Evolutionary Game-theoretic Modeling of Massive Distributed Renewable Energy Deployment Towards Low-carbon Distribution Networks

Xuanyi Xiao, Jianbing Yin, Lin Chen, Mingchang Wang, Yi Zhao, and Zhiyi Li,
Senior Member, IEEE

Abstract—This paper proposes an evolutionary game-theoretic model of massive distributed renewable energy deployment in order to shed light on the self-organization sustainable developments of renewable energies in distribution networks towards low-carbon targets. Since neighboring buses can interact in terms of energy exchanges, the return matrices of individual buses in the evolutionary game are established based on profiles of loads and renewable energy generation. More specifically, an evolutionary strategy is proposed based on the return matrices for individual buses to determine whether or not to deploy renewable energies in the next round of the game. Then, a dynamic model is derived for analyzing the renewable energy penetration rate in the distribution network throughout the multi-round evolutionary game. In theory, this model can reveal the self-organization process of renewable energy deployment in the distribution network. With this model, the distribution network operator would be aided in designing the incentives for buses deploying renewable energies toward a pre-defined low-carbon target. Numerical results on an actual 141-bus system and a synthetic 2000-bus system have demonstrated the validity and efficiency of the proposed model.

Index Terms—Distribution network, renewable energy deployment, self-organization, evolutionary game.

NOMENCLATURE

A. Indices and Sets

\( \Omega^t \) Set of time periods
\( \Omega_{i}^{(i)} \) Set of neighboring buses of bus \( i \)
\( h \) Time period index

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<td>( i )</td>
<td>Bus index</td>
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<td>( S1, S2 )</td>
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B. Parameters

\( \lambda^D, \lambda^U \) Renewable cost and electricity price
\( \lambda^{SN}, \lambda^{DS} \) Prices at which a bus sells electricity to neighboring buses and buys electricity from neighboring buses with incentives
\( \lambda^{SN}, \lambda^{DS} \) Electricity price, renewable cost, and selling price
\( k \) Parameter in sigmoid function
\( p^{DS}, p^{DN} \) Renewable generation of buses and neighboring buses
\( p^{LS}, p^{LN} \) Load of buses and neighboring buses

C. Variables

\( \lambda^{UI}, \lambda^{DS} \) Incentives toward low-carbon distribution networks
\( \rho \) Proportion of buses deploying renewable energies in distribution networks
\( \rho^U \) Low-carbon target in distribution networks
\( P_{S1}, P_{S2} \) Probabilities of the buses with and without renewable energy deployment
\( R, \tilde{R} \) Return matrices without and with incentives
\( ST_i \) Strategy of bus \( i \) (1 indicating renewable deployment and 0 indicating no renewable deployment)
\( U_{S1}, U_{S2} \) Bus returns with and without renewable energy deployment
\( \bar{U}_{S1}, \bar{U}_{S2} \) Average bus returns with and without renewable energy deployment

I. INTRODUCTION

To avoid the worst consequences of climate change, the global energy system must rapidly reduce its greenhouse gas emissions. Accordingly, the development of clean energy technologies has been dramatically scaled up [1]-[3]. The integration of distributed renewable energies, including photovoltaic (PV) power and wind power, into distribution
networks has shown a positive effect to bring down carbon emissions [4], [5]. With the growing deployment of distributed renewable energies, it is of significance to understand the inherent driving force and the pertinent evolution pattern of interconnecting renewable energies to the systems as well as the difference of multiple solutions given the detailed structure of distribution networks [6].

To this end, this paper attempts to reveal the self-organization sustainable developments of renewable energies toward low-carbon distribution networks, where the low-carbon target is quantified by the renewable energy penetration rate of the distribution network. Given the potential self-organization pattern of renewable energy deployment, this paper also attempts to shed light on the policies for renewable energy deployment which can boost the low-carbon development of distribution networks.


Due to the small capacity and geographical dispersity of renewable energies, their deployment is often driven by end users that represent different decision entities except for the distribution network operator. In other words, the conventional centralized decision-making fashion would be hardly effective in the near future. In fact, distributed optimization methods are becoming a hot topic to overcome the challenges, where a large number of entities co-exist in the decision-making on renewable energy deployment. Reference [14] studied the interactions between renewable energy deployment and distribution network reinforcements in a competitive market. Reference [15] studied the cooperative planning of renewable energies in interconnected microgrids, which could reduce the overall system cost. Reference [16] proposed a multi-agent framework for deploying renewable energies and distribution lines to enhance the reliability of distribution networks. Reference [17] proposed an optimal interconnection model for community microgrids, which provided economical solutions to improve reliability. Reference [18] proposed a chance-constrained stochastic conic model for renewable energy deployment in network microgrids in order to balance the operational cost and risk-hedging capability.

Meanwhile, although many countries and regions have made roadmaps of renewable energy integration to achieve low-carbon targets [19], [20], the evolution patterns of distribution networks toward low-carbon systems are not clear in theory. In that regard, the impacts of individual decisions on the holistic evolution of distribution networks have not been well studied in existing works. In addition, most of the existing works rely on mathematical programming models for renewable energy deployment in distribution networks. However, the solution process of such models for large-scale distribution networks is rather time-consuming.

Hence, the research gap in the existing literature is that a systematic and efficiently-solvable model is absent which can reveal the inherent driving force and evolution process of renewable energy deployment in large-scale distribution networks. This paper provides a new perspective on renewable energy deployment to fill this research gap.

Renewable energies in geographically close regions usually have similar output profiles. However, if a bus and its neighboring buses are deployed with the same type of renewable energies, they would suffer the same curtailment issues of renewable energy generation. Hence, in order to realize the complementary energy generation patterns and increase the economic return of renewable energy deployment, each bus representing an individual decision-making entity may have an opposite tendency to deploy renewable energies relative to its neighbors. This phenomenon thus lays the basis for the game strategies of deploying renewable energies among individual buses in the distribution network.

There are several studies on game theory in the distribution network. Reference [21] proposed a game-theoretic model for distribution network participants, which reflected the conflicts among these participants and enhanced the system reliability. Reference [22] proposed a Nash game model for electric vehicle charging schedules in distribution networks. The model was solved by alternating direction method of multipliers with blockchain implementation for enabling trusted coordination. Reference [23] formulated a robust optimization model based on game theory to present the competition of market participants considering uncertainties in the distribution network. Reference [24] proposed a Stackelberg game-based model for peer-to-peer transactive energy in the distribution network and the existence and the uniqueness of the game equilibrium point were proved. Reference [25] proposed an energy scheduling model for integrated energy distribution systems based on the Stackelberg game to configure a more reasonable solution. The above works studied the application of the Nash game [21]-[23] and the Stackelberg game [24], [25] in renewable energy trading and management in distribution networks. These models are effective for facilitating energy trading among renewable energy owners or cost-effective renewable energy integration. However, these models cannot show the self-organization process of renewable energy deployment in large-scale distribution networks so that they have policies for renewable energy deployment in large-scale distribution networks toward low-car-
bon targets.

Besides, the evolutionary game theory arises from the interpretation of ecological phenomena, which is envisioned to describe individual behaviors under conflicts [26]. This theory has already been explored and employed in the research field of power engineering, such as modeling the behaviors of residential users [27] and generation enterprises [28]. In contrast to the equilibrium point in traditional game theory, the concept of equilibrium in evolutionary game theory is the evolutionarily stable strategy along with the stationary point, which can capture the holistic tendency of whole system evolution. Hence, the evolutionary game theory is regarded as a promising tool to analyze the sustainable strategic roadmap of renewable energy deployment in distribution networks toward low-carbon targets.

To this end, this paper attempts to fill the abovementioned research gap by making the following contributions.

1) A new perspective is provided for revealing the inherent rules and patterns regarding massive renewable energy deployment in distribution networks, where an evolutionary game-theoretic model instead of conventional mathematical programming is proposed to reveal the self-organization process of renewable energy deployment while providing the design of incentives for facilitating renewable energy deployment toward low-carbon targets.

2) Return matrices for individual buses (decision-making entities) participating in the game of renewable energy deployment are established based on the profiles of load and renewable energy generation, and an evolutionary strategy for those buses is then proposed. On this basis, the impact of individual decisions on the holistic evolution of distribution networks is theoretically revealed.

3) The dynamic model regarding the change of renewable energy penetration rates is derived in distribution networks, when incentives are designed and embedded in the evolutionary game. It is shown that if the incentives are properly designed, the dynamic model would converge to the predetermined target value.

4) Numerical results on an actual 141-bus system and a synthetic 2000-bus system show that the proposed model can show the self-organization of renewable energy deployment in large-scale distribution networks and provide the basis for the policy of renewable energy deployment incentives toward low-carbon targets.

The rest of this paper is organized as follows. The evolutionary game of renewable energy deployment is presented in Section II. The details of the evolutionary game of renewable energy deployment with incentives are presented in Section III. Numerical results are presented in Section IV using an actual 141-bus system and a synthetic 2000-bus system. Section V concludes this paper.

II. EVOLUTIONARY GAME OF RENEWABLE ENERGY DEPLOYMENT

A. Assumptions

Since a practical distribution network is usually well organized with enough schedulable resources, voltage and power flow constraints are assumed to be satisfied in this paper. Each bus is regarded as a decision-making entity that decides whether or not to deploy renewable energies. This means that two strategies S1 and S2 representing with and without deploying renewable energies can be chosen by a bus during the evolutionary game. A bus can exchange energy with neighboring buses. Because this paper studies the deployment of massive renewable energies at the system level rather than the individual bus level, the load and renewable energy capacity on each bus are assumed to be identical and the bus load is assumed to be equal to the average bus load in the system. Some studies have used individual average parameters to study the performance of a system [26] - [29]. Similarly, the uncertainty and intermittence of renewable energies are not considered, since this paper focuses on massive renewable energy deployment and corresponding incentives at the system level where the above factors have minor impacts. Besides, many regions have set targets for the renewable energy penetration rate to achieve low-carbon distribution networks or carbon-neutral zones. Hence, the low-carbon target is regarded as the target of renewable energy penetration rate in the distribution network, which is in turn defined as the number of buses deploying renewable energies divided by the total number of buses in the distribution network.

B. Return Matrix

Here we attempt to show the developments of massive renewable energies in large-scale distribution networks. A bus is a decision-making entity that decides whether to deploy renewable energies. The same type of renewable energies in a region have similar output profiles and neighboring buses are usually deployed with the same type of renewable energies for similar site spaces and resources. If a bus and its neighboring bus both decide to deploy renewable energies, renewable energies cannot be consumed during the period when the renewable output level is high and the load level is low. This lowers the profits of these buses with renewable energies. Hence, a bus may have an opposite tendency on whether to deploy renewable energies relative to its neighbors’ decisions to maximize its returns. This characteristic of renewable energy deployment can be modeled by the return matrix.

The average cost per kilowatt-hour λ^{DS} of renewable output is used to present the renewable cost and to calculate the return matrices of buses. It includes the installation and operation costs of renewable energies. The return matrix of a bus in the evolutionary game is established as:

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$ (1)

$$R_{11} = \sum_{h \in D^c} \left( p_{h}^{DS} - \left( p_{h}^{DS} - p_{h}^{LS} \right)^{+} \right) \left( \lambda_{h}^{U} - \lambda_{h}^{DS} \right) - \sum_{h \in D^s} \left( p_{h}^{DS} - p_{h}^{LS} \right)^{+} \lambda_{h}^{DS}$$ (2)
where \( ()^\dagger \) is an operator that \( x^\dagger = \max \{ x, 0 \} \), \( R_{11} \) presents the return of a bus when both itself and its neighboring buses deploy renewable energies. The return matrix is calculated using the predicted load and renewable output data. Then, the return matrix would be normalized, which means that the element with the largest absolute value in the matrix is 1 or -1. The first item in (2) represents the return generated by the generation of renewable energies, and the second item represents the cost of electricity that cannot be consumed. \( R_{12} \) presents the return of a bus when it deploys the renewable energies while its neighboring buses not. The first two terms in (3) are the same as those in (2) and the last term in (3) represents the return from selling electricity to neighboring buses. \( R_{21} \) presents the return of a bus when it does not deploy the renewable energies and its neighboring bus deploys renewable energies. \( R_{22} \) presents the return of a bus when both itself and its neighboring buses do not deploy renewable energies.

**C. Evolutionary Strategy**

Each bus decides its strategy in the next round of the game based on the observation of its neighbors’ strategies in the current round of the game. A bus tends to choose the strategy that brings more return. However, many factors affect the decision of a bus besides the return, which is the embodiment of the bounded rationality of individuals in sociology. The sigmoid function is considered to be appropriate for modeling this behavior [29], which is shown in Fig. 1.

![Illustration of sigmoid function](image)

Mathematically, the probabilities of bus \( i \) choosing the two strategies in the next round are stated as:

\[
P_{i,\text{SL}} = \frac{1}{1 + \exp \left( - \left( U_{i,\text{SL}} - U_{i,\text{SU}} \right) / k \right)}
\]

\[
P_{i,\text{SU}} = \frac{1}{1 + \exp \left( - \left( U_{i,\text{SU}} - U_{i,\text{SL}} \right) / k \right)}
\]

where \( \exp() \) is the exponential function with the natural constant.

According to the property of the sigmoid function, a bus has a higher probability of choosing a strategy that benefits it more and it is obvious that \( P_{i,\text{SL}} + P_{i,\text{SU}} = 1 \). The returns of bus \( i \) adopting the two strategies in the next round are stated as:

\[
U_{i,\text{SL}} = \sum_{j \in \Omega^{(i)}} ST_j R_{11} + \sum_{j \in \Omega^{(i)}} \left( 1 - ST_j \right) R_{12}
\]

\[
U_{i,\text{SU}} = \sum_{j \in \Omega^{(i)}} ST_j R_{21} + \sum_{j \in \Omega^{(i)}} \left( 1 - ST_j \right) R_{22}
\]

Because this paper studies the evolution of renewable energy deployment in large-scale distribution networks, the difference between the returns of buses adopting the two strategies can be approximately replaced by the average difference between the returns of all buses adopting the two strategies. Thus, evolutionary strategy (6)-(9) can be approximately substituted by (10)-(13) in the subsequent analysis. The probabilities of bus \( i \) choosing the two strategies in the next round are approximately stated as:

\[
P_{i,\text{SL}} = \frac{1}{1 + \exp \left( - \left( \bar{U}_{i,\text{SL}} - \bar{U}_{i,\text{SU}} \right) / k \right)}
\]

\[
P_{i,\text{SU}} = \frac{1}{1 + \exp \left( - \left( \bar{U}_{i,\text{SU}} - \bar{U}_{i,\text{SL}} \right) / k \right)}
\]

The average returns of buses in distribution networks with and without renewable energies are shown as:

\[
\bar{U}_{\text{SL}} = \rho R_{11} + (1 - \rho) R_{12}
\]

\[
\bar{U}_{\text{SU}} = \rho R_{21} + (1 - \rho) R_{22}
\]

The evolutionary strategies (6)-(9) and (10)-(13) would be compared and the effectiveness of the approximation would be verified in the numerical results. Without loss of generality, (10)-(13) would be used for the analysis below.

**D. Dynamic Model of Renewable Energy Penetration**

The individual strategies in evolutionary games for renewable energy deployment have been studied above. Then, individual decisions and system performance need to be linked to analyze how bus decisions affect the renewable energy penetrations in distribution networks. Substituting (12) and (13) into (10), the dynamic model of renewable energy penetrations in distribution networks is obtained as:

\[
\dot{\rho} = \frac{1}{1 + \exp \left( \left( \bar{R}_{12} + \bar{R}_{21} - \bar{R}_{11} - \bar{R}_{22} \right) \rho + \bar{R}_{22} - \bar{R}_{12} \right) / k} - \rho
\]

This is a nonlinear time-invariant dynamic system in which the parameters are elements of the return matrix. This dynamic model shows the self-organization of renewable energy deployment in distribution networks. Furthermore, modifying elements of the return matrix would change the performance of this dynamic model. Thus, the evolutionary game-based model shows the development of renewable energy deployment in distribution networks from the perspective of both individuals and systems, which is critical to studying the incentives in the subsequent section.
III. Evolutionary Game of Renewable Energy Deployment with Incentives

A. Return Matrix with Incentives

To achieve low-carbon power and energy systems, some regions tend to make timetables for renewable energy penetrations. Although the dynamic model (14) can be stable, it often fails to converge to the target value. So certain incentives should be properly designed when the following two scenarios are considered.

Scenario 1: renewable energy penetration rate is below the low-carbon target. Due to the rapid development of renewable technologies in recent years, the cost of renewable energies has been significantly reduced. For example, in China, the investment costs of PV power have dropped by 68% compared with those in 2012 thanks to the considerable feed-in-tariff support for PV industries since 2006 [30], [31]. However, the costs of PV per kWh are still higher than electricity prices in some regions because of resource constraints and the costs of site usage and employment. Thus, when the renewable energy penetration rate is below the low-carbon target, a reasonable increment in electricity price can be adopted by the distribution network operator to incentivize buses to deploy renewable energies. The electricity price in this scenario can be modified as:

\[ \bar{\lambda}_h^U = \lambda_h^U + \lambda_h^D \]  

(15)

Considering the above incentive, the return matrix in this scenario is modified as:

\[ \bar{R} = \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} \\ \bar{R}_{21} & \bar{R}_{22} \end{bmatrix} \]  

(16)

\[ \bar{R}_{11} = \sum_{h \in D^p} \left[ p_h^{DS} - (p_h^{DS} - p_h^{LS})^+ \right] (\lambda_h^U - \lambda_h^{DS}) - \sum_{h \in D^x} (p_h^{DS} - p_h^{LS})^+ \lambda_h^{DS} \]  

(17)

\[ \bar{R}_{12} = \sum_{h \in D^p} \left[ p_h^{DS} - (p_h^{DS} - p_h^{LS})^+ \right] (\lambda_h^U - \lambda_h^{DS}) - \sum_{h \in D^x} (p_h^{DS} - p_h^{LS})^+ \lambda_h^{DS} + \sum_{h \in D^x} (p_h^{DS} - p_h^{LS})^+ \lambda_h^{NS} \]  

(18)

\[ \bar{R}_{21} = \sum_{h \in D^x} (p_h^{DS} - p_h^{LS})^+ (\lambda_h^U - \lambda_h^{NS}) \]  

(19)

\[ \bar{R}_{22} = 0 \]  

(20)

Scenario 2: renewable energy penetration rate is higher than the low-carbon target. If the renewable energy costs are adequately low relative to the electricity price, almost every bus may tend to deploy renewable energies. Because of the uncertainty and intermittence of renewable energies, excessive renewable energy penetrations would threaten the security operation of distribution networks. So distribution network operators could add renewable taxes to buses with renewable energies, which can be regarded as an increment in renewable energy costs. The increase in renewable energy costs certainly leads to an increase in energy trading prices among buses. Hence, the renewable energy costs and energy trading prices among buses in this scenario are modified as:

\[ \bar{\lambda}_h^{DS} = \lambda_h^{DS} + \lambda_h^{DS} \]  

(21)

\[ \bar{\lambda}_h^{SN} = \left( \bar{\lambda}_h^{DS} + \lambda_h^U \right) / 2 \]  

(22)

The incentive should satisfy that the increased renewable cost is not higher than the electricity price, i.e., \( \lambda_h^{DS} + \lambda_h^{DS} \leq \lambda_h^U \). The transaction price between buses should not be higher than the electricity price. So after adding incentives, the transaction price between buses is set to be the middle value of renewable cost and electricity price, as presented in (22). Considering such incentives, some elements of the return matrix \( \bar{R} \) in this scenario are modified as:

\[ \bar{R}_{11} = \sum_{h \in D^x} \left[ p_h^{DS} - (p_h^{DS} - p_h^{LS})^+ \right] (\lambda_h^U - \lambda_h^{DS}) - \sum_{h \in D^x} (p_h^{DS} - p_h^{LS})^+ \lambda_h^{DS} \]  

(23)

\[ \bar{R}_{12} = \sum_{h \in D^x} \left[ p_h^{DS} - (p_h^{DS} - p_h^{LS})^+ \right] (\lambda_h^U - \lambda_h^{DS}) - \sum_{h \in D^x} (p_h^{DS} - p_h^{LS})^+ \lambda_h^{DS} + \sum_{h \in D^x} (p_h^{DS} - p_h^{LS})^+ \lambda_h^{SN} \]  

(24)

\[ \bar{R}_{21} = \sum_{h \in D^x} (p_h^{DS} - p_h^{LS})^+ (\lambda_h^U - \lambda_h^{NS}) \]  

(25)

Thus, whether the self-organization of the massive renewable energy deployment results with too low or too high renewable energy penetration rates, the distribution network operator can adjust the renewable energy penetration rate by adding incentives to buses. The following subsection will show how to determine the incentives to achieve the low-carbon targets in distribution networks.

B. Dynamic Model with Incentives

Considering the return matrix with incentives, the dynamic model of renewable energy penetration rates with incentives is obtained by substituting (15)-(20) or (16), (20), and (21)-(25) into (14) as follows:

\[ \dot{\rho} = \frac{1}{1 + \exp \left( \frac{\bar{R}_{12} + \bar{R}_{21} - \bar{R}_{11} - \bar{R}_{22}}{\bar{R}_{12} - \bar{R}_{11} - \bar{R}_{22}} \right) \rho + \bar{R}_{22} - \bar{R}_{12}} - \rho \]  

(26)

Each incentive corresponds to a dynamic model of renewable penetrations in distribution networks. The right-hand side of (26) is a nonlinear function of \( \rho \), named \( f(\rho) \). The low-carbon target in distribution networks \( \rho = \rho^T \) is a stationary point if the following equation holds [32]:

\[ f(\rho^T) = 0 \]  

(27)

Because the elements of return matrices with incentives are linear functions of \( \lambda^U \) or \( \lambda^{DS} \), respectively, the following equations are obtained by substituting (15)-(20) or (16), (20), and (21)-(25) into (27):

\[ g_1(\lambda^U) = \frac{1}{1 + \exp (a_1 \lambda^U + b_1)} - \rho^T = 0 \]  

(28)

\[ g_2(\lambda^{DS}) = \frac{1}{1 + \exp (a_2 \lambda^{DS} + b_2)} - \rho^T = 0 \]  

(29)

\[ a_1 = \frac{1}{k} \sum_{h \in D^x} \left( \rho^T (p_h^{DS} - p_h^{LS})^+ - (p_h^{DS} - (p_h^{DS} - p_h^{LS})^+ \right) \]  

(30)
The renewable costs are high and close to the electricity price from distribution network operators. If renewable energies are deployed on neighboring buses, the curtailment of renewable consumption damages the interests of these buses, as shown in (36), \( R_{11} = -1 \). Even if only one of the two neighboring buses deploys renewable energies, it still benefits less, as shown in (36), \( R_{12} = 0.2813 \) and \( R_{21} = 0.0674 \). This would result in a less renewable energy penetration rate. The renewable energy penetration rate during the 200 rounds of the evolutionary game is shown by the dashed line in Fig. 2, which shows the self-organization of renewable energy deployment in this case. It can be observed in Fig. 2 that the renewable energy penetration rates oscillate below 50%. The average renewable energy penetration rate is about 42.2% during the last 50 rounds of the evolutionary game.

![Fig. 2. Comparison of renewable energy penetration rates during evolutionary game in different cases of high renewable cost scenario.](image)

Because the low-carbon target is set to be 50% renewable energy penetration rate in the system, incentives for more renewable energy deployment should be added. Case 2 of the high renewable cost scenario is considered below. The incentive is calculated using (34) as \( \lambda^{U} = 0.11 \) CNY/kWh. Then, the return matrix with incentives calculated using (16)-(20) is as follows:

\[
\begin{bmatrix}
-0.6076 & 1 \\
0.3923 & 0
\end{bmatrix}
\]

(37)

In case 2, the electricity price increases from 0.60 to 0.71 CNY/kWh. Buses with renewable energies can buy less electricity from the distribution network operator. So, with the increase in electricity price, buses with renewable energies benefit more in case 2 than those in case 1. As shown in (37), \( \bar{R}_{12} = 1 \) and \( \bar{R}_{21} = 0.3923 \), which are higher than those in case 1. This would increase the renewable energy penetration in the system. The renewable energy penetration rates during the evolutionary game in case 2 are shown by the solid line in Fig. 2. It can be observed in Fig. 2 that the renewable energy penetration rates in case 2 oscillate at about 50% and are generally higher than those in case 1. The average renewable energy penetration rate in case 2 is about 50.1% during the last 50 rounds of the evolutionary game, which achieves the low-carbon target in the 141-bus system. This result shows that when renewable energy penetration rate is lower than the low-carbon target, the increment of electricity price calculated by (34) can provide an incentive for buses to deploy renewable energies and meet the low-carbon target.

\[
\begin{align*}
& b_1 = \frac{1}{k} \sum_{k \in U} \left[ p_k^{DS} - p_k^{LS} \right]^+ \lambda_k^{SN} - \left[ p_k^{DN} - p_k^{LN} \right]^+ \lambda_k^{DS} - \lambda_k^{SN} \\
& a_2 = \frac{1}{k} \sum_{k \in U} \left[ \frac{1}{2} \rho \left[ \left( p_k^{DS} - p_k^{LS} \right)^+ \lambda_k^{SN} - \left( p_k^{DN} - p_k^{LN} \right)^+ \lambda_k^{DS} - \lambda_k^{SN} \right] - \frac{1}{2} \rho \left( p_k^{DS} - p_k^{LS} \right)^+ - p_k^{DS} \right] \\
& b_2 = \frac{1}{k} \sum_{k \in U} \left[ \frac{1}{2} \rho \left[ \left( p_k^{DS} - p_k^{LS} \right)^+ \lambda_k^{SN} - \left( p_k^{DN} - p_k^{LN} \right)^+ \lambda_k^{DS} - \lambda_k^{SN} \right] - \frac{1}{2} \rho \left( p_k^{DS} - p_k^{LS} \right)^+ - p_k^{DS} \right]
\end{align*}
\]

(31)

(32)

(33)

When the self-organization of renewable energy deployment results in a penetration rate lower or higher than the target, the distribution network operator could obtain the following incentives and add them to the transactions of electricity among buses, respectively.

\[
\lambda^{U} = g_1^{-1}(0) \tag{34}
\]

\[
\lambda^{DS} = g_2^{-1}(0) \tag{35}
\]

Hence, the distribution network operator could add incentives \( \lambda^{U} \) or \( \lambda^{DS} \) in two scenarios, respectively, for renewable energy deployment to achieve the low-carbon targets in the distribution networks \( \rho = \rho^T \).

IV. NUMERICAL RESULTS

The simulation program is developed in the environment of MATLAB 2021a. An actual 141-bus system [33] and a synthetic 2000-bus system [34] are simulated to validate the proposed model. The parameter of the sigmoid function is set to be \( k = 1 \) and the maximum number of game rounds is set to 200. The following two cases are considered.

1) Case 1: self-organization of renewable energy deployment, i.e., the evolutionary game of renewable energy deployment without incentives.

2) Case 2: renewable energy deployment towards a low-carbon target, i.e., the evolutionary game of renewable energy deployment with incentives.

Because renewable energy costs and low-carbon targets are important factors affecting the deployment of massive renewable energies, these two factors are studied in the case studies.

A. Actual 141-bus System

The 141-bus system is an actual distribution system in the metropolitan area of Caracas and details of the system are presented in [33]. In this subsection, the low-carbon target is set to be 50% renewable energy penetration rate in the system. The electricity price is set to be \( \lambda^{U} = 0.60 \) CNY/kWh.

1) High Value of Renewable Energy Costs

The parameters of renewable energies are set to be \( \lambda^{DS} = 0.55 \) CNY/kWh and \( \lambda^{SN} = \lambda^{SN} = 0.57 \) CNY/kWh. First, case 1 of the high renewable cost scenario is considered. The return matrix without incentives calculated using (1) is as follows:

\[
R = \begin{bmatrix}
-1 & 0.2813 \\
0.0674 & 0
\end{bmatrix}
\]

(36)
2) Low Value of Renewable Energy Costs

The parameters of renewable energies are set to be \( \lambda^{DS} = 0.3 \) CNY/kWh and \( \lambda^{NS} = 0.32 \) CNY/kWh. First, case 1 of the low renewable cost scenario is considered. The return matrices without and with incentives calculated using (1)-(5) are as follows:

\[
R = \begin{bmatrix}
0.5803 & 1 \\
0.4302 & 0 
\end{bmatrix} \quad (38)
\]

\[
\widetilde{R} = \begin{bmatrix}
-0.6702 & 1 \\
0.3749 & 0 
\end{bmatrix} \quad (39)
\]

In case 2, the renewable taxes are set to be 0.17 CNY/kWh, which means that the renewable cost increase by 0.17 CNY/kWh. With the increase in renewable energy costs, buses are less willing to deploy renewable energies in case 2 than those in case 1. The renewable energy penetration rates during the evolutionary game in case 2 of a low renewable cost scenario are shown by the solid line in Fig. 3. It can be observed from Fig. 3 that the renewable energy penetration rates in case 2 oscillate at about 50% and are generally lower than those in case 1. The average renewable energy penetration rate during the last 50 rounds of the evolutionary game is about 50.6%, which achieves the low-carbon target in the 141-bus system. This result shows that when renewable energy penetration may exceed the low-carbon target, adding renewable taxes calculated by (35) can hinder buses to deploy excessive renewable energies with the low-carbon target satisfied.

![Fig. 3. Comparison of renewable energy penetration rates during evolutionary game in different cases of low renewable cost scenario.](image)

The results show that whether the renewable energy costs are high or low, the proposed model can achieve the low-carbon targets in distribution networks by adding incentives to the renewable energy deployment of buses. Because the scale of the 141-bus system is relatively small, it is difficult to obtain a stable renewable energy deployment scheme using the proposed model. The actual distribution networks may contain thousands of buses and a large number of renewable energies, so a 2000-bus system is simulated below.

B. Synthetic 2000-bus System

The 2000-bus system is a synthetic system based on actual data and details of the system presented in [34]. The parameters are set to be \( \lambda^{DS} = 0.6 \) CNY/kWh, \( \lambda^{NS} = 0.45 \) CNY/kWh, and \( \lambda^{SS} = 0.55 \) CNY/kWh. The return matrix without incentives is calculated using (1)-(5) as follows:

\[
R = \begin{bmatrix}
-0.3239 & 1 \\
0.1204 & 0 
\end{bmatrix} \quad (40)
\]

The renewable energy penetration rates during evolutionary games in case 1 are shown by the dashed line in Fig. 4 and the average renewable energy penetration rate during the last 50 rounds of the game is about 54.5%. This means that self-organization of renewable energy deployment in the 2000-bus system results in about 54.5% renewable energy penetration rate.

![Fig. 4. Comparison of renewable energy penetration rates during evolutionary game in different cases.](image)

Next, case 2 with two different low-carbon targets is illustrated to elucidate that the different incentives can be calculated to achieve different targets by using the proposed model.

1) High Value of Low-carbon Target

The low-carbon target is set to be 60% renewable energy penetration rate in the system. Because the self-organization of renewable energy deployment results in a 54.5% renewable energy penetration rate, which is below the low-carbon target, the incentive is calculated by using (34) as \( \lambda^{DS} = 0.17 \) CNY/kWh. Then, the return matrix with incentives calculated using (16)-(20) is as follows:

\[
\widetilde{R} = \begin{bmatrix}
0.2957 & 1 \\
0.2866 & 0 
\end{bmatrix} \quad (41)
\]

The elements of return matrix with incentives are significantly larger than those without incentives. This shows that buses profit more from deploying renewable energies in this scenario, which incentivizes more renewable energy deployment. The renewable energy penetration rates in this scenario during the game are shown by the red line in Fig. 4. The average renewable energy penetration rate during the last 50 rounds of the evolutionary game is about 60.2%, which achieves the low-carbon target in this scenario.

2) Low Value of Low-carbon Target

The low-carbon target is set to be 45% renewable energy penetration rate in the system. Because the self-organization of renewable energy deployment results in a 54.5% renewable energy penetration rate, which is higher than the low-carbon target, the incentive is calculated using (35) as \( \lambda^{DS} = 0.07 \) CNY/kWh. Then, the return matrix with incentives calculated using (23)-(25) is as follows:

\[
\widetilde{R} = \begin{bmatrix}
-1 & 0.5373 \\
0.1032 & 0
\end{bmatrix} \quad (42)
\]

The elements of return matrix with incentives are lower
than those in return matrix without incentives. This shows that buses profit less from deploying renewable energies in this scenario, which can hinder excessive renewable energy deployment. The renewable energy penetration rates in this scenario during the game are shown by the blue line in Fig. 4. The average renewable energy penetration rate during the last 50 rounds of the evolutionary game is about 44.7%, which achieves the low-carbon target in this scenario.

The results of renewable energy deployment in different cases on the 2000-bus system are compared in Fig. 5, where 1 and 0 represent the renewable energy deployment and no renewable energy deployment on a bus, respectively. It shows that the renewable energy deployment increases from Fig. 5(a) to Fig. 5(c). These indicate that the distribution network operator could add different incentives to the renewable energy deployment of buses to achieve different low-carbon targets in the distribution network.

The average simulation time of the case study on the 2000-bus system is 7.2 s. Hence, the proposed model is efficiently-solvable for large-scale distribution networks.

C. Comparison of Different Evolutionary Strategies

We use strategies A and B to represent the adoption of (6)-(9) and (10)-(13), respectively. In Section IV-A and IV-B, strategy A is used for case studies, where a bus determines its decision based on the decisions of its neighboring buses. This means that the only information a bus can get is whether its neighboring buses plan to deploy renewable energies. To adopt strategy B, a bus needs to know the average profit of buses with or without renewable energies in the system for its decision. In Sections II and III, strategy A is approximately substituted by strategy B for analysis to get incentives and the effectiveness of such incentives towards low-carbon targets has been verified in Section IV-A and IV-B. In this subsection, we will directly compare the evolutionary strategies A and B and show the effectiveness of the approximation for analysis, especially in large-scale distribution networks. The low-carbon target in distribution networks is set to be 50% renewable energy penetration rate for all cases in this subsection.

Comparison of renewable energy penetration rates during the evolutionary game in 141-bus and 2000-bus systems using different strategies is shown in Fig. 6(a) and (b), respectively. The mean and variance values of renewable energy penetration rates in the last 50 rounds of evolutionary games are presented in Table I.

![Fig. 6. Comparison of renewable energy penetration rates during evolutionary game in 141-bus and 2000-bus systems using different strategies. (a) 141-bus system. (b) 2000-bus system.](image)

TABLE I

<table>
<thead>
<tr>
<th>System</th>
<th>Strategy</th>
<th>Penetration rate (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>141-bus system</td>
<td>Strategy A</td>
<td>50.5</td>
</tr>
<tr>
<td></td>
<td>Strategy B</td>
<td>50.4</td>
</tr>
<tr>
<td>2000-bus system</td>
<td>Strategy A</td>
<td>50.5</td>
</tr>
<tr>
<td></td>
<td>Strategy B</td>
<td>50.0</td>
</tr>
</tbody>
</table>

It can be observed from the comparison of results in the two systems that the amplitude of renewable energy penetration rate oscillations is significantly smaller in larger distribution networks. This indicates that the proposed model can obtain more stable and predictable renewable energy deployment schemes in large-scale distribution networks than those obtained in small-scale distribution networks.

The mean values of renewable energy penetration rates in the 141-bus system using strategies A and B are 50.5% and
50.4%, respectively. This shows that both strategies can let the renewable energy penetration rate close to the low-carbon target in the 141-bus system. The variance values of renewable energy penetration rates in the 141-bus system using strategies A and B are 0.0239 and 0.0022, respectively. In both two systems, the renewable energy penetration rates during 200 rounds of evolutionary games obtained by strategy A are more stable than those obtained by strategy B. This shows that if buses could get global information, the renewable energy deployment schemes would be more stable and predictable.

It is noted that the mean and variance values of renewable energy penetration rates in the 2000-bus system using strategy A are 50% and 0.02%, respectively. This shows that the low-carbon target in the 2000-bus system is accurately achieved, and the oscillation of renewable energy penetration rates is very small during the game. Such results indicate that the proposed model could almost get deterministic and predictable schemes of renewable energy deployment to achieve the low-carbon targets in large-scale distribution networks. The larger the distribution network is, the more effective the model is to reflect the self-organization of renewable energies and to incentivize the buses deploying renewable energies toward low-carbon distribution networks.

V. CONCLUSION

This paper proposes an evolutionary game of massive renewable energy deployment in large-scale distribution networks. The evolutionary strategies for buses are proposed to determine whether to deploy renewable energies in the next round of games. The dynamic model of renewable energy penetrations is derived based on the evolutionary strategy, which provides the basis for making policies towards low-carbon targets in distribution networks. Based on numerical results in an actual 141-bus system and a synthetic 2000-bus system, the following conclusions are obtained.

1) The proposed model provides a new perspective for renewable energy deployment, which can reflect both individual decisions in a self-organizing manner and the evolution characteristics of the whole system in the process of massive renewable energy deployment.

2) The distribution network operator can adjust the renewable energy penetration rates in the distribution network by adding incentives for the renewable energy deployment of buses. The proposed model provides the basis for determining incentives for renewable energy deployment toward low-carbon targets.

3) The proposed model is effective in large-scale distribution networks. The larger the system scale is, the more predictable renewable energy deployment schemes the proposed model can get.

4) Referring to the comparison of different strategies, if the buses could get global information, stable and predictable renewable deployment schemes can be obtained and low-carbon targets in distribution networks can be accurately achieved.

The model proposed in this paper is put forward under the assumption of simplification, so more practical conditions need to be considered in subsequent studies, including: (1) evolutionary games with multiple strategies in distribution networks need to be studied to allow buses to choose different renewable capacities; (2) the application of the evolutionary game in the heterogeneous network for renewable energy deployment needs to be studied to consider the difference of bus return matrices.

REFERENCES


Xuanyi Xiao received the B.Sc. and Ph.D. degrees in electrical engineering from Hunan University, Changsha, China, in 2013 and 2022, respectively. He was a Visiting Scholar with the Illinois Institute of Technology, Chicago, USA, from 2018 to 2019. He is currently a Postdoctoral Researcher with the College of Electrical Engineering, Zhejiang University, Hangzhou, China. His research interests include smart distribution systems and cyber-physical systems.

Jianbing Yin is currently with the State Grid Hangzhou Power Supply Company, Hangzhou, China. His research interests include power system planning and operation.

Lin Chen is currently with the State Grid Hangzhou Power Supply Company, Hangzhou, China. Her research interests include power system analyses and optimization.

Mingchang Wang is currently with the State Grid Hangzhou Power Supply Company, Hangzhou, China. His research interests include power system planning and economics.

Yi Zhao is currently with the School of Electric Power, Shenyang Institute of Engineering, Shenyang, China. He has extensive teaching and research experience in the areas of electrical engineering and automation. His research interests include power system planning, operations, and economics.

Zhiyi Li received the Ph.D. degree from the Department of Electrical and Computer Engineering, Illinois Institute of Technology, Chicago, USA, in 2017. From August 2017 to May 2019, he was a Senior Research Associate with the Robert W. Galvin Center for Electricity Innovation, Illinois Institute of Technology. Since June 2019, he has been a Tenure-Track Associate Professor with the College of Electrical Engineering, Zhejiang University, Hangzhou, China. His research interests include cyber-physical systems and power system optimization.