

Optimal Location Allocation Strategy of Gas-fired Unit in Transmission Network

Jiale Fan and Xiaoyang Tong

Abstract—The gas-fired generation has recently become an important power source for power systems. The increasing integration of gas-fired units (GFUs) brings a problem of location allocation strategy for power system planners. This paper proposes a bi-level maximum-minimum optimal placement model of GFUs to improve the static voltage stability in the transmission network. In the first stage, the locations of installed GFUs are optimized to improve the static voltage stability margin. The optimal installed capacity of GFUs is determined to minimize the operation costs and power losses in the second stage. The proposed mixed-integer nonlinear programming (MINLP) model is solved by second-order cone programming relaxations. Numerical results in the IEEE 118-bus test system demonstrate the effectiveness of the proposed method and the static voltage stability can be improved.

Index Terms—Integrated energy systems, gas-fired unit (GFU), static voltage stability, optimal power flow.

I. INTRODUCTION

THE gas-fired generation has become a significant power source for power systems due to its low cost and environmental benefits. It was reported that gas-fired generation accounted for nearly 42% of the total installed generation capacity in the United States in 2015 [1]. The growing capacity of installed gas-fired units (GFUs) and the development of power-to-gas technologies increase the closeness of the interactions between power systems and natural gas systems. Some researchers have focused on the coordinated modeling, optimal controlling, and planning problem of the power-gas integrated energy (IE) system [2]–[7].

In practice, there are mainly two types of GFUs applied to power systems: ① GFUs with low operation cost and sufficient power supplies, which are mainly used in the transmission system to provide power adequacy [8]–[10]; ② GFUs with small capacities but quick ramping abilities, which are widely used in the distribution system to mitigate the fluctuations of intermittent renewable resources [11] and reverse power flows [12]. In summary, GFUs are regarded as the most important coupling component of the IE system, and it is of vital importance to optimize their placement.

However, most researchers did not consider GFU placement in IE systems and only coupled them by energy conversion constraints in the formulations [4], [5], [9], [10]. Furthermore, although the optimal location allocation of GFUs has become an important issue for power system planners with the widespread integration of GFUs, it has not been fully studied in the existing literature. Thus, the goal of this study was to find the optimal location allocation results of GFUs considering static voltage stability.

The optimal location allocation problem of power system components is also known as the optimal siting and sizing problem. It has attracted widespread attention in the literature. In [13], a decomposable stochastic program was implemented to optimize the location allocation of both line switches and compensators. In [14], a two-stage robust optimization model was proposed to find the optimal siting and sizing of renewable resources.

Generally speaking, the optimal locations of the system components are represented by binary variables, and the optimization model usually contains nonlinear operation constraints. The optimal location allocation problem is inherently a mixed-integer nonlinear programming (MINLP) model, which is difficult to solve. To overcome such adversities, the most common approaches utilize decomposition methods, the linearized approach, or just ignore the nonlinear constraints. For instance, [15] applied Benders decomposition and DC power flow model to linearize the planning problem. However, the DC model could not deal with voltage or reactive power variables [16].

In addition, most of the optimal planning models aim to minimize operation costs [13] and power losses [17], [18], but rarely consider steady-state stability concerns. However, the voltage instability risk increases owing to the increasing power demands, integration of intermittent renewable generations, and the retirement of large synchronous generators such as coal-based ones.

One of the widely utilized criteria for static voltage stability analysis is the loadability margin (LM) index [19]. This index quantifies the distance between the current operation point (COP) and the critical voltage unstable operation point. The relevant definitions and investigations can be found in [20]–[22]. It is worth mentioning that the LM index can be easily incorporated into the optimal power flow (OPF) problem, and the voltage stability constrained OPF (VSC-OPF) approach can yield the same results as the continuation power flow (CPF) method [19], [23], [24]. Accord-

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ingly, the VSC-OPF approach has been regarded as an effective method to assess the static voltage stability of power systems. In this study, we incorporate the voltage stability constraint by using the LM index in the proposed model.

Researchers tried to improve the stability margin by optimizing the location and allocation of distributed generation (DG) units in [25] such as wind power and solar units. However, power system operators could not ensure that the renewable sources operated at the desired level owing to the presence of large uncertainties. Moreover, the locations and available capacity of solar and wind energy based DGs are critically limited by geographical conditions. These factors make it difficult to improve the static voltage stability of power systems by optimizing the capacity and location of renewable DGs with current technologies. In contrast, GFUs are dispatchable and less dependent on the natural environment. Thus, GFUs can be planned in a way that benefits power system operations once the gas supply is sufficient and natural gas networks are geographically close to the power grids.

In summary, the previous studies that focused on the optimal location allocation problem identified the following challenges:

- 1) The nonlinearity of the optimal location allocation problem causes computational difficulties.
- 2) The optimization model rarely considers the power system stability constraints.
- 3) Non-dispatchable resources do not benefit power systems.

To address the aforementioned challenges, a bi-level maximum-minimum model is proposed to optimize the placement strategy of GFUs. The contributions of this study are as follows:

1) A bi-level maximum-minimum optimization model is proposed to obtain the optimal placement strategy of GFUs. This model aims to find the optimal location of GFUs while improving the static voltage stability margin, and the optimal allocation of GFUs while decreasing the operation costs and power losses. We mainly focus on the static voltage stability using the LM index. The dynamic voltage stability of the transient stage of power systems is not discussed.

2) We resort to convex relaxations to transform the original MINLP model into a mixed-integer second-order cone programming (MISOCP) model. Extensive simulations on various standard test systems reveal the superiority of computational efficiency of the proposed method and its capability to find high-quality feasible solutions.

3) The impacts of different GFU penetration ratios and the minimum desired value of LM on the proposed approach are quantitatively analyzed through extensive cases.

The remainder of this paper is organized as follows. The basic structure and major concerns are addressed in Section II with the detailed formulations. Section III introduces the solution methodology and the evaluation method. Simulation results are presented and analyzed in Section IV. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

A. Structure of Proposed Bi-level Optimization Model

It is acknowledged that the LM index is mainly determined by two points of a power system, i.e., the COP and the loadability limit point (LLP). Both points represent a specific operation point during the operation of a power system. To better represent the improvement of the LM index by the proposed method, the proposed optimization model only considers the load data of system at the current time point instead of the long-term optimization in an annual unit. We assume that the total installed capacity of GFUs is deterministic, which means that if the capacity of a single gas turbine is fixed, the total number of installed GFUs is certain as well. Moreover, as the purpose of this study is to optimize the placement of GFUs in a manner that benefits the power system, the investment costs of gas turbines are not considered during the planning stage.

The overall structure of the proposed approach is given in Fig. 1. Differing from most published studies which optimize the location allocation of power facilities simultaneously, we divide this optimization problem into two parts: ① an upper-level optimization problem, which is solved by improving the static voltage stability by determining the optimum installed buses x_i of GFUs; ② a lower-level problem which is solved by decreasing the operation costs and power losses by determining the optimum installed capacity of GFUs, i.e., installed number variable y_i multiplies the capacity of a single gas turbine. Since the power flow constraints are essential and fundamental for the power system planning problems, we formulate the power flow equations in both two stages.

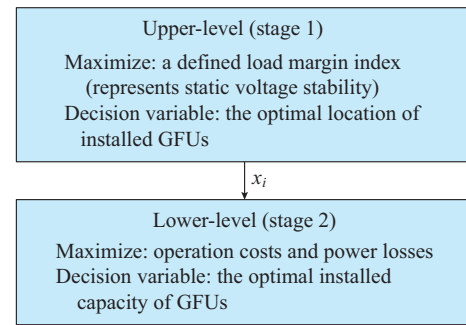


Fig. 1. Overall structure of proposed approach.

The major reason to utilize a bi-level optimization model is that determining the integer decision variables x_i and y_i in a single-level optimization problem brings computational difficulties in practice. Although the variables x_i and y_i can be integrated into a new integer variable, the system loads will increase to their maximum critical values during the simulation to find the maximum loadability of the system λ^{LLP} , leading to the improvements in the demands for active power generations including the power generation of GFUs. Thus, the optimal allocation of GFUs may be larger than the planned under normal load conditions. Moreover, choosing the voltage stability as the upper-level objective function en-

courages the selection of buses that are more sensitive to the load margin.

B. Stage 1: Optimizing Locations of GFUs to Improve Static Voltage Stability

1) Objective Function

The static voltage stability of power systems can be analyzed by the P - V curve of a certain bus. As shown in Fig. 2, the x-axis represents the loading scaling factor λ , which is the ratio of the power load to the base power load. The LMs (LM1 and LM2) measure the distance from the value of λ at the COP (λ^{COP}) to the value of λ at the LLP (λ^{LLP1} , λ^{LLP2}). The LM index is widely utilized to evaluate static voltage stability. Owing to the stable real power injection of GFUs, the voltage at the COP will increase, and the maximum loadability increases from λ^{LLP1} to λ^{LLP2} . It is apparent that greater integration of GFUs means a larger λ^{LLP} for the same power system.

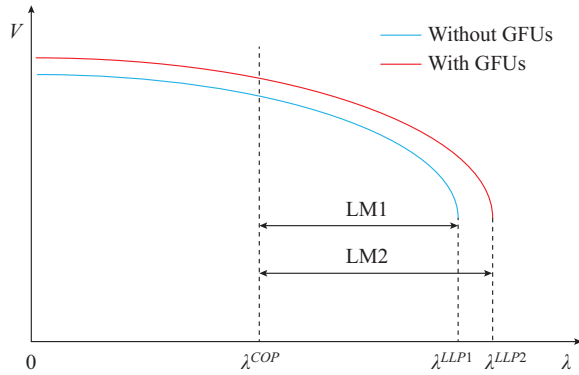


Fig. 2. Impact of GFUs on maximum loadability and voltage stability margin.

Most of the existing studies assume that each load bus has the same λ , which means all the load buses have the same direction of load increase. In other words, when the power system reaches the LLP, all the PQ buses have the same value of λ^{LLP} . However, this typically does not hold true in practice, as the load growth directions and amplitudes for the PQ buses locating in different areas exhibit different patterns. To address that, we assume that each load bus has its individual maximum loadability λ_i^{LLP} . Thus, for the entire power system, we have $\lambda^{LLP} = [\lambda_1^{LLP}, \lambda_2^{LLP}, \dots, \lambda_n^{LLP}]^T$, where n is the number of power system buses. Let $\mathbf{P}^{L0} = [P_1^{L0}, P_2^{L0}, \dots, P_n^{L0}]^T$ represent the base load vector. To improve the LM of the entire system, a modified maximum loadability index λ^{index} is proposed as follows:

$$\lambda^{index} = \frac{(\lambda^{LLP})^T \mathbf{P}^{L0}}{\sum_{i=1}^n P_i^{L0}} \quad (1)$$

The proposed index λ^{index} is decided by each λ_i^{LLP} of the PQ buses and total base power loads, while the traditional one assumes that λ_i^{LLP} is the same for all the PQ buses. This formulation ensures that the load bus with the largest load distribution factor has the largest weight when maximizing

λ^{index} . The rationale behind this is that the static voltage stability margin will be increased by improving the voltages on the buses that have large power demand.

Finally, the objective function for stage 1 is formulated to improve the static voltage stability margin:

$$\max \lambda^{index} \quad (2)$$

2) Constraints

The constraints in the stage 1 are mainly related to power flow equations as well as physical limits. Considering that the nonlinear AC power flow equations bring computational difficulties, and that the DC power flow model can not be utilized in the static voltage stability studies as it ignores the voltage variable of power systems, we adopt the convex relaxation approach addressed in [27]-[29]. The superiority of the convex relaxation formulation over other formulations was depicted in [27]. In particular, researchers in [29] recommended that, for mesh networks, one should either use the chordal relaxation or the second-order cone programming (SOCP) relaxation, trading off tightness and the required computational effort. For this reason, we adopt the SOCP relaxation.

1) Let N_b denote the set of power system nodes, L denote the set of branches, l denote the number of branches. Assuming that $\forall i \in N_b, \forall (i, j) \in L$, the power flow constraints in a rectangular coordinate system can be represented as:

$$|V_i|^2 = e_i^2 + f_i^2 = C(i, i) \quad (3)$$

$$|V_i| |V_j| \cos \theta_{ij} = e_i e_j + f_i f_j = C(i, j) \quad (4)$$

$$|V_i| |V_j| \sin \theta_{ij} = e_i f_j - e_j f_i = S(i, j) \quad (5)$$

$$C(i, j) = C(j, i) \quad (6)$$

$$S(i, j) = -S(j, i) \quad (7)$$

$$C^2(i, j) + S^2(i, j) = C(i, i)C(j, j) \quad (8)$$

where V_i is the voltage variable; e_i and f_i are the real part and imaginary part of the voltage vector, respectively; θ_{ij} is the angular phase difference between bus i and bus j ; $C(i, j)$ is the element of matrix $\mathbf{C}$ with the dimension of $n \times n$ at COP; and $S(i, j)$ is the element of matrix $\mathbf{S}$ with the dimension of $l \times 2$ at COP. Applying the SOCP relaxation to (8) yields:

$$C^2(i, j) + S^2(i, j) + \left(\frac{C(i, i) - C(j, j)}{2} \right)^2 \leq \left(\frac{C(i, i) + C(j, j)}{2} \right)^2 \quad (9)$$

Then, the active power flow for branch (i, j) can be expressed as:

$$\begin{aligned} P_{ij} &= V_i V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) - t_{ij} V_i^2 G_{ij} \\ &= G_{ij} C(i, j) + B_{ij} S(i, j) - t_{ij} G_{ij} C(i, i) \end{aligned} \quad (10)$$

where P_{ij} is the real power flow of branch (i, j) ; G_{ij} and B_{ij} are the real and imaginary parts of the admittance matrix of power systems, respectively; and t_{ij} is the ratio of the transformer at branch (i, j) .

Finally, the nonlinear and non-convex power flow equations are transformed into the SOCP formulation. On the other hand, to find the maximum λ^{index} , the power system needs to operate at LLP while satisfying the physical constraints.

With the assumption that the active/reactive power demand has the same load parameter at COP/LLP, $\forall i \in N_b$, the power balance equations are expressed as follows:

$$\hat{P}_i^G + \hat{P}_i^{gas} - \lambda_i^{LLP} P_i^{L0} = \sum_{j=1}^n (G_{ij} \hat{C}(i,j) - B_{ij} \hat{S}(i,j)) \quad (11)$$

$$\hat{Q}_i^G - \lambda_i^{LLP} Q_i^{L0} = \sum_{j=1}^n (-B_{ij} \hat{C}(i,j) - G_{ij} \hat{S}(i,j)) \quad (12)$$

where $\hat{P}_i^G$ and $\hat{Q}_i^G$ are the active and reactive power generations of the i^{th} thermal generator at LLP, respectively; $\hat{P}_i^{gas}$ is the active power generation of the i^{th} GFU at LLP; $\hat{C}(i,j)$ is a new element representing $C(i,j)$ at LLP; $\hat{S}(i,j)$ is another element representing $S(i,j)$ at LLP; and Q_i^{L0} is the base reactive power of the load at bus i . Then, the equations related to the pre-defined variables at LLP can be expressed by the following constraints:

$$V_{i,\min}^2 \leq \hat{C}(i,i) \leq V_{i,\max}^2 \quad (13)$$

$$\hat{C}(i,j) = \hat{C}(j,i) \quad (14)$$

$$\hat{S}(i,j) = -\hat{S}(j,i) \quad (15)$$

where $V_{i,\min}$ and $V_{i,\max}$ are the lower and upper bounds of the voltage magnitude at bus i , respectively.

$$\hat{C}^2(i,j) + \hat{S}^2(i,j) + \left(\frac{\hat{C}(i,i) - \hat{C}(j,j)}{2} \right)^2 \leq \left(\frac{\hat{C}(i,i) + \hat{C}(j,j)}{2} \right)^2 \quad (16)$$

2) Physical limits:

$$P_{i,\min}^G \leq \hat{P}_i^G \leq P_{i,\max}^G \quad (17)$$

$$Q_{i,\min}^G \leq \hat{Q}_i^G \leq Q_{i,\max}^G \quad (18)$$

$$x_i P_{i,\min}^{gas} \leq \hat{P}_i^{gas} \leq x_i P_{i,\max}^{gas} \quad (19)$$

$$\sum_{i=1}^n x_i P_{i,\max}^{gas} \leq f^{gas} P_L^{peak} \quad (20)$$

$$-P_{ij,\max} \leq \hat{P}_{ij} \leq P_{ij,\max} \quad (21)$$

$$\lambda_i^{LLP} \geq \lambda^{des} \quad (22)$$

where $P_{i,\min}^G$ and $P_{i,\max}^G$ are the minimum and maximum active power generations of the i^{th} thermal generator, respectively; $Q_{i,\min}^G$ and $Q_{i,\max}^G$ are the minimum and maximum reactive power generations of the i^{th} thermal generator, respectively; $P_{i,\min}^{gas}$ and $P_{i,\max}^{gas}$ are the minimum and maximum active power generations of a single gas turbine, respectively; x_i is a binary variable representing whether GFUs are installed at bus i or not; f^{gas} is the penetration coefficient of GFUs; P_L^{peak} is the peak load of the power system; $P_{ij,\max}$ is the maximum transfer capacity of branch (i,j) ; $\hat{P}_{ij}$ is the active power flow of branch (i,j) at LLP; and λ^{des} is the minimum desired load scaling factor.

Constraints (17) and (18) represent the operation limits of thermal generators, and constraint (19) is utilized to determine the optimal location of a GFU. Note that for stage 1, only the locations of GFUs are optimized, and their final optimum locations and capacities are determined by both location variable x_i and installed number variable y_i obtained in the stage 2. Only when $x_i = 1$ and $y_i > 0$ are satisfied simultaneously is the GFU installed at bus i . The maximum total in-

stalled capacity of GFUs is limited by (20), while constraint (21) is used to ensure the thermal stability of the transmission lines. Constraint (22) is the static voltage stability limit at bus i , and it ensures that the maximum loadability is larger than λ^{des} for each PQ bus.

C. Stage 2: Optimizing Allocation of GFUs to Reduce Costs and Power Losses

1) Objective Function

In the stage 2, the goal is to minimize the operation costs of thermal generators, gas fuel costs, and power losses P_{loss} while maintaining optimal GFU allocations. It is represented as follows:

$$\min(\sum_{i=1}^n F_i^G(P_i^G) + F_g(G^{gas}) + \omega P_{loss}) \quad (23)$$

$$F_i^G = a_{1i}(P_i^G)^2 + a_{2i}P_i^G + a_{3i} \quad (24)$$

$$F_g = kG^{gas} \quad (25)$$

$$G^{gas} = \rho^{gas} \sum_{i=1}^n P_i^{gas} \quad (26)$$

$$P_{loss} = \sum_{i=1}^n (P_i^G + P_i^{gas} - P_i^{L0}) \quad (27)$$

where F_i^G is the cost function of the thermal generator i , which is typically represented by a quadratic equation with coefficients a_{1i} , a_{2i} , and a_{3i} ; F_g is a linear function that characterizes the fuel cost of natural gas; k is the corresponding coefficient; G^{gas} is the total natural gas consumption of GFUs; and ρ^{gas} is the conversion efficiency of GFU generation. The last part of the objective function in (23) represents that the power losses with a weight coefficient ω , which ensures that the power loss is an important part of the objective function.

2) Constraints

As the locations of GFUs have been determined in the stage 1, only the installed capacities (i.e., allocation) of GFUs need to be optimized. Furthermore, it is assumed that the power demand at the COP equals the base power load, i.e., $\lambda^{COP} = 1$. The following power balance equations $\forall i \in N_b$ are obtained:

$$P_i^G + P_i^{gas} - P_i^{L0} = \sum_{j=1}^n (G_{ij} C(i,j) - B_{ij} S(i,j)) \quad (28)$$

$$Q_i^G - Q_i^{L0} = \sum_{j=1}^n (-B_{ij} C(i,j) - G_{ij} S(i,j)) \quad (29)$$

Similar to (16)-(19) in the stage 1, the SOCP formulations at the COP are utilized, and (6), (7), (9), and (30) are obtained.

$$V_{i,\min}^2 \leq C(i,i) \leq V_{i,\max}^2 \quad (30)$$

The physical constraints are:

$$P_{i,\min}^G \leq P_i^G \leq P_{i,\max}^G \quad (31)$$

$$Q_{i,\min}^G \leq Q_i^G \leq Q_{i,\max}^G \quad (32)$$

$$y_i P_{i,\min}^{gas} \leq P_i^{gas} \leq y_i P_{i,\max}^{gas} \quad (33)$$

$$y_i \geq 0 \quad (34)$$

$$\sum_{i=1}^n y_i P_{i,\max}^{gas} \leq f^{gas} P_L^{peak} \quad (35)$$

$$P_i^G + P_i^{gas} \leq P_{i,\max}^{inj} \quad (36)$$

$$-P_{ij,\max}^G \leq P_{ij} \leq P_{ij,\max} \quad (37)$$

where $P_{i,\max}^{inj}$ is the maximum power injection of bus i ; and P_{ij} is the active power flow of branch (i, j) at the COP.

Constraints (31) and (32) represent the physical limits of thermal generators, while constraints (33) and (34) are utilized to determine the installed number y_i of GFUs. Note that the optimal allocation of GFUs at the determined bus i is determined by $y_i P_{i,\max}^{gas}$. Constraint (35) limits the maximum installed capacity of GFUs; constraint (36) denotes that the active power injection of thermal generators and GFUs at bus i could not exceed the maximum power injection $P_{i,\max}^{inj}$; and the thermal stability of transmission lines is constrained by (37).

III. SOLUTION PROCEDURE AND EVALUATION OF PROPOSED METHOD

In the proposed bi-level optimization model, the optimal locations of GFUs are determined in the stage 1 while improving the voltage stability margin. The obtained results are further used for the optimization model of stage 2 to obtain the optimal allocations of GFUs while minimizing the power losses and total generation costs. Formally, the overall formulation for the optimal location allocation of GFUs can be summarized as follows.

1) The optimization problem in the stage 1:

$$\begin{cases} \max \lambda_{index} \\ \text{s.t. (10)-(22)} \end{cases} \quad (38)$$

The decision variable is $\{\lambda_{LLP}^G, \hat{P}_i^G, \hat{Q}_i^G, \hat{P}_i^{gas}, x_b, \forall i \in N_b\}$.

2) The optimization problem in the stage 2:

$$\begin{cases} \min(\sum_{i=1}^n F_i^G(P_i^G) + F_g(G^{gas}) + \omega P_{loss}) \\ \text{s.t. (28)-(38)} \end{cases} \quad (39)$$

The decision variable is $\{y_i, P_i^G, Q_i^G, P_i^{gas}, \forall i \in N_b\}$.

The proposed MISOCP formulations can be solved by the widely used commercial software ‘‘Gurobi’’. The detailed solution procedures are shown in Fig. 3. The final optimal placements of GFUs are determined by the value of $x_i y_i P_{i,\max}^{gas}$. Two evaluation indices are presented to assess the obtained results at each stage. To assess the improvement of the voltage stability margin by the proposed method, optimization problem 1 [31] is utilized to calculate the first evaluation index λ_{LLP}^G (the maximum loadability of the whole power system rather than each PQ bus):

$$\begin{cases} \max \lambda_{LLP}^G \\ \text{s.t. (16)-(21), (24)} \\ \hat{P}_i^G + \hat{P}_i^{gas} - \lambda_{LLP}^G P_i^{L0} = \sum_{j=1}^n (G_{ij} \hat{C}(i, j) - B_{ij} \hat{S}(i, j)) \\ \hat{Q}_i^G - \lambda_{LLP}^G Q_i^{L0} = \sum_{j=1}^n (-B_{ij} \hat{C}(i, j) - G_{ij} \hat{S}(i, j)) \\ 0 \leq \hat{P}_i^{gas} \leq P_i^{capacity} \\ \lambda_{LLP}^G \geq \lambda_{des} \end{cases} \quad (40)$$

where $P_i^{capacity}$ is the maximum installed capacity of GFUs on bus i , and it equals the result of $x_i y_i P_{i,\max}^{gas}$.

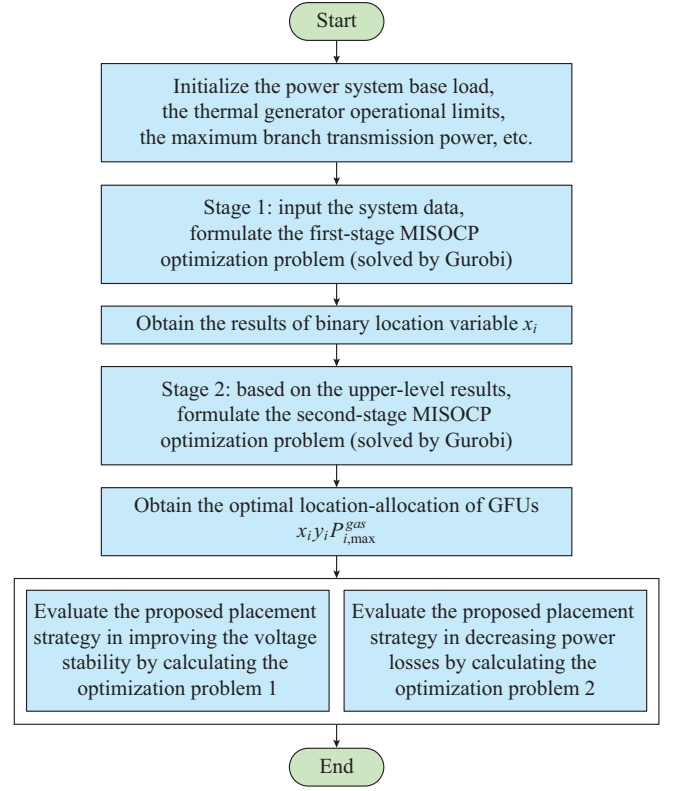


Fig. 3. Solution procedure of proposed model.

To assess the benefits of the proposed method in minimizing the operation costs and power losses, optimization problem 2 is utilized to calculate the second evaluation index:

$$\begin{cases} \min(\sum_{i=1}^n F_i^G(P_i^G) + F_g(G^{gas})) \\ \text{s.t. (28)-(33), (38)} \\ 0 \leq P_i^{gas} \leq P_i^{capacity} \end{cases} \quad (41)$$

When the optimum results are calculated, the second evaluation index P_{loss} can be calculated by:

$$P_{loss} = \sum_{i=1}^n P_i^G + \sum_{i=1}^n P_i^{gas} - \sum_{i=1}^n P_i^{L0} \quad (42)$$

IV. SIMULATION RESULTS

To evaluate the performance of the proposed approach, extensive simulations are carried out on the modified IEEE 118-bus test system. The data of the thermal generators, active/reactive power demands, and limits on transmission lines and bus voltage can be found in [32]. The parameter settings are as follows: the cost coefficient k is 8 \$/MWh; the conversion coefficient ρ^{gas} is 1.25; the weight coefficient ω is 20; and the peak load of this test system is assumed to be 6363 MW. To verify the effectiveness of the proposed method, the following three cases are designed for comparison.

1) Case 1: the IEEE 118-bus test system is not integrated

with GFUs.

2) Case 2: the IEEE 118-bus test system is integrated with GFUs, but both the locations and allocations of GFUs are determined randomly. This is the widely adopted strategy in [4], [5], [9], [10].

3) Case 3: the IEEE 118-bus test system is integrated with GFUs, and the locations allocations of GFUs are determined by the proposed method.

The proposed formulation is implemented by using the 64-bit Gurobi with default termination criteria in the Yalmip toolbox [33] on a PC with an Intel Core-i5 processor at 2.9 GHz and 4 GB of RAM.

A. Computational Issues

In the simulations, the maximum penetration f^{gas} of GFUs is 30% of the peak load (which is set to be 1.5 times of the base load); λ^{des} is 1.2. The maximum capacity of a single gas turbine $P_{i, \max}^{gas}$ is assumed to be 100 MW.

In Case 2, the location allocation plan of GFUs is randomly determined by planners. Thus, the solution time of the decision progress could be regarded as zero. On the other hand, the CPU time for determining the optimal location allocation of GFUs is 177.2664 s in Case 3. Considering that the scale of the IEEE 118-bus test system and the planning problem are usually determined off-line, the solution time is acceptable for power system planners.

To evaluate the computation efficiency of the proposed formulation, we solve the original MINLP problem, i.e., the objective function is the total operation costs and the constraints are in terms of the nonlinear AC formulations, with the global solver “Baron” [34]. The simulations are tested on the IEEE 14-bus, 30-bus, 57-bus, and 118-bus systems and the results are shown in Table I. It shows that for the IEEE 118-bus test system, the computation time exceeds the maximum calculation time of the “Baron” solver. Thus, no solution can be founded. Compared with the MINLP formulation, the proposed MISOCP formulation can effectively reduce the solution time and generate high-quality feasible solutions whose gap is less than 0.01%. Although the MISOCP formulation can improve the computation efficiency, it can not always be used to obtain a global optimal solution with a zero dual gap. This problem can be addressed by introducing the valid inequalities to strengthen the relaxation [35]. As for large-scale power systems, e.g., a power system with

thousands of buses, it is possible and necessary to apply the distributed algorithm in future work.

TABLE I
COMPUTATION TIME OF DIFFERENT FORMULATION

Test system	CPU time (s)		Optimal solution	
	MINLP (Baron)	MISOCP (Gurobi)	MINLP (Baron)	MISOCP (Gurobi)
IEEE 14-bus	14.12	0.8086	Yes	Yes
IEEE 30-bus	873.36	1.7203	Yes	Yes
IEEE 57-bus	1000.03	164.8258	Yes	Yes
IEEE 118-bus		177.2664	No	Yes

B. Results of Maximum Load Parameter

The penetration percentage f^{gas} of GFUs is 30%, λ^{des} is 1.2, and the maximum capacity of a single gas turbine $P_{i, \max}^{gas}$ is assumed to be 100 MW. The optimal location allocation plans of GFUs in Case 3 are calculated by Gurobi with the following results: 3(1), 7(1), 11(1), 20(1), 29(1), 40(1), 42(1), 45(1), 54(3), 60(1), 75(2), 78(1), 82(1), 95(1), 107(1), and 114(1). The first number represents the locations of GFUs, and the second number represents the installed numbers. For example, 54(3) means that GFUs of 300 MW are installed at bus 54.

More details about the simulation results of Case 3 are displayed in Fig. 4, which shows that most of GFUs are installed on buses that have high power demands. To assess the benefits of using the proposed approach to improve the voltage stability margin, the results for optimization problem 1 and its associated index are calculated. Note that in Case 2, 5 GFU placement plans are assessed, and each plan is determined randomly. The average value is taken as the final result. The aim is to be fair with existing approaches. The calculated values of voltage stability index λ^{LLP} for Cases 1, 2, 3 are 1.9535, 1.9804, and 2.1846, respectively. It is observed that λ^{LLP} in Case 2 and Case 3 is larger than that in Case 1, which means the static voltage stability is improved by integrating GFUs in Case 2 and Case 3. In addition, λ^{LLP} in Case 3 in which GFUs are optimally placed is obviously more improved than that in Case 2, in which GFUs are placed randomly. These results verify the effectiveness of the proposed method.

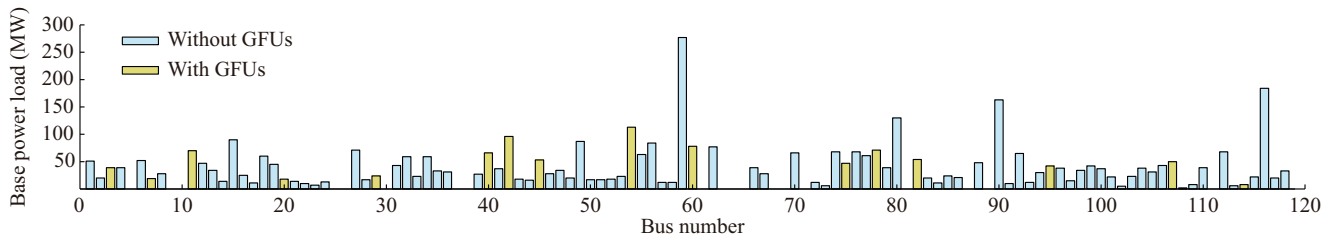


Fig. 4. Base power load of each bus in IEEE 118-bus test system with and without GFUs.

The aforementioned results are obtained by assuming $P_{i, \max}^{gas}$ as 100 MW for each GFU. To evaluate the impact of $P_{i, \max}^{gas}$ on λ^{LLP} of the placement plan, different values of $P_{i, \max}^{gas}$

are considered and tested in the proposed bi-level optimization problem. The penetration percentage level f^{gas} remains 30%, i.e., the maximum installed capacity of GFUs is still

1909 MW.

The results of index λ^{LLP} for the stage 1 placement plan and stage 2 results are compared and shown in Table II. It should be noted that λ^{LLP} is calculated by optimization problem 1. It is observed that when $P_{i, \max}^{gas}$ is 100 MW, the placements of GFUs in the stage 1 are different from the final ones, and the corresponding λ^{LLP} is slightly lower than that of the final plan. This is because after minimizing the total costs and power losses in the stage 2, the optimal installed capacity of GFUs may be adjusted to achieve better voltage stability improvement. For example, the final optimum capacity installed at bus 54 is 300 MW while this value is 100 MW in the stage 1. As a result, λ^{LLP} can be increased if GFUs are installed at the buses whose voltages are sensitive to active power injections. On the other hand, further increase in $P_{i, \max}^{gas}$ for each GFUs does not change the final results, as the stage 1 and final results have the same placement plans. This means that λ^{LLP} is mainly determined by the optimization problem in the stage 1, which validates the effectiveness of the proposed method.

TABLE II
RESULTS OF λ^{LLP} WITH RESPECT TO DIFFERENT $P_{i, \max}^{gas}$

$P_{i, \max}^{gas}$ (MW)	Placement plan in stage 1	Final placement plan of proposed method	λ^{LLP} in stage 1	λ^{LLP} of final plan
100	3, 7, 11, 20, 29, 40, 41, 42, 45, 52, 54, 60, 75, 78, 79, 82, 95, 107, 114	3, 7, 11, 20, 29, 40, 42, 45, 54(3), 60, 75(2), 78, 82, 95, 107, 114	2.1175	2.1846
200	20, 40, 42, 45, 52, 54, 75, 78, 82, 95	20, 40, 45, 52, 54(2), 75, 78, 82, 95	2.2274	2.2274
300	20, 42, 45, 54, 75, 82, 95	20, 42, 45, 54, 75, 82, 95	2.2597	2.2597
400	20, 42, 54, 75, 95	20, 42, 54, 75, 95	2.2871	2.2871
500	20, 54, 75, 95	20, 54, 75, 95	2.3415	2.3415

C. Results of Operation Costs and Power Losses

To assess the benefits of the proposed approach in reducing operation costs and power losses, the second evaluation indices of Cases 1-3 are calculated. The results are displayed in Table III.

TABLE III
RESULTS OF OPTIMIZATION PROBLEM 2 FOR THREE CASES

Case	Generation of thermal generator (MW/h)	Cost of thermal generator (\$/h)	Generation of GFUs (MW/h)	Cost of GFUs (\$/h)	Total operation cost (\$/h)	Total power loss (MW/h)
1	4319.4	129961			129961	77.4
2	2421.5	61967	1900	19000	80967	79.5
3	2376.1	60466	1900	19000	79466	34.1

Table III demonstrates that by adopting the proposed approach, the system operation costs and power losses can be reduced. For the power losses, in particular, their values in Case 3 are obviously lower than those in Case 2. This is because the power losses are considered an important part of the objective function in (23) in the planning stage. Thus,

even if the total power loads and physical networks are the same, the generation of GFUs at different locations leads to different power flow conditions. As a result, the power losses are different. It is worth pointing out that the generations of GFUs in Cases 2 and 3 are the same. This is because the system operator prefers to operate GFUs at their maximum capacities as their cost is much lower than that of the thermal generators. As a result, once Cases 2 and 3 have the same GFU penetration coefficient, their outputs are the same. However, the optimal locations of GFUs allow the proposed approach to achieve a better performance in reducing system power losses (see the comparison of the results obtained in Cases 2 and 3 in Table III). Owing to the integration of low-cost GFUs, the thermal generations in Cases 2 and 3 decrease significantly, and the system economic benefits are improved. The average voltage magnitudes for Cases 1, 2, 3 are 1.0413, 1.0411, and 1.0457, respectively. As a result, no voltage issues are found for the proposed approach.

D. Impacts of λ^{des} and f^{gas} on Simulation Results

In the proposed approach, the performances of the final location allocation results of GFUs are assessed by λ^{LLP} and total system costs. To calculate them, two important parameters are involved, namely, the penetration percentage f^{gas} and the pre-set minimum desired loading scaling factor λ^{des} . In this section, we carry out simulations to investigate the impacts of these parameters on the final results.

First of all, the impacts of different values of λ^{des} on the maximum loadability λ^{LLP} are assessed by carrying out simulations for Case 1 through optimization problem 1. The results are shown in Table IV, from which we find that the maximum loadability of the system does not change with the variations in λ^{des} . This does not come as a surprise as the system stability is determined by its inherent properties if no additional external devices or actions are involved. Note that λ^{des} is set by the operator according to the system characteristics.

TABLE IV
 λ^{LLP} FOR DIFFERENT VALUES OF λ^{des} IN CASE 1

λ^{des}	λ^{LLP}
1.2	1.9535
1.4	1.9535
1.6	1.9535
1.8	1.9535
2.0	Infeasible

Secondly, different values of the penetration percentage f^{gas} for Cases 2 and 3 are tested. According to the variations in f^{gas} , the optimization problems 1 and 2 are leveraged to calculate λ^{LLP} and P_{loss} , respectively. Note that the base power demands are fixed irrespective of the value of f^{gas} .

Figure 5 shows the variations in λ^{LLP} with respect to the penetration percentage f^{gas} of GFUs. It is observed that the value of λ^{LLP} increases with f^{gas} in both Case 2 and Case 3, whereas the value of λ^{LLP} obviously increases with f^{gas} in Case 2 but not very obviously in Case 3. This is because,

with the increase in the GFU penetration percentages, the total system power generation increases. As a result, the maximum load that the system can support increases, leading to an improved LM for Case 2. In contrast, the proposed location and allocation approach in Case 3 always guarantees the placement of GFUs on the load buses that are sensitive to the real power to improve the system LM, i.e., large value of λ^{LLP} , irrespective of f^{gas} . On the other hand, the value of λ^{LLP} in Case 3 is always larger than that of Case 2. This indicates that the optimal placement of GFUs allows us to achieve a better voltage stability margin compared with the random placement strategy such as in Case 2. Another result that merits attention is that, when f^{gas} is further increased, the difference between the results obtained in Cases 2 and 3 is reduced. This is because with an increase in the GFU penetrations, the benefits of flexibility provided by location allocation of GFUs are reduced. As a result, when the penetration level is large enough, the impacts of location allocation of GFUs on the system voltage stability margin is less important.

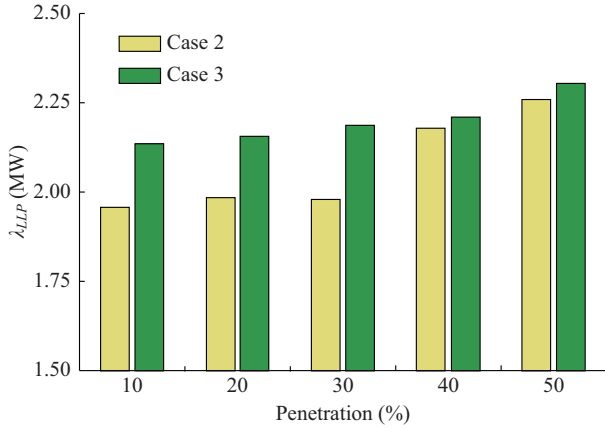


Fig. 5. λ^{LLP} for different penetrations.

Figure 6 shows the variations in P_{loss} with respect to f^{gas} . It is clear that for different GFU penetration ratios, the proposed approach achieves a better performance in reducing the system losses.

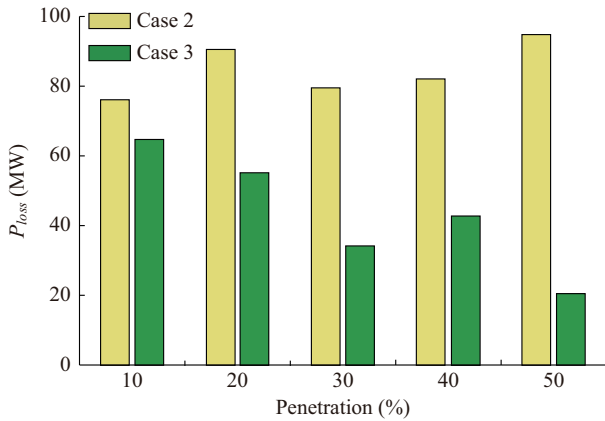


Fig. 6. Power losses for different penetrations.

Note that the value of f^{gas} represents the penetration per-

centage of the installed capacity of GFU accounting for the total real power generations and that the base power demands are unchangeable during the simulation. This explains why the total power losses for Case 2 with different GFU penetrations produce similar results. By contrast, in Case 3, the total power loss is an important part of the objective function when determining the optimal allocation of GFUs. Moreover, P_{loss} is obviously reduced with an improvement of f^{gas} . This is because the optimal capacity of GFUs affects the active power injection of the power system, which directly affects the power flows of the power system with the increases in f^{gas} , leading to a significant change in P_{loss} . As the power demands are fixed in optimization problem 2, accordingly, the decrease in power losses indicates that the requirements of the total power generation of the thermal generators have decreased, leading to the improvement of economic benefits.

V. CONCLUSION

In this study, a bi-level optimization model considering static voltage stability constraints is proposed to obtain the placement strategy of GFUs. This model is formulated as an MISOCP model and implemented for three cases on the IEEE 14-bus, 30-bus, 57-bus, 118-bus test systems. The main conclusions are summarized as follows:

- 1) The proposed method transforms the original MINLP problem into a bi-level MISOCP formulation by using convex relaxations. Thus, the problem can be solved in a computationally efficient manner.
- 2) Compared with determining location allocation of GFUs randomly, the proposed approach can improve the static voltage stability margin and reduce the operation costs by decreasing power losses.
- 3) With an increase in the penetration ratio of GFUs, the LM value in the proposed method slowly grows, while the power losses can be further reduced. Thus, when the total installed capacity of GFUs is lower than 50% of the peak load, the proposed method can have a better result than the traditional methods.

We will focus on the application of the distributed algorithm and tighter relaxations in future research.

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