the VSC stations do not adopt a constant V_{DC} control strategy, the corresponding DC bus is neither a constant P bus nor a V_{DC} bus. However, DC power mismatch equations are necessary to solve the DC power flow variables. Hence, a novel bus type has to be defined, i.e., DC tie bus, to classify the DC interlinking buses in Cases 1-3 and 6-12. The DC tie bus shares the similar power mismatch equation as the constant P bus, and the power injected into the DC interlinking bus needs to be updated during the PFC iterations.

If a VSC station operates in a constant V_{DC} mode or a droop-controlled mode (Cases 4-5 and 7-12), the corresponding PCC bus is neither a P - Q bus nor a P - V_{AC} bus because the converter losses and the active power injected into the PCC are unknown before the PFC. However, AC power mismatch equations are necessary to solve the AC power flow variables. Hence, two novel bus types, i.e., P - V tie bus and P - Q tie bus, are defined to classify the PCC bus if the VSC station adopts V_{AC} and Q_{AC} control strategies in the reactive current channel, respectively. The P - V tie bus and P - Q tie bus share the similar power mismatch equations as the P - V bus and P - Q bus, respectively, and the active power injected into the PCC bus needs to be iteratively updated during the PFC iterations.

III. COMPREHENSIVE POWER FLOW FORMULATION OF VSC-MTDC SYSTEM

A. Motivation

As described in Section II, since the output power of droop-controlled units and the power injected into the PCC or the DC voltage of droop-controlled VSC stations are coupled with AC frequencies, the frequencies of different asynchronous systems inevitably become variables. In addition, the converter losses cannot be pre-specified before PFC. To update the power injected into the PCC and the DC interlinking buses, additional state variables are introduced to represent the converter losses.

To solve these additional state variables, extra mismatch equations need to be formulated. The mismatch equations of the entire VSC-MTDC system are extended to four groups: AC power mismatch equations, DC power mismatch equations, active power-balance mismatch equations and droop-controlled mismatch equations.

B. Power Flow Formulation

A unified power flow formulation is described mathematically by a set of linear and nonlinear equations $\mathbf{f}(\mathbf{x})$ in a polar form, as shown in (8).

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}_{AC}, \mathbf{x}_{DC}, \mathbf{x}_{cl}) = [\mathbf{f}_{AC}^T \quad \mathbf{f}_{DC}^T \quad \mathbf{f}_{ACDC}^T]^T = \mathbf{0} \quad (8)$$

$$\begin{cases} \mathbf{x} = [\mathbf{x}_{AC}^T & \mathbf{x}_{DC}^T & \mathbf{x}_{cl}^T] \\ \mathbf{x}_{AC} = [\delta^T & \mathbf{V}^T & \omega_s^T]^T \\ \mathbf{x}_{DC} = V_{DC} \\ \mathbf{x}_{cl} = \mathbf{P}_{cl} \end{cases} \quad (9)$$

where $\mathbf{f}$ is the vector containing mismatch equations including AC mismatch equation $\mathbf{f}_{AC}$, DC mismatch equation $\mathbf{f}_{DC}$ and VSC station mismatch equation $\mathbf{f}_{ACDC}$; $\mathbf{x}$ is the vector containing the iterative power flow variables, which are divided into 3 parts, i.e., AC iterative variable $\mathbf{x}_{AC}$, DC iterative variable $\mathbf{x}_{DC}$ and converter iterative variable $\mathbf{x}_{cl}$; $\mathbf{x}_{AC}$ contains AC bus voltage phase angle δ , voltage magnitude $|\mathbf{V}|$ and system frequency ω_s ; $\mathbf{x}_{DC}$ contains DC bus voltage V_{DC} ; and $\mathbf{x}_{cl}$ contains converter loss $\mathbf{P}_{cl}$.

Collecting all the AC active power mismatch equation $\Delta \mathbf{P}_{AC}$, reactive power mismatch equation $\Delta \mathbf{Q}_{AC}$, DC power mismatch equation $\Delta \mathbf{P}_{DC}$, active power-balance mismatch equation $\Delta \mathbf{P}_{ACDC}$ and droop-controlled mismatch equation $\Delta \mathbf{D}_{ACDC}$, yields the detailed power flow equation in (10) expressed by a modified Jacobian matrix $\mathbf{J}$. The detailed expressions of these mismatch equations are given in the following subsections.

$$\begin{bmatrix} \Delta \mathbf{P}_{AC} \\ \Delta \mathbf{Q}_{AC} \\ \Delta \mathbf{P}_{DC} \\ \Delta \mathbf{P}_{ACDC} \\ \Delta \mathbf{D}_{ACDC} \end{bmatrix} = \begin{bmatrix} \mathbf{J}_{P\delta} & \mathbf{J}_{PV} & \mathbf{J}_{P\omega} & \mathbf{J}_{PV_{dc}} & \mathbf{J}_{PL} \\ \mathbf{J}_{Q\delta} & \mathbf{J}_{QV} & \mathbf{J}_{Q\omega} & \mathbf{J}_{QV_{dc}} & \mathbf{J}_{QL} \\ \mathbf{J}_{P_{dc}\delta} & \mathbf{J}_{P_{dc}V} & \mathbf{J}_{P_{dc}\omega} & \mathbf{J}_{P_{dc}V_{dc}} & \mathbf{J}_{P_{dc}L} \\ \mathbf{J}_{B\delta} & \mathbf{J}_{BV} & \mathbf{J}_{B\omega} & \mathbf{J}_{BV_{dc}} & \mathbf{J}_{BL} \\ \mathbf{J}_{D\delta} & \mathbf{J}_{DV} & \mathbf{J}_{D\omega} & \mathbf{J}_{DV_{dc}} & \mathbf{J}_{DL} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |\mathbf{V}|/|\mathbf{V}| \\ \Delta \omega_s \\ \Delta V_{DC} \\ \Delta \mathbf{P}_{cl} \end{bmatrix} \quad (10)$$

Equation (10) is capable of analyzing VSC-MTDC systems with arbitrary network topologies and versatile control strategies of VSC stations. If multiple VSC stations are integrated into one AC system, all except one of the corresponding connected DC bus voltages need to be taken out of the matrix in (10) to maintain an equal number of variables and equations.

1) Mismatch Equations of AC Subsystem

For a general AC bus, the power mismatch equations are:

$$\begin{cases} \Delta P_i = P_{Gi} + P_{si} - P_{Li} - P_{calc,i} & i \in \mathbf{B}_{AC} \\ \Delta Q_i = Q_{Gi} + Q_{si} - Q_{Li} - Q_{calc,i} & i \in \mathbf{B}_{AC} \end{cases} \quad (11)$$

where P_{Gi} and Q_{Gi} are the generated active power and reactive power, respectively; P_{si} and Q_{si} are the active power and reactive power injected into the PCC bus i from the connected VSC station, respectively, which are equal to zero if the bus i is not connected to a VSC station; P_{Li} and Q_{Li} are the load active power and reactive power, respectively; and $P_{calc,i}$ and $Q_{calc,i}$ are the calculated active power and reactive power at bus i , respectively, which can be obtained by:

$$\begin{cases} P_{calc,i} = |V_i| \left| \sum_{j=1}^{N_{AC}} Y_{ij}(\omega_s) \right| |V_j| \cos(\delta_i - \delta_j - \theta_{ij}(\omega_s)) \\ Q_{calc,i} = |V_i| \left| \sum_{j=1}^{N_{AC}} Y_{ij}(\omega_s) \right| |V_j| \sin(\delta_i - \delta_j - \theta_{ij}(\omega_s)) \end{cases} \quad (12)$$

where $|V_i|$ and δ_i are the voltage magnitude and phase angle of bus i , respectively; θ_{ij} and Y_{ij} are the phase of line impedance and admittance between bus i and bus j , respectively; and N_{AC} is the number of AC buses.

For VSC-MTDC systems, AC buses $\mathbf{B}_{AC}$ are classified into 6 types: P - V buses $\mathbf{B}_{PV}$, P - Q buses $\mathbf{B}_{PQ}$, V - δ buses $\mathbf{B}_{V\delta}$, AC

generator droop buses $\mathbf{B}_{ACd}$, P - V tie buses $\mathbf{B}_{PVt}$, and P - Q tie buses $\mathbf{B}_{PQt}$. The denotation $\mathbf{B}$ represents the set of buses. The first two types of buses are the same with the conventional AC power flow approach [11]. The difference between a V - δ bus and a slack bus is that the V - δ bus provides only voltage magnitude and phase angle references, but does not hold a constant system frequency. The P - V tie buses and P - Q tie buses share the similar mismatch equations with the corresponding P - V and P - Q buses, respectively. If the droop-controlled strategies are adopted, SGs, wind turbines (WTs), photovoltaics (PVs) and energy storage systems (ESSs) can be modelled as AC generator droop buses [23]-[26]. The difference between AC generator droop buses and traditional AC buses is that the four AC variables of each AC generator droop bus and the system frequency are unknown before the PFC. To solve this problem, implicit functions are used to express the generated power by power flow variables ω_s and $|V_i|$, as shown in (13). The expressions in (13) are based on the assumption of the inductive impedance of transmission lines. Other droop relations based on the resistive and complex impedance refer to [24].

$$\begin{cases} P_{Gi} = P_{Gi,0} + m_{pi}(\omega_{s0} - \omega_s) \\ Q_{Gi} = Q_{Gi,0} + m_{qi}(|V_0| - |V_i|) \end{cases} \quad (13)$$

where $|V_0|$ and ω_{s0} are the nominal voltage magnitude and frequency, respectively; $P_{Gi,0}$ and $Q_{Gi,0}$ are the nominal active power and reactive power, respectively; and m_{pi} and m_{qi} are the frequency and voltage droop coefficients, respectively.

Therefore, the mismatch equations of AC subsystems can be summarized by (14), where the detailed expression of f_{AC} is shown in (15).

$$f_{AC}(\mathbf{x}_{AC,i}) = \mathbf{0} \quad i \in \mathbf{B}_{AC} \quad (14)$$

$$f_{AC,i} = \begin{cases} P_{Gi} + P_{si} - P_{Li} - P_{calc,i} & i \in \mathbf{B}_{AC} - \mathbf{B}_{V\delta} \\ Q_{Gi} + Q_{si} - Q_{Li} - Q_{calc,i} & i \in \mathbf{B}_{PQ} + \mathbf{B}_{PQt} + \mathbf{B}_{ACd} \\ P_{Gi} - P_{Gi,0} - m_{pi}(\omega_{s0} - \omega_s) & i \in \mathbf{B}_{ACd} \\ Q_{Gi} - Q_{Gi,0} - m_{qi}(|V_0| - |V_i|) & i \in \mathbf{B}_{ACd} \end{cases} \quad (15)$$

2) Mismatch Equations of DC Subsystem

For a general DC bus, the power mismatch equation is:

$$\Delta P_{DCi} = P_{G,DCi} + P_{c,DCi} - P_{L,DCi} - P_{calc,DCi} \quad i \in \mathbf{B}_{DC} \quad (16)$$

where $P_{G,DCi}$ is the generated active power; $P_{c,DCi}$ is the active power injected into the DC interlinking bus i from the connected VSC station, which equals to 0 if the bus i is not connected with a VSC station; $P_{L,DCi}$ is the active power demand; and $P_{calc,DCi}$ is the calculated active power at the bus i that can be obtained by:

$$P_{calc,DCi} = V_{DCi} \sum_{j=1}^{N_{DC}} G_{DC,ij} V_{DCj} \quad (17)$$

where V_{DCi} , $G_{DC,ij}$ and N_{DC} are the DC voltage of the DC bus i , the conductance between the DC buses i and j , and the number of DC buses, respectively.

DC buses $\mathbf{B}_{DC}$ are classified into 4 types: V_{DC} buses $\mathbf{B}_P$, P

buses $\mathbf{B}_P$, DC tie buses $\mathbf{B}_{Pt}$, and DC generator droop buses $\mathbf{B}_{DCd}$. V_{DC} buses and P buses are the same with the conventional DC power flow approach [27]. The DC tie buses share the similar mismatch equations with P buses, where $P_{c,DCi}$ is obtained by (4). In addition, PVs and ESSs can be modelled as DC generator droop buses if they adopt the voltage droop-controlled strategies. An implicit function is used to express the generated power by DC voltage as:

$$P_{G,DCi} = P_{G,DCi,0} + m_{dpci}(V_{DCi,0} - V_{DCi}) \quad (18)$$

where $V_{DCi,0}$ and $P_{G,DCi,0}$ are the nominal DC voltage and active power, respectively; and m_{dpci} is the voltage droop coefficient.

Therefore, the mismatch equations of DC subsystems can be summarized by (19), where the detailed expression of f_{DC} is described by (20).

$$f_{DC}(\mathbf{x}_{DC,i}) = \mathbf{0} \quad i \in \mathbf{B}_{DC} \quad (19)$$

$$f_{DC,i} = \begin{cases} P_{G,DCi} + P_{c,DCi} - P_{L,DCi} - P_{calc,DCi} & i \in \mathbf{B}_{DC} - \mathbf{B}_V \\ P_{G,DCi} - P_{G,DCi,0} - m_{dpci}(V_{DCi,0} - V_{DCi}) & i \in \mathbf{B}_{DCd} \end{cases} \quad (20)$$

3) Mismatch Equations of VSC Stations

The mismatch equations of VSC stations consist of two parts: active power-balance mismatch equations and droop-controlled mismatch equations.

The active power-balance mismatch equation is shown in (21), and the corresponding detailed expression is described in (22).

$$\Delta P_{ACDC}(\mathbf{x}_{AC,i}, \mathbf{x}_{DC,i}) = \mathbf{0} \quad i \in \mathbf{B}_{VSC} \quad (21)$$

$$\Delta P_{ACDC,i} = P_{si} + P_{cLi} + P_{c,DCi} \quad (22)$$

where $\mathbf{B}_{VSC}$ is the PCC or DC interlinking buses that are connected to VSC stations; and P_{si} , P_{cLi} , $P_{c,DCi}$ are the power injected into the PCC calculated by (12), the converter losses calculated by (5), and the power injected into the DC interlinking bus calculated by (17), respectively.

The droop-controlled mismatch equations of the VSC stations adopting droop strategies are presented in (23). Different types of droop relations such as $V_{DC}-\omega_s$, $P_s-\omega_s$, ω_s-V_{DC} , ω_s-P_s , $V_{DC}-P_{DC}$ and $V_{DC}-I_{DC}$, are expressed by (24)-(29), respectively.

$$\Delta \mathbf{D}_{ACDC}(\mathbf{x}_{AC,i}, \mathbf{x}_{DC,i}) = \mathbf{0} \quad i \in \mathbf{B}_{Pt} \quad (23)$$

$$\Delta V_{Doi} = [V_{DCi,0} + k_{Vdoi}(\omega_{si} - \omega_{sl,0})] - V_{DCi} \quad (24)$$

$$\Delta P_{Doi} = [P_{sl,0} + k_{Pdoi}(\omega_{si} - \omega_{sl,0})] - P_{si} \quad (25)$$

$$\Delta \omega_{DVi} = [\omega_{sl,0} + k_{\omega Vdci}(V_{DCi} - V_{DCi,0})] - \omega_{si} \quad (26)$$

$$\Delta \omega_{DPI} = [\omega_{sl,0} + k_{\omega Pdi}(P_{si} - P_{sl,0})] - \omega_{si} \quad (27)$$

$$\Delta V_{DPI} = [V_{DCi,0} + k_{Vdcdci}(P_{DCi} - P_{DCi,0})] - V_{DCi} \quad (28)$$

$$\Delta V_{Dli} = [V_{DCi,0} + k_{Vdldci}(I_{DCi} - I_{DCi,0})] - V_{DCi} \quad (29)$$

where $\mathbf{B}_{Pt}$ is the PCC buses or the DC interlinking buses that

are connected to the VSC stations adopting droop strategies, $\mathbf{B}_{P_i} \subseteq \mathbf{B}_{VSC}$; V_{DCi} , P_{si} , ω_{si} are the DC bus voltage, the power injected into the PCC and AC system frequency at the interlinking bus i , respectively; $V_{DCi,0}$, $P_{si,0}$, $\omega_{si,0}$ are the nominal DC bus voltage, the power injected into the PCC and the AC system frequency at the interlinking bus i , respectively; and $k_{V_{DC} \omega_{si}}$, $k_{P_{si} \omega_{si}}$, $k_{\omega_{si} V_{DC}}$, $k_{\omega_{si} P_{si}}$, $k_{V_{DC} P_{si}}$, $k_{V_{DC} I_{DC}}$ are the corresponding coefficients of V_{DC} - ω_{si} , P_{si} - ω_{si} , ω_{si} - V_{DC} , ω_{si} - P_{si} , V_{DC} - P_{si} and V_{DC} - I_{DC} droop relations, respectively. In addition, VSC stations can also adopt nonlinear droop relations between V_{DC} or P_{si} and ω_{si} , P_{DC} or I_{DC} [17]. Equation (23) can be generalized to represent nonlinear droop-controlled mismatch equations.

Therefore, the mismatch equations of VSC stations are summarized by (30), where the detailed expression of f_{ACDC} is described by (31).

$$f_{ACDC}(\mathbf{x}_{AC,i}, \mathbf{x}_{DC,i}) = \mathbf{0} \quad i \in \mathbf{B}_{VSC} \quad (30)$$

$$f_{ACDC,i} = \begin{cases} \Delta P_{ACDC,i} & i \in \mathbf{B}_{VSC} \\ \Delta D_{ACDC,i} & i \in \mathbf{B}_{P_i} \end{cases} \quad (31)$$

C. Summary

The flow chart of the proposed power flow formulation in VSC-MTDC systems is depicted in Fig. 4, where the power flow of VSC-MTDC systems without droop-controlled strategies can be calculated by a unified approach described in [16].

The proposed power flow formulation is solved by the numerical methods such as a globally convergent trust region method [20], [25], [28]. Note that if the variables of some VSC stations exceed the limits, the corresponding VSC stations need to switch to the constant operation modes [17].

The variables and mismatch equations for solving the power flow of VSC-MTDC systems are summarized in Table II, where N_P , N_{P_i} and N_{DCd} are the numbers of P , DC tie and DC droop buses, respectively; N_{P_i} , $N_{P_{PQ}}$, $N_{P_{PV}}$, $N_{P_{PQ}}$, N_{ACd} are the

numbers of P - V , P - Q , P - V tie, P - Q tie and AC droop buses, respectively; and N_{VSC} and N_{INT} are the numbers of VSC stations and asynchronous AC systems, respectively.

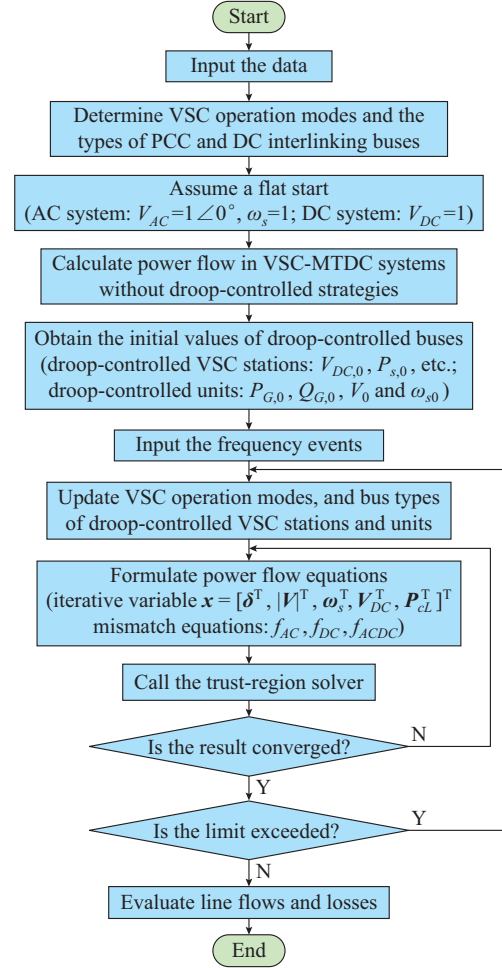


Fig. 4. Flow chart of the proposed power flow formulation of VSC-MTDC systems with cross-regional frequency responses.

TABLE II
SUMMARY OF POWER FLOW FORMULATION IN VSC-MTDC SYSTEMS

Subsystem	Bus type	No. of buses	Specified quantity	Unknown variable	Iterative variable	No. of equations
DC grid	P	N_P	P_i	V_i	P_i	N_P
	DC tie	N_{P_i}		V_i, P_i	V_i	N_{P_i}
	DC droop	N_{DCd}		V_i, P_i	V_i	N_{DCd}
AC grid	V - δ	$N_{V\delta}$	$\delta_i, V_i $	P_i, Q_i		0
	P - V	N_{PV}	$P_i, V_i $	δ_i, Q_i	δ_i	N_{PV}
	P - Q	N_{PQ}	P_i, Q_i	$\delta_i, V_i $	$\delta_i, V_i $	$2N_{PQ}$
	P - V tie	$N_{P_{PV}}$	$P_i, V_i $	δ_i, Q_i	δ_i	$N_{P_{PV}}$
	P - Q tie	$N_{P_{PQ}}$	P_i, Q_i	$\delta_i, V_i $	$\delta_i, V_i $	$2N_{P_{PQ}}$
	AC droop	N_{ACd}		δ_i, V_i , P_i, Q_i	$\delta_i, V_i $	$2N_{ACd}$
VSC station				P_{cli}	P_{cli}	N_{VSC}
				ω_{si}	ω_{si}	N_{INT}

IV. CASE STUDY

A. System Configurations and Parameters

A 6-terminal VSC-HVDC test system is applied in this pa-

per, as described in Fig. 5. The AC grids 1 and 5 (abbreviated to AC1 and AC5) are load centers, and AC2-AC4 are integrated with wind farms (WFs) and ESS stations. The test system has the following characteristics: ① VSC operates in numerous operation modes such as P - V_{AC} (VSC station 1), P -

Q (WFs and ESSs), V_{DC} - Q (VSC station 5), V_{DC} - V_{AC} (VSC stations 1 and 4) and V_{AC} - F (VSC stations 2, 3, 6); ② different subsystems present diversified grid configurations such as the point-to-point integration of RESs (AC2-AC4), and one important load center integrated with multiple VSC stations (AC5); ③ slack buses are absent in both AC and DC subsystems. The design of the test system aims at reducing the frequency deviations of AC1 and AC5 by the cross-regional frequency responses of the units in AC2-AC4. Larger frequency deviations of AC2-AC4 could be acceptable because there do not exist important loads.

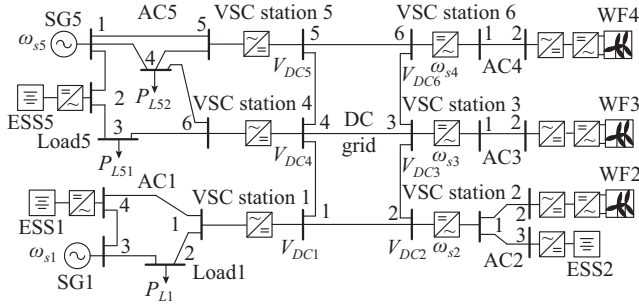


Fig. 5. 6-terminal VSC-HVDC test system.

The parameters of the test system can be found in Table III and Table IV. The resistance and reactance of AC cables are $R_{AC} = 0.01 \Omega/\text{km}$ and $X_{AC} = 0.04 \Omega/\text{km}$ (nominal frequency is 50 Hz), respectively. The resistance of DC cables R_{DC} is equal to $0.02 \Omega/\text{km}$. For VSC stations, $Z_c = 0.0001 + j0.15$, $Z_r = 0.001 + j0.1$. VSC stations are represented by an average model, and their AC filters are ignored. The base capacity of the entire system S_{base} is equal to 300 MVA. The base AC and DC voltages are 220 kV and 400 kV, respectively. For each droop generator bus connected to SGs and WTs, $m_p = 20$, $m_q = 0$.

The seven cases listed in Table V are studied. Case 1 provides a benchmark that the units and VSC stations do not provide any frequency responses, and AC frequencies are set to the nominal values. In Cases 2.1, 2.2, 3.1, 3.2, 4.1 and 4.2, different droop strategies are adopted. In AC1, VSC station 1 adopts the V_{DC} - ω_s (Cases 2.1 and 2.2), P_s - ω_s (Cases

3.1 and 3.2) or V_{DC} - P_{DC} (Cases 4.1 and 4.2) droop strategy, and SG1 adopts the P_m - ω_s droop strategy, where P_m is the input mechanical power of SGs. In AC2-AC4, VSC stations adopt the ω_s - V_{DC} droop strategy and WFs adopt the P_e - ω_s droop strategy, where P_e is the output electromagnetic power. In AC5, VSC stations adopt the V_{DC} - ω_s droop strategy, and SG5 adopts the P_m - ω_s droop strategy. Different locations of load changes are also considered. In Cases 2.1, 3.1 and 4.1, the load power is increased by 0.2 p.u. at AC5 bus 3 ($\Delta P_{L51} = 0.2$ p.u.). In Cases 2.2, 3.2 and 4.2, the load power is increased by 0.2 p.u. at AC1 bus 2 ($\Delta P_{L1} = 0.2$ p.u.). In addition, Case 1 provides the initial references to all droop-controlled units and VSC stations in Cases 2.1-4.2.

TABLE III
LINE DATA OF TEST SYSTEM

Subsystem	Line	Length (km)	Subsystem	Line	Length (km)
AC1	1-2	50.0	AC2	1-2	50.0
	2-3	15.0		1-3	35.0
	3-4	30.0	AC3	1-2	40.0
	1-4	25.0	AC4	1-2	45.0
AC5	1-2	35.0	DC	1-2	184.4
	2-3	75.0		2-3	78.3
	3-6	20.0		3-4	131.1
	1-4	60.0		1-4	101.0
	1-5	35.0		4-5	119.0
	4-5	50.0		5-6	210.0
	4-6	80.0		3-6	195.2

TABLE IV
DROOP SETTINGS OF VSC STATIONS

VSC	Droop	Coefficient
1	V_{DC1} - ω_{s1} , P_{s1} - ω_{s1} or V_{DC1} - P_{DC1}	$k_{V_{dc\omega}} = 1.5$, $k_{P_{\omega}} = -1.5$ or $k_{V_{dc}P_{dc}} = -1.5$
2	ω_{s2} - V_{DC2}	$k_{\omega V_{dc}} = 1.2$
3	ω_{s3} - V_{DC3}	$k_{\omega V_{dc}} = 1.2$
4	V_{DC4} - ω_{s4}	$k_{V_{dc\omega}} = 1.5$
5	V_{DC5} - ω_{s5}	$k_{V_{dc\omega}} = 1.5$
6	ω_{s4} - V_{DC6}	$k_{\omega V_{dc}} = 1.2$

TABLE V
STUDY CASES

Case No.	Case
1.0	No droop; AC1 bus 2: $P_{L1} = 0.8$ p.u.; AC5 bus 3: $P_{L51} = 1$ p.u.; AC5 bus 4: $P_{L52} = 2$ p.u.
2.1	VSC station 1: V_{DC} - ω_s droop; AC5 bus 3: $\Delta P_{L51} = 0.2$ p.u.
2.2	VSC station 1: V_{DC} - ω_s droop; AC1 bus 2: $\Delta P_{L1} = 0.2$ p.u.
3.1	VSC station 1: P_s - ω_s droop; AC5 bus 3: $\Delta P_{L51} = 0.2$ p.u.
3.2	VSC station 1: P_s - ω_s droop; AC1 bus 2: $\Delta P_{L1} = 0.2$ p.u.
4.1	VSC station 1: V_{DC} - P_{DC} droop; AC5 bus 3: $\Delta P_{L51} = 0.2$ p.u.
4.2	VSC station 1: V_{DC} - P_{DC} droop; AC1 bus 2: $\Delta P_{L1} = 0.2$ p.u.

B. Validation of Proposed Power Flow Approach

According to the bus type classification introduced in Section II, the types of the PCC buses that are connected to the

VSC stations 1-6 are P - V tie, V - δ , V - δ , P - V tie, P - Q tie and V - δ , respectively. Taking Case 2.1 as an example, Table VI compares the results of the proposed power flow approach

and the steady-state results obtained by the time-domain simulation. In Table VI, the maximum errors of voltage magnitude, phase angle and system frequency are less than 0.0017,

0.0007 and 0.00006, respectively. The closely-matched results validate the effectiveness of the proposed approach.

TABLE VI
POWER FLOW SOLUTION OF CASE 2.1

Subsystem	Bus type	Calculation results by proposed approach					Simulation results by Matlab/Simulink				
		$ V $	δ	P_{cal}	Q_{cal}	ω_s	$ V $	δ	P_{cal}	Q_{cal}	ω_s
AC1	P - V tie	1.000	0	0.132	-0.023	0.9985	1.000	0	0.124	-0.019	0.9985
	P - Q	0.998	-0.020	-0.800	0		0.998	-0.019	-0.800	0	
	Droop	1.000	0.005	0.689	-0.119		1.000	0.005	0.698	-0.121	
	P - Q	1.001	0.002	0	0		1.000	0.002	0	0	
AC2	V - δ	1.000	0	-1.020	-0.016	0.9977	1.000	0	-1.019	-0.016	0.9977
	Droop	1.019	0.125	1.047	0		1.019	0.125	1.046	0	
	P - Q	1.001	0	0	0		1.001	0	0	0	
AC3	V - δ	1.000	0	-1.032	-0.018	0.9975	1.000	0	-1.032	-0.018	0.9976
	Droop	1.016	0.101	1.049	0		1.016	0.101	1.049	0	
AC4	V - δ	1.000	0	-1.029	-0.008	0.9976	1.000	0	-1.029	-0.008	0.9976
	Droop	1.018	0.113	1.048	0		1.018	0.113	1.048	0	
AC5	Droop	1.000	0	0.382	-0.035	0.9982	1.000	0	0.383	-0.032	0.9982
	P - Q	1.000	-0.010	0	0		1.000	-0.010	0	0	
	P - Q	0.995	-0.029	-1.200	0		0.995	-0.029	-1.200	0	
	P - Q	0.986	-0.085	-2.000	0		0.986	-0.085	-2.000	0	
	P - Q tie	0.999	0.028	1.226	0		0.999	0.028	1.225	0	
	P - V tie	1.000	0.026	1.678	0.044		1.000	0.026	1.678	0.042	
DC	DC Tie	0.998		-0.132			0.998		-0.124		
	DC Tie	1.006		1.019			1.008		1.018		
	DC Tie	1.005		1.031			1.006		1.030		
	DC Tie	0.994		-1.681			0.993		-1.681		
	DC Tie	0.992		-1.227			0.991		-1.227		
	DC Tie	1.005		1.028			1.007		1.028		

C. Impact of Droop Coefficients and Relations in VSC Stations on Cross-Regional Frequency Regulation

1) Droop Coefficients of VSC Stations

To investigate the impact of droop-controlled VSC stations on the power flow and load allocation in VSC-MTDC systems, different droop coefficients of ω_s - V_{DC} (VSC stations 2, 3 and 6) and V_{DC} - ω_s (VSC stations 1, 4 and 5) are compared in Figs. 6-9. With the increase of $k_{\omega V_{dc}}$ and $k_{V_{dc}\omega_s}$, the system frequency deviations of AC1 and AC5 are reduced, and those of AC2-AC4 are increased as shown in Fig. 6 and Fig. 8. Also, the power allocation of SGs in AC1 and AC5 is transferred to WFs in AC2-AC4, as shown in Fig. 7 and Fig. 9.

In Fig. 8, if $k_{V_{dc}\omega_s}$ is set to zero, the units in AC1-AC4 cannot perceive the frequency event occurred in AC5, and only SG5 provides the frequency responses, resulting in a large steady-state frequency deviation of AC5. Thus, a larger $k_{V_{dc}\omega_s}$ may facilitate other AC subsystems providing cross-regional frequency responses. With the increase of $k_{V_{dc}\omega_s}$ of VSC stations 1, 4 and 5, the frequencies of AC2-AC4 are decreased, and the generated power of SG1 is first increased to provide the frequency response to the frequency event in AC5, and then decreased due to the increasing output power of WF2-WF4.

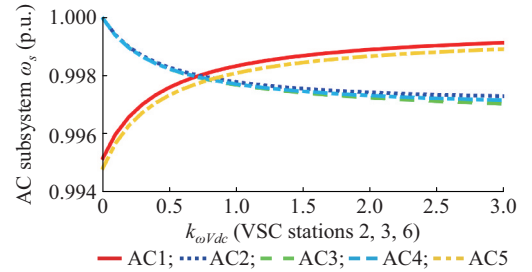


Fig. 6. Frequency of AC subsystems with different $k_{\omega V_{dc}}$ in Case 2.1 ($\Delta P_{L51} = 0.2$ p.u.).

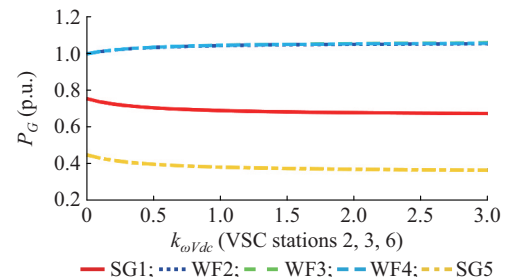


Fig. 7. Generated power of SGs and WFs with different $k_{\omega V_{dc}}$ in Case 2.1 ($\Delta P_{L51} = 0.2$ p.u.).

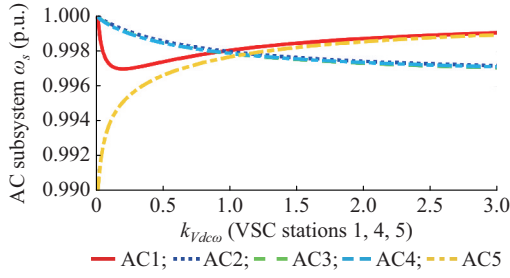


Fig. 8. Frequency of AC subsystems with different k_{Vdco} in Case 2.1 ($\Delta P_{L1} = 0.2$ p.u.).

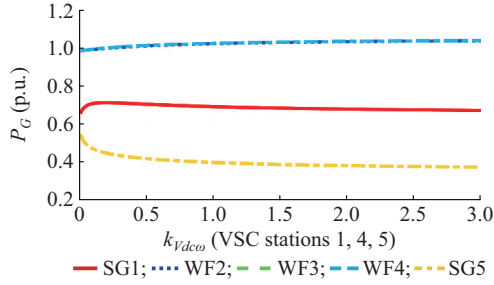


Fig. 9. Generated power of SGs and WFs with different k_{Vdco} in Case 2.1 ($\Delta P_{L1} = 0.2$ p.u.).

2) Droop Relations of VSC Stations

VSC stations adopting different droop strategies may present the distinctive cross-regional frequency regulation capabilities in VSC-MTDC systems.

The VSC station adopting a $V_{DC}-\omega_s$ droop strategy takes both AC and DC variables into account. The units in the corresponding connected AC subsystem can provide responses to the frequency events in other asynchronous AC subsystems, and the units in other AC subsystems can also participate in the frequency regulations of this AC subsystem. In Case 2.2, with the increase of k_{Vdco} of VSC station 1, the units in other subsystems increase their output power and provide the responses to the frequency event in AC1, as shown in Figs. 10 and 11.

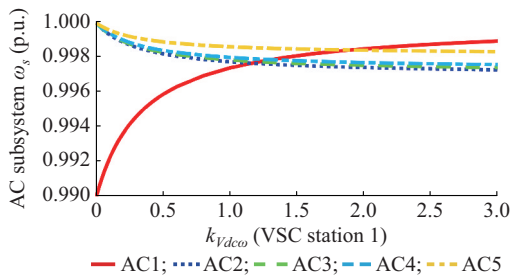


Fig. 10. Frequency of AC subsystems with different k_{Vdco} in Case 2.2 ($\Delta P_{L1} = 0.2$ p.u.).

The VSC station adopting a $P_s-\omega_s$ droop strategy takes only AC variables into account. The units in the corresponding connected AC subsystem cannot respond to the frequency events in other AC subsystems, but the units in other AC subsystems are capable of providing frequency responses to this subsystem. In Cases 3.1 and 3.2, the VSC station 1 adopts a $P_s-\omega_s$ droop strategy. In Case 3.1, SG1 does not provide any response to the frequency event in AC5 no matter what value $k_{P\omega}$ of the VSC station 1 is taken, as shown in Fig. 12. However, in Case 3.2, with the increase of $|k_{P\omega}|$ of the VSC station 1, the units in other subsystems increase their output power and provide responses to the frequency event in the AC1, as shown in Fig. 13 and Fig. 14.

vide any response to the frequency event in AC5 no matter what value $k_{P\omega}$ of the VSC station 1 is taken, as shown in Fig. 12. However, in Case 3.2, with the increase of $|k_{P\omega}|$ of the VSC station 1, the units in other subsystems increase their output power and provide responses to the frequency event in the AC1, as shown in Fig. 13 and Fig. 14.

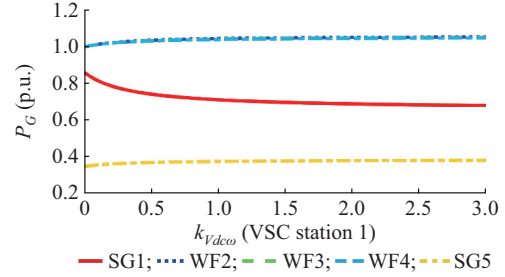


Fig. 11. Generated power of SGs and WFs with different k_{Vdco} in Case 2.2 ($\Delta P_{L1} = 0.2$ p.u.).

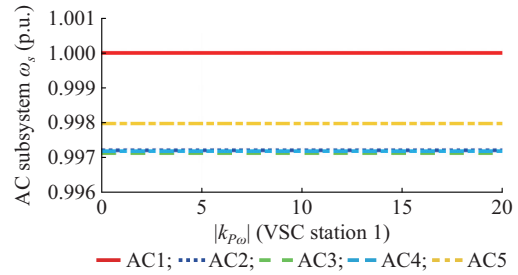


Fig. 12. Frequency of AC subsystems with different $k_{P\omega}$ in Case 3.1 ($\Delta P_{L1} = 0.2$ p.u.).

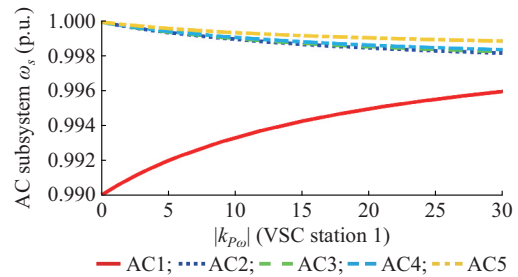


Fig. 13. Frequency of AC subsystems with different $k_{P\omega}$ in Case 3.2 ($\Delta P_{L1} = 0.2$ p.u.).

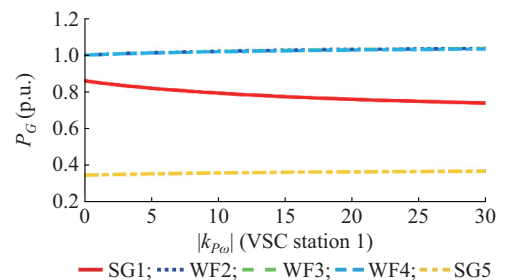


Fig. 14. Generated power of SGs and WFs with different $k_{P\omega}$ in Case 3.2 ($\Delta P_{L1} = 0.2$ p.u.).

The VSC station adopting a $V_{DC}-P_{DC}$ droop strategy takes only DC variables into account. In Cases 4.1 and 4.2, the VSC station 1 adopts a $V_{DC}-P_{DC}$ droop strategy. In Case 4.1, a frequency event occurs in AC5. With the increase of $|k_{Vdcd}|$

of VSC station 1, only the frequency of the AC1 is restored, but the frequency of the AC5 is deteriorated even though the WFs in the AC2-AC4 increase their output power, as shown in Fig. 15 and Fig. 16. In particular, the V_{DC} - P_{DC} droop strategy only focuses the power allocation in DC systems and ignores the frequency performances of the AC subsystems. In Case 4.2, the units in other subsystems do not provide any response to the frequency event of the AC1 no matter what value k_{VdcPdc} is taken, as shown in Fig. 17.

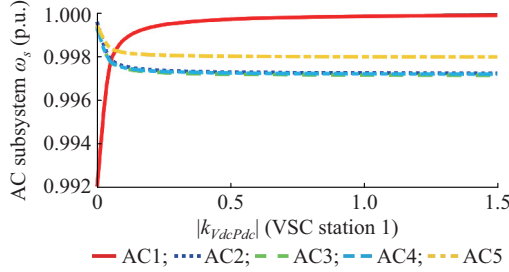


Fig. 15. Frequency of AC subsystems with different k_{VdcPdc} in Case 4.1 ($\Delta P_{L1} = 0.2$ p.u.).

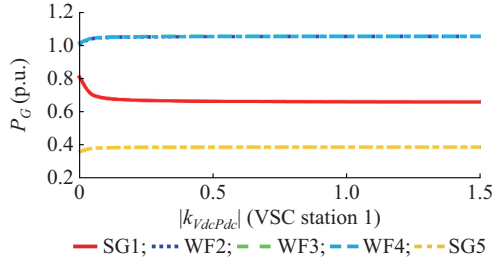


Fig. 16. Generated power of SGs and WFs with different k_{VdcPdc} in Case 4.1 ($\Delta P_{L1} = 0.2$ p.u.).

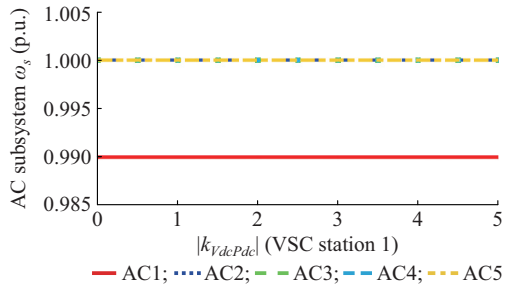


Fig. 17. Frequency of AC subsystems with different k_{VdcPdc} in Case 4.2 ($\Delta P_{L1} = 0.2$ p.u.).

Compared to the P_s - ω_s droop strategy in Fig. 13 and the V_{DC} - P_{DC} droop strategy in Fig. 17, the V_{DC} - ω_s droop strategy adopted by VSC stations in Fig. 10 enhances the cross-regional frequency regulation performance. Therefore, the proposed power flow formulation in this paper may further facilitate the determination of the droop relations and droop coefficients of VSC stations according to the different requirements of the planning and operation in VSC-MTDC systems.

V. CONCLUSION

A central challenge of analyzing VSC-MTDC systems lies in the formulation and computation of power flow with the coupling relations between AC frequencies, DC voltages, the

power injected into the PCC of VSC stations and the output power of the units. This paper revisits the conventional assumptions associated with power flow problems, and introduces a suitable approach of frequency-dependent power flow formulations. The proposed approach can be generalized to analyze the steady states of VSC-MTDC systems with arbitrary system topologies, operation modes and the frequency droop strategies adopted by VSC stations.

The time-domain simulation results obtained by MATLAB/Simulink validate that the proposed approach can precisely analyze the power flow under different operation conditions. Simulation results reveal that the V_{DC} - ω_s droop strategy adopted by VSC stations presents the best cross-regional frequency regulation performance compared with the P_s - ω_s and V_{DC} - P_{DC} droop strategies. The results also illustrate that larger droop coefficients of V_{DC} - ω_s and P_s - ω_s may enhance the cross-regional frequency regulation performance. In contrast, the V_{DC} - P_{DC} droop strategy may deteriorate the cross-regional frequency regulation in VSC-MTDC systems because it focuses only on the power allocation of DC systems, regardless of the frequency performances of the AC systems.

The proposed model can further aid the design, operation and planning of VSC-MTDC and hybrid LCC-VSC HVDC systems. The operation modes and droop coefficients of VSC stations can be optimized to trade off the economic dispatch and operation stability of the entire system. In addition, improved trust-region and Newton-Raphson methods can be investigated to solve the power flow problem of larger systems with complicated topologies.

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